It is straightforward to show that

$$(1) \qquad \hat{\sigma}_j = \bar{y}_{+j+}, \quad \hat{\alpha}_j = \bar{y}_{+j+}\log[N_{+j}/p_{+j}].$$

This gives some idea about the magnitudes of σ and α and provides an alternative initial estimate for the likelihood; the method mentioned by Smith based on the Gumbel distribution also comes from taking $k_j = 0$. From (1), $\hat{\alpha}$ decreases as p_{+j} increases. This may happen even if k_j is not fixed at zero, so that one must be careful in how to deal with missing values at hours when the ozone levels are typically low.

Finally I have some comments referring to Sections 4 and 5. The mean exceedance rates are based on a unit of a cluster and so one must take into account the average number of days in a cluster in order to compare with the quantities specified by air quality standards. Chock (1982) raises the question of whether it is reasonable to count an adverse multiday meteorological event several times as having exceeded a threshold.

Smith points out that the analysis needs to be repeated at other sites to get a firm indication of a downward trend in crossing rates at high levels. Walker (1985) reports on ozone trends in California and Texas over a period of 10 years and concludes that there is little evidence that annual average ozone or average peak ozone has been reduced. Walker's analysis is not an extreme value analysis, but he does mention two confounding factors for the ozone trend that are relevant here. These are trends in analytical methodology (for measurements) and data quality assurance. The EPA made ultraviolet photometry the basic calibration procedure for all official ambient ozone monitors in 1979, and data prior to this year are generally adjusted (calibrated) in order to study trends from 1973 on. Was this true of the ozone data in this analysis? Concerning the data quality trend, Walker states that more recently many high values are invalidated as outliers where earlier they were accepted. The methodology in this paper has wide applicability but one must be careful with potential confounding factors in making conclusions.

In conclusion, Professor Smith is to be commended for an excellent paper that develops statistical methodology for an important application and mentions important areas of developing research.

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Comment

Ishay Weissman

I enjoyed Richard Smith's study of ground-level ozone data using extreme value theory. Smith should be commended for undertaking this project, and congratulated for his lucid analysis and exposition.

After describing the data, Smith gives some theoretical background, just enough for the reader who is not an expert in extreme value theory to understand the analysis that follows. The paper as a whole was written in a free-flowing format that makes it interesting and enjoyable to read. The author applied simple descriptive methods (tables, histograms, boxplots, etc.) as well as sophisticated ones (generalized extreme value models, generalized Pareto models with and without trend, etc.). The latter have been developed to a large extent by Smith himself in earlier works.

Due to time pressure, I will only make a few short comments.

1. EXTREME VALUE ASPECTS

I totally agree with Smith's decision to concentrate on high exceedances. Ozone as well as other pollutants become serious health-hazards when they exceed certain levels (thresholds). The current ozone standard, as Smith puts it, permits no more than three exceedances above 12 parts per 100 million in any 3-year period. Hence, looking at high exceedances is only

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natural. It is not clear, however, what the standard says about the length of time for which the violation occurs. For the people in Houston, Texas it makes a big difference whether a single measurement exceeds the permitted threshold or whether the violation lasts for many hours or days. How does the Air Control Board count the violations if they are interrupted by a few hours: as one or more?

2. EXTREMAL INDEX

Smith chose a cluster-interval of 72 hours. Every such interval, with its 72 hourly measurements, is represented by the largest value—the peak. An index that measures the average length of a cluster of exceedances is the *extremal index* θ ($1/\theta$ being the mean cluster size). Smith introduced θ in the theoretical section but did not use it in the analysis of the ozone data.

It should be emphasized that the rate of exceedances, reported by Smith, is in fact the rate of 72-hour intervals with at least one exceedance. Two periods with the same rates could still have different θ values. In view of the fact that the data did not exhibit very conclusive improvement in time (i.e., increasing rate of exceedances over 8 and 12 parts per 100 million, decreasing rate of exceedances over 16 plus), the comparison of θ values could add another dimension for judgment whether or not the situation in Houston has improved or worsened.

Smith himself did use the extremal index ones. In Smith (1984) he studies wave heights in the English Channel. The extremal index θ is discussed and estimated together with the other parameters. In a recent paper of Leadbetter, Weissman, de Haan and Rootzén (1989), the extremal index of stationary dependent sequences is discussed. Asymptotically, under some regularity conditions, the value of θ is not influenced by the choice of the threshold level, the cluster-interval or cluster-definition. But for finite sample sizes, the estimation of θ is influenced by them.

3. ESTIMATION OF *N*-YEAR RETURN VALUES

Estimates of N-year return values are reported in Table 3. We observe very little variability due to the choice of threshold level and cluster interval—much less variability than exhibited by the estimation of the natural parameters. A similar phenomenon was observed in Smith and Weissman (1985) when extreme value methods were applied to Kimball's (1960) data. The conclusion was that "tail percentiles of a distribution can be estimated more accurately than the endpoint itself." Notice that here, too, under the present model, the upper endpoint $\mu + \sigma/k$ is finite and unknown.

4. POSSIBLE EXTENSIONS

It is not clear whether or not readings of other environmental variables are available for the period under study in Houston. If there are not any, stop reading here. If there are, obviously more information could have been extracted. These variables could either be used as explanatory variables for the ozone variable or their joint distribution with ozone could be analyzed by multivariate extreme value methods. Richard Smith has been the driving force in developing these methods and it would no doubt be illuminating for him to apply them here also.

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Comment

Nozer D. Singpurwalla

Richard Smith's paper upholds the spirit of *Statistical Science*; it is a thorough exposition of current statistical methodology embedded in the description and analysis of an important scientific problem. The text of his paper consists of four themes, the main ones being an up-to-date summary of the use of extreme value theory and its ramifications for environmental data analysis, and a conclusion regarding the downward trend in extreme values of ozone concentrations in Houston. The other themes pertain to a description of the Houston ozone data and an ap-

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