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# Comment: Representing and Communicating Uncertainty

**Robert L. Winkler**

## 1. INTRODUCTION

We live in a world fraught with uncertainties, and these uncertainties are often communicated via qualitative expressions. Since such expressions are lacking in precision, it is helpful to know what different people might mean when they use specific expressions. Therefore, recent studies of quantitative interpretations of linguistic expressions of uncertainty are valuable to anyone who hears or uses such expressions. The paper by Mosteller and Youtz and its predecessors can help us understand how people represent uncertainty and how the process of representing and communicating uncertainty might be improved.

In these comments I focus on some issues that I view as important in the representation and communication of uncertainty. I discuss sources of variability in interpretations of qualitative expressions of uncertainty in Section 2, with emphasis on differences

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among individuals. Some advantages of qualitative expressions of uncertainty are contrasted with advantages of numerical expressions in Section 3. A Bayesian view of an individual's reaction to expressions of uncertainty is described briefly in Section 4, and a few thoughts on codifications of linguistic expressions of uncertainty are offered in Section 5.

## 2. VARIABILITY IN INTERPRETATIONS OF QUALITATIVE EXPRESSIONS OF UNCERTAINTY

Much of the analysis and discussion in the paper by Mosteller and Youtz focuses on *averages* of quantitative interpretations of the qualitative expressions of uncertainty that are considered. One message that is conveyed is that the averages from different studies for a given expression tend to be similar. For example, the authors say that "the variation of the averages for most of the expressions was modest" and "our emphasis is more on the near constancy of opinions as illustrated in Table 1" (Table 1 presents averages from different studies). Some evidence on variability in interpretations among the science writers is presented in the form of quartiles and interquartile ranges, but the impression that is left with the reader is that there is a great deal of agreement on the probability values associated with various qualitative expressions.

Other writers have taken a different tack, stressing the differences among individuals in their interpretations. Considering the communication of uncertainty in weather forecasts, Murphy (1985, page 367) writes as follows:

> "Traditionally, uncertainty in weather forecasts has been expressed in terms of verbal modifiers such as 'chance' or 'likely.' Are these verbal expressions of uncertainty interpreted by the general public in an appropriate and consistent manner? This question has been addressed in several studies related to terminology in weather forecasts (e.g., Bickert, 1967; Abrams, 1971; Rogell, 1972; McBoyle, 1974) and in laboratory experiments conducted by behavioral psychologists (e.g., Lichtenstein and Newman, 1967; Budescu and Wallsten, 1981; Beyth-Marom, 1982). These investigators all reached the same general conclusion—a large amount of variability exists when individuals are asked to assign numerical values to such verbal expressions and the amount of overlap among terms is substantial."

TABLE 1

*Frequency of response in different intervals when students in an MBA class in decision analysis were asked to interpret qualitative expressions of uncertainty in terms of numerical probabilities (in percentages)*

| | 0–4 | 5–9 | 10–14 | 15–19 | 20–24 | 25–29 | 30–34 | 35–39 | 40–44 | 45–49 | 50–54 | 55–59 | 60–64 | 65–69 | 70–74 | 75–79 | 80–84 | 85–89 | 90–94 | 95–99 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Much better than even chance | 1 | | | | | | | | | | 1 | | | 1 | 5 | 13 | 1 | | 1 | | |
| Possibility | | 1 | 5 | 2 | 2 | 5 | 2 | | 1 | | 2 | 1 | | | | 1 | | | | | 1 |
| High likelihood | 1 | | | | | | | | 1 | | 1 | | 1 | 1 | 2 | 2 | 7 | 3 | 4 | | |
| Probability relatively constrained | | | 2 | 3 | 4 | 2 | 5 | 2 | 3 | | | | | | | | | | | | 1 |
| Probability very high | 1 | | | | | | | | | | | | | | 2 | | 3 | 8 | 7 | 2 | |
| Very unlikely | 3 | 7 | 8 | 1 | 3 | | 1 | | | | | | | | | | | | | | |
| Slight chance | 2 | 7 | 8 | 1 | 3 | | | 1 | 1 | | | | | | | | | | | | |
| Possibility low | 1 | 2 | 4 | 4 | 6 | 2 | 1 | 1 | 1 | 1 | | | | | | | | | | | |
| Chances better than even | 1 | | | | | | | | | | 1 | 4 | 14 | 1 | 2 | | | | | | |
| No serious probability | 14 | 5 | 2 | 2 | | | | | | | | | | | | | | | | | |
| Little chance | 3 | 6 | 5 | 2 | 3 | 3 | | 1 | | | | | | | | | | | | | |
| Probable | | | | | | | | | 1 | | | | 6 | 2 | | 7 | 4 | 2 | | | 1 |
| Unlikely | 2 | 3 | | 3 | 6 | 5 | 3 | 1 | | | | | | | | | | | | | |
| Good chance | | | | | | | 1 | | 2 | | | | 4 | 1 | 7 | 2 | 5 | | | | 1 |
| Quite unlikely | 2 | 7 | 6 | 1 | 3 | | 3 | 1 | | | | | | | | | | | | | |
| Improbable | 7 | 4 | 6 | 4 | 2 | | | | | | | | | | | | | | | | |
| High probability | 1 | | | | | | 1 | | | | | | | | 2 | 1 | 4 | 4 | 10 | | |
| Chance | 1 | | 4 | | 3 | 5 | 1 | | 1 | | 4 | 1 | | 1 | | 1 | | | | | 1 |
| Very improbable | 7 | 9 | 3 | 2 | 1 | | | | | | | | | | | | | 1 | | | |
| Likely | | | | | | | | 1 | | | | 1 | 4 | 1 | 5 | 5 | 3 | 1 | 1 | | 1 |
| Probably | | | | | | | | | | | | | 4 | | 1 | 8 | 4 | 1 | 3 | 1 | 1 |

To emphasize the distinction between qualitative and quantitative expressions of uncertainty, I regularly have students provide the probabilities they associate with a series of qualitative expressions. Some results from a recent class are given in Table 1, and these results are typical for MBA students who have elected to take my course in decision analysis. The data are uncensored and, as Mosteller and Youtz note, respondents occasionally get turned around, or perhaps they don't all take the task seriously. For example, one response to "much better than even chance" was a probability in the 0–4 range (in terms of percentages). Even if such seemingly absurd responses are ignored, there is quite a bit of variability in the interpretations of most expressions. Interpretations of "the event will probably occur" ranged from 60 to 100, and expressions such as "there is a possibility that the event will occur" and "there is a chance that the event will occur" spanned virtually the entire range of values. Students are invariably surprised to see the wide range of responses, and those skeptical about probabilities become somewhat less so as they see the potential for miscommunication via qualitative expressions. As I point out in class, the variability in interpretations is enough to make an important difference in many inferential and decision-making problems.

In general, then, similarity in average interpretations provides no guarantee that qualitative expressions of uncertainty will be interpreted as intended by those providing them. There are many sources of variability in interpretations, and the between-group variability seems to be the least important. Even if groups agree on average, the individuals within a group may disagree considerably. Furthermore, in addition to variability between individuals, an individual's own uncertainty or ambiguity could add another layer of variability. Adding factors such as context effects (the questionnaire given to my students was context-free) could complicate matters further.

## 3. QUALITATIVE EXPRESSIONS VERSUS PROBABILITY ASSESSMENTS

There are many considerations leading to the use of qualitative expressions of uncertainty. Some people are naturally distrustful of numbers and simply prefer qualitative terms. That's not surprising, since it is consistent with our educational system. In school, uncertainty is often suppressed as students face an emphasis on learning facts. Probabilities are seldom encountered, and any treatment of uncertainty is generally informal. Also, people may feel vague about probabilities of many real-world events. Such vagueness may lead to the use of qualitative expressions instead of numbers, which may seem too precise. This works both ways, of course; the use of qualitative expressions instead of probabilities might be viewed as a contributing factor to ambiguity. Because people are not trained to think in terms of probabilities, and because they may not encounter probability forecasts too often in everyday life, they simply may feel uncomfortable with probabilities.

Foundational issues involving the interpretation of probability may also be relevant. Mosteller and Youtz state that "People often say that they would prefer to use the *actual numbers* if they were available rather than use qualitative expressions" (emphasis mine). But what are these "actual numbers"? This claim suggests that there are "true" probabilities for events. But the concern here is with the form of expression of uncertainty by *individuals,* and the appropriate view of probability is subjective, with an individual's probability for an event representing his or her degree of belief concerning the occurrence of that event. To the extent that training or experience leads people to think of probabilities in terms of scientific, objective values, it is only natural that they will be inhibited in expressing their subjective uncertainty numerically. This is an unfortunate state of affairs, and statisticians, unfortunately, are not without blame.

Some public support for probabilities can be found, as the following quote from Murphy (1985, page 368) indicates.

> "The reactions of members of the general public to the precipitation probability forecasting program and their preferences regarding the mode of expression of uncertainty in forecasts are also of interest here. The PoP program initially encountered some resistance from both the forecasters and the public, but it is now generally agreed that these probabilities are an important and integral part of public weather forecasts in the U.S. (e.g., Bickert, 1967; American Telephone and Telegraph Company, 1971). Moreover, a recent nationwide statistical survey of 1300 members of the general public reveals that 70 percent of the participants prefer numerical probabilities to verbal modifiers as descriptors of uncertainty in forecasts of precipitation occurrence (M.S.I. Services, Inc., 1981). In addition, requests by the public in Canada for precipitation probability forecasts played a major role in the recent decision by the Atmospheric Environment Service to initiate a nationwide PoP program in July 1982 (Grimes, 1982)."

Perhaps familiarity with probabilities of precipitation on a day-to-day basis through the media makes people more comfortable with probabilities in this particular context. If the public were exposed to prob-

abilities on a regular basis in a wide variety of contexts, reluctance to use probabilities to express uncertainty might be lessened.

An important issue in the choice of whether to use qualitative or quantitative expressions of uncertainty involves effective communication. Wallsten (1989, page 2) states, "Perhaps the most important potential cost of communicating uncertainty linguistically is that of misunderstanding between forecaster and decision maker." Mosteller and Youtz also note the risk of misunderstanding with qualitative expressions of uncertainty. A numerical value is more precise and less likely to be misunderstood. Of course, as mentioned above, this very precision may make people uncomfortable if they feel somewhat vague about their probabilities. A reasonable alternative is to report not just a single number, but a range of probability values, thereby communicating some vagueness or uncertainty without the miscommunication that can occur when qualitative expressions of uncertainty are used.

The question of whether to use probabilities or linguistic representations of uncertainty can be viewed from both descriptive and normative perspectives. Looking at the issue descriptively, I suspect that qualitative terms generally get the nod. If left to their own devices, most people tend to use qualitative terms, being used to dealing with such expressions and finding them easier to use than probabilities. From a normative standpoint, however, I think there are clear advantages to probabilities. They offer more effective, ungarbled communication; they should be more informative to a decision maker; and they would seem to make the person issuing the expression of uncertainty a bit more accountable.

The primary interest in effective and informative communication involves important problems; the normative advantages ascribed above to probabilities are of less concern in minor problems. Consider the following statements:

"Graf is likely to beat Navratilova tomorrow."

"It is likely to rain today."

"The return on this project is likely to be above 30 percent."

"Exposure to ozone is likely to cause lung damage."

Given a choice, I will always prefer to hear probabilities. Unless I had a large bet riding on the outcome of the tennis match (a very *un*likely event), however, I would be satisfied with the qualitative expression in the first statement. And even though I am used to hearing probabilities of rain, the second statement would be acceptable unless my choice of activities for the day were sensitive to moderate variations in the probability of rain. (When weather forecasts are issued to the general public, some users will generally be sensitive and will find probabilities more useful.) In the third statement, if I were contemplating an investment in the project I would surely ask, "*How* likely?" With the last statement, I would once again want to know how likely the event is, and I would also want a more precise definition of "lung damage." Before we can think seriously about expressing uncertainty concerning an event in terms of probabilities, we need to define that event carefully. Vagueness about the exact nature of an event may leave us with no choice other than to use a somewhat vague qualitative expression of uncertainty (see Wallsten, 1990).

## 4. A BAYESIAN VIEW

However a person's uncertainty about an event is expressed, the statement provided by the person represents information to a recipient. The recipient then combines this information with any previously available information and updates his or her opinions about the event. If little prior information is available, the statement may be given a great deal of weight, whereas if other evidence has been received, the statement may play a minor role in the recipient's revised opinion. This seems to be a reasonable description of how expressions of uncertainty are received and used.

The revision process can be formalized in terms of a Bayesian model. For someone who has received statement $S$ about event $E$, for example, the relevant probability is

$$P(E \mid S) = \frac{P(E)P(S \mid E)}{P(E)P(S \mid E) + P(\bar{E})P(S \mid \bar{E})},$$

where $\bar{E}$ represents the complement of $E$ and P($E$) and $P(\bar{E})$ are the prior probabilities. Alternatively, if we think about $p = P(E)$ with a prior distribution $f(p)$ (recognizing, apropos of the discussion in Section 3 concerning the interpretation of probability, that $p$ is not a "true," objective probability), then the revision process yields a revised distribution for $p$,

$$f(p \mid S) = \frac{f(p)l(S \mid p)}{\int_0^1 f(p)l(S \mid p)\, dp},$$

where $l(S \mid p)$ is the likelihood function.

In either representation of the problem, $S$ can be a qualitative expression or a probability. Thinking in terms of the latter representation, I speculate that the likelihood function $l(S \mid p)$ would typically be "tighter" when $S$ is a probability than when $S$ is a linguistic expression. This expresses more formally the notion that quantitative expressions of uncertainty tend to provide more effective and informative communication.

Analyses of quantitative interpretations of qualitative terms often work with frequency distributions of

probabilities $P(E \mid S)$ or of means $E(p \mid S)$ across different individuals, sometimes considering the entire distributions and sometimes concentrating on selected summary measures. Frequency distributions of probabilities and the acceptability functions of Mosteller and Youtz (which are somewhat in the spirit of approval voting, as discussed in Brams and Fishburn, 1983) could be helpful in assessing likelihood functions. As for the prior distribution, $f(p)$ might be assumed to be diffuse in context-free situations. In specific contexts, however, both the prior distribution and the likelihood could be context-dependent. When specific contexts are of interest, it is difficult to disentangle the influences of the prior probabilities and the likelihoods, and the assessment of likelihoods in practice can be a difficult task. If, as is quite common, we obtain probabilities or linguistic statements from two or more individuals, potential dependence among the individuals complicates matters further (e.g., see Genest and Zidek, 1986; Clemen, 1989). For the purposes of the discussion here, however, the Bayesian model is presented simply to illustrate a way of thinking about the reaction to expressions of uncertainty, and questions of implementation are beyond the scope of these comments.

## 5. ON CODIFICATIONS

Mosteller and Youtz mention the development of codifications of probability expressions, but the purpose of these planned codifications is not clear to me. Even if the person whose judgments are of interest understands and uses a codification, those hearing the resulting qualitative expression of uncertainty would interpret it in their own manner anyway. It seems that the variability from person to person in interpretations of qualitative expressions means that codifications will not improve the communication problem. If all of the recipients are trained in the appropriate interpretations according to the codification, perhaps miscommunication can be avoided. But then both the issuer and user of the expression of uncertainty know and think about the codification, hence about probabilities. This means that they are going to all the trouble of assessing and thinking about probabilities without the advantages of using them, and they need to learn prescribed interpretations of various words when an unambiguous language already exists in the form of probabilities.

Linguistic expressions of uncertainty are and will continue to be encountered widely, and analyses of how people interpret such expressions are valuable. What I question is the benefit of encouraging further use of qualitative expressions as opposed to probabilities. My preference would be to see greater training in and use of probabilities in place of the more vague qualitative expressions of uncertainty.

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