natural. It is not clear, however, what the standard says about the length of time for which the violation occurs. For the people in Houston, Texas it makes a big difference whether a single measurement exceeds the permitted threshold or whether the violation lasts for many hours or days. How does the Air Control Board count the violations if they are interrupted by a few hours: as one or more?

2. EXTREMAL INDEX

Smith chose a cluster-interval of 72 hours. Every such interval, with its 72 hourly measurements, is represented by the largest value—the peak. An index that measures the average length of a cluster of exceedances is the *extremal index* θ ($1/\theta$ being the mean cluster size). Smith introduced θ in the theoretical section but did not use it in the analysis of the ozone data.

It should be emphasized that the rate of exceedances, reported by Smith, is in fact the rate of 72-hour intervals with at least one exceedance. Two periods with the same rates could still have different θ values. In view of the fact that the data did not exhibit very conclusive improvement in time (i.e., increasing rate of exceedances over 8 and 12 parts per 100 million, decreasing rate of exceedances over 16 plus), the comparison of θ values could add another dimension for judgment whether or not the situation in Houston has improved or worsened.

Smith himself did use the extremal index ones. In Smith (1984) he studies wave heights in the English Channel. The extremal index θ is discussed and estimated together with the other parameters. In a recent paper of Leadbetter, Weissman, de Haan and Rootzén (1989), the extremal index of stationary dependent sequences is discussed. Asymptotically, under some regularity conditions, the value of θ is not influenced by the choice of the threshold level, the cluster-interval or cluster-definition. But for finite sample sizes, the estimation of θ is influenced by them.

3. ESTIMATION OF *N*-YEAR RETURN VALUES

Estimates of N-year return values are reported in Table 3. We observe very little variability due to the choice of threshold level and cluster interval—much less variability than exhibited by the estimation of the natural parameters. A similar phenomenon was observed in Smith and Weissman (1985) when extreme value methods were applied to Kimball's (1960) data. The conclusion was that "tail percentiles of a distribution can be estimated more accurately than the endpoint itself." Notice that here, too, under the present model, the upper endpoint $\mu + \sigma/k$ is finite and unknown.

4. POSSIBLE EXTENSIONS

It is not clear whether or not readings of other environmental variables are available for the period under study in Houston. If there are not any, stop reading here. If there are, obviously more information could have been extracted. These variables could either be used as explanatory variables for the ozone variable or their joint distribution with ozone could be analyzed by multivariate extreme value methods. Richard Smith has been the driving force in developing these methods and it would no doubt be illuminating for him to apply them here also.

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Comment

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Richard Smith's paper upholds the spirit of *Statistical Science*; it is a thorough exposition of current statistical methodology embedded in the description and analysis of an important scientific problem. The text of his paper consists of four themes, the main ones being an up-to-date summary of the use of extreme value theory and its ramifications for environmental data analysis, and a conclusion regarding the downward trend in extreme values of ozone concentrations in Houston. The other themes pertain to a description of the Houston ozone data and an ap-

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proach towards the analyses of these data. The four themes are tactfully interwoven with each other, enforcing a standard for statistical papers in the physical sciences; however, this is achieved at the cost of requiring of the reader several passes to gain appreciation of its content. Of particular emphasis is an advocacy of the point-process approach for studying environmental data.

Whereas I feel convinced of the merits of the point-process approach in working with such data, and find the authors decomposition of the entire series into M-day periods with the representation $\mu_{ij} = \alpha_j + i\beta_j$ reasonable, the material in Section 5 leaves me with a sense of uneasiness. This does not imply a nonconcurrence with the author's conclusion regarding the downward trend in extreme values; Table 5 supplies sufficient evidence for this. Notwithstanding the usual Bayesian's objections to an exclusive reliance on the likelihood function and its associated paraphernalia for guiding the nature of the data analysis, the approach to declustering is of concern. The author is cognizant of this latter issue and has attempted to soothe criticism by way of Table 3, but a matter that may be germane is the appropriateness of the asymptotic theory of equation (3.6) for different choices of the cluster interval. The Bayesian position for concluding the absence of a long term trend would be to incorporate prior information about the β_j's based on a knowledge of the efforts of the regulatory boards and use the ensuing posteriors to infer trend. In this connection, the author's model, as exemplified by equations (4.1) and (4.2), is ideal for a Bayesian development; (4.1) is reminiscent of Lindley and Smith's (1972) set up for linear models. The above matter is germane because a downward trend in the crossing rates of high levels, especially levels much higher than the official 12 parts per hundred million, may not be of much solace to the regulatory agencies. Incidentally, would an illustration analogous to that of Figure 6, but with a threshold of 16 and above, indicate a downward trend in the mean excess level? Also, the ordinate of Figure 6 suggests three additions to an otherwise exhaustive list of references; these are Barlow (1972), Barlow and Singpurwalla (1974) and Mittal (1978). Finally, is there a connection between the Poisson point-process model of equation (3.6) and the gamma process model of Ferguson and Klass (1972)? If not, would the latter be a viable alternative to the former?

Whereas an analysis of the type undertaken by Richard Smith and his references can be classified as being "passive," in the sense that the role played by the statistician is one of *measurement* and *monitoring*, the models described by Smith can be put to more "active" use in the context of *decision making* and *control*. It appears that the potential for such a role has not been fully recognized in many areas of the biological, engineering and physical sciences, including those pertaining to the environmental sciences. Thus, for example, the setting of air quality standards, such as the U.S. Environmental Protection Agency's no more than three exceedances of 12 parts per 100 million in any 3-year period, should be based on decision theoretic considerations which would balance the risks of health hazards versus the economic hardships caused by stringent air pollution requirements. The point-process model described in this paper would play an essential role in undertaking the decision theoretic considerations. In general, the possibility of harnessing the very attractive results of extreme value theory in the broader context of decision making under uncertainty, in engineering design and environmental control, remains to be explored. Furthermore, if the task of regulatory bodies is to introduce measures to reduce the frequency and level of high exceedances, then a methodology that could be of natural value to them would be that of control theory. Control theory has a statistical foundation, in the sense that corrective action is taken prior to the occurrence of an undesirable event, whose forecast is based on an extrapolation of a time series. Here again, Richard Smith's model, as exemplified by his equations (4.1) and (4.2), would provide the necessary ingredient for a more general development of the kind say in Smith and Miller (1986), but with a control component. Thus is appears to me that the decomposition (4.1) may have broader implications than its original intent of simplicity.

As a concluding remark, if someone like myself who has an interest in extreme value theory, but who does not claim any specialization in it, were to ask if this paper has anything enlightening to say, then my answer would be a most emphatic yes. Of course, being an admirer of Richard Smith's work, I have come to expect interesting contributions from him, of which this paper is another example.

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