like most of our other concepts, a radial one, not characterized by necessary and sufficient conditions. I would not describe Shafer as re-unifying probability. I would say he is just reminding us what it is and has been since around 1660.

8. THE INSTITUTIONAL REUNIFICATION OF PROBABILITY

The diversity of statistics is one of its strengths. Any attempt to restore an hegemenous department of statistics could only harm the subject. Yes, let statisticians (those who identify themselves as such) again be more open, more willing to learn from other departments, more willing to think hard about the problems, both practical and conceptual, that arise whenever we try to reason with precision short of deduction, or to assess plans for deciding under uncertainty. If a department of statistics, frightened by the proliferation of its expertise, turns inward and dedicates itself to pure mathematics, it will lose its reason for existence. But statistics departments should not try to reclaim old territory. Let statistical thinking be done in many houses. Why should Shafer be so keen to "co-opt" people from other disciplines? Won't "co-operation" do? Why should there be one department that provides all the basic teaching in statistics? Contrary to the belief of Shafer and David S. Moore, statistics is not one of the liberal arts. It is part of logic, and logic, I remind you, is one third of the trivium of logic, grammar and rhetoric. I quite disagree with my own colleagues who want all students to take a basic course in logic and critical thinking in our philosophy department. I urge for others what I urge at home. Don't try to claim everything for yourself. I teach an elementary course on inductive logic and probability, which is much enriched by the fact that some of the students have picked up a little statistics in pharmacy, in physics, in archaeology, in computer science. The friction is great. Had they all learned their little statistics in the same department, from the same teachers, I would probably quit teaching the course; I don't want to teach serried ranks of bland and uniform young people.

There is all too much "reclaiming" in Shafer's vision of his subject. Most departments of statistics at research universities grant the Ph.D. Would Shafer want us philosophers to reclaim "our" degree? Shafer is something of a philosopher (rather more than something, in fact). I am delighted that such a philosopher is located in a School of Business. I do not want to co-opt him but to learn from him—as I have always done.

Comment

David S. Moore

Glenn Shafer alleges that our discipline is in some disarray, not only institutionally but intellectually. He traces this disarray to the "balkanization" of probability and urges as a solution a conceptual reunification of interpretations of probability. In presenting his case, he offers a most interesting glimpse at the recent surge of work on the history of probability and statistics. How shall we react to Shafer's diagnosis and to his proposed therapy? Subjectively, of course. For my part, I commend him for calling our attention to our history, accept with some hesitations his allegation of institutional disarray and remain unconvinced that whatever intellectual disarray (I would call it ferment) we face is a disease needing the treatment he proposes.

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OUR INSTITUTIONAL VITALITY

Shafer notes the extensive growth of both teaching and research about probability and statistics in other disciplines and the considerable contributions made by scholars in these fields. All true and all to the good. No fundamental intellectual method can be confined within a neat institutional framework.

The case of mathematics is instructive. Research that only the narrow-minded would distinguish from research in mathematics has long been carried out by scholars in many fields. A recent sample survey finds that over half of all students studying advanced mathematics are enrolled in courses taught outside of mathematics departments (Garfunkel and Young, 1990). Mathematics is simply too important to be left to mathematicians. Mathematics has undergone the fragmentation that Shafer laments institutionally as well as in research and teaching. This ought not to surprise us. The differentiation of once unified functions among diverse institutions is an essential

component of modernity, in the intellectual no less than in the social sphere. What was once natural philosophy is now 20 natural sciences. In the past half-century not only statistics but operations research and computer science have separated, both intellectually and institutionally, from mathematics. It is quite true that some of the institutional separation was due to the intolerance of mathematicians, but the intellectual differentiation is genuine, the ties between those disciplines remain close, and science as a whole is the stronger for the emergence of the new specialties.

Statistics, like mathematics, is a fundamental intellectual method too important to be confined by organizational boundaries. Like mathematics, statistics is subject to the forces of differentiation. There is a certain inevitability to the fragmentation of teaching and research. Yet Shafer is right in expressing concern over the future of statistics departments in universities.

Statistics is not mathematics, and statisticians cannot afford to share the isolated grandeur that many mathematicians continue to see as their ideal state. A leading mathematician, responding to the findings of Garfunkel and Young, is quoted (Turner, 1990) as saying, "Mathematicians want to train people to be good at math. It isn't their job to teach them to be good at using it." When I lie awake at night pondering life's mysteries, I ask what it might mean to be good at mathematics without being good at using it. But I am certain that it is meaningless to claim to be good at statistics without being good at using it. Because of its nature as the science of data and uncertainty, statistics requires for its vitality close and continuing ties with other disciplines. An overemphasis on statistics-as-mathematics is (despite the great importance of mathematical tools in statistics) the chief barrier to these ties. As I have had a chance to wax rhetorical on the issue (Moore, 1988), I will simply record general agreement with Shafer's suggestions.

I am, however, more optimistic than Shafer about current trends. Statistics as a field and statistics departments as organizations have responded to changing circumstances, especially to new computing technology, in encouraging ways. More fundamentally, our institutional health will in the long run be determined most of all by our intellectual health.

OUR INTELLECTUAL VITALITY

Glenn Shafer paints "a picture of intellectual vitality." He then worries about the effects, not simply institutional but also intellectual, of the resulting rampant growth. Observers differ in their perceptions of much simpler scenes, so I will simply record my quite different perceptions of this complex scene. Shafer seems to see the beginnings of a narrow-minded senility; I see a basically strong maturity, in need of self-criticism but not about to ossify. I do not, for example, consider the continuing discussion of the reasoning of inference to be "sterile." Both Bayesians and non-Bayesians have advanced interesting ideas, such as conditional confidence procedures (Brownie and Kiefer, 1977; Kiefer, 1977) and Bayesian methods that are insensitive to the choice of prior (Berger, 1984, 1985, 1990). I know no "frequentist" statistician who sees "Bayesians in other disciplines as cranks" simply because they are Bayesians or in other disciplines. Most non-Bayesians are ecumenists who support the use of Bayesian methods when the informed judgment of a decision-maker is relevant, as is surely often the case in business and engineering.

Replacing one political metaphor by another, I suggest that the science of data and uncertainty is not a Balkan congeries of warring ministates but a federal assembly whose union rests on both common concerns and a body of commonly accepted mathematics. Discord and cooperation are both present, while the union as a whole is remarkably productive. The mathematics of probability theory offers to this federal union a common language, so that we can dispute about ideas and not about grammar. Probability theory provides a quantitative means of reasoning about uncertainty, not the only such means but surely the most successful to date. The mathematical structure allows the derivation of subtle and beautiful conclusions from relatively modest assumptions. I was impressed on first acquaintance, and remain impressed now, that so profound a fact as the law of the iterated logarithm could follow from a simple description of independent coin tosses.

Not the least of the attractions of mathematics is that it allows us to temporarily ignore the question of interpretation. This is also, as Shafer notes, a seductive weakness. In his concern over diversity of interpretation, Shafer may understate the value of the common grammar. But he is right to remind us that there is more to the advance of probabilistic thinking than mathematics.

When Shafer urges us to lift our heads from parsing the Book of Kolmogorov, he does so most attractively by calling attention to the historians' exploration of the origins and development of concepts (deliberately plural) of probability. I second his implicit suggestion that teachers and researchers make themselves familiar with the struggles of better minds than ours to understand and quantify uncertainty. An excellent starting point is *The Empire of Chance* (Gigerenzer, Swijtink, Porter, Daston, Beatty and Krüger, 1989), in which six randomly ordered authors give a narrative

overview more accessible than the specialized histories. The authors include Lorraine Daston and Theodore Porter, whose work Shafer cites in his brief survey.

The authors of *The Empire of Chance* (pages xiv–xv) consider their book "a study in the interactive effects of quantification." On the one hand, probabilistic methods entering a new field brought with them analogies and ideas from other sciences where probability had earlier appeared. "These encounters of probability and statistics with science have in no case been neutral—mere translations of extant ideas and methods into the language of mathematical probability." On the other hand, "... probability theory was as much modified by its conquests as the disciplines it invaded." The historians do offer support for Shafer's emphasis on the scientific importance of conceptual interpretation of the mathematics.

Yet I remain unconvinced by the last step in Shafer's argument, that a conceptual reunification of probability is needed. The present variety of interpretations seems appropriate to the variety of situations to which probabilistic thinking is applied. As in the earlier history, interpretations from one situation may prove to advance (or temporarily divert) progress in another. Statistics departments may or may not serve as foci for communications among varied scientific cultures, but the conversations will in any event take place. Shafer in fact appears to advocate not a conceptual reunification but a reformulation of the mathematics that makes varied interpretations more natural. His present essay gives little detail about his proposed program, so my lack of conviction may stem from mere ignorance. It is not clear to me that the present common mathematical framework has impeded either growth or communication and so is in need of reformulation. Since "warranted belief" and "sequences of experiments" are not common to the many interpretations of probability, I am content with a mathematical structure that does not make them fundamental. A formal structure that goes beyond providing a basis for mathematical deductions might well be counterproductive. No axiomatic statement of what counts as warranted belief, please.

ANOTHER LESSON FROM HISTORY

I want to urge that in the midst of our reflections on the importance of concepts as well as technique in research and applications of statistics and probability we reflect also on their importance in teaching. One of the clearest impressions left by a reading of the historians' accounts is that *probability is a very hard idea.* If original minds from Pascal to Galton seem a bit confused, some patience with our students' confusion is in order. If the formal mathematical structure must in practice be supplemented by conceptual interpretation, then teaching that concentrates on formal proofs is inadequate to convey understanding.

The difficulty of probability concepts is of course attested to from sources other than history. Psychologists (e.g., Tversky and Kahnemann, 1983) have documented the surprising extent to which intuitive thinking about uncertainty conflicts with the rules of probability theory. The common underestimation of the probability of runs in purely random sequences and the common surprise at the occurrence of one of very many possible "coincidences" are examples of our faulty intuition about chance. Education researchers have looked intensely at students' understanding of probability and at attempts to improve their learning. They find that our students, even those who can successfully solve most textbook problems, have a very limited understanding of probability and statistics. Garfield and Ahlgren (1988), after a useful survey of this research, conclude that "teaching a conceptual grasp of probability still appears to be a very difficult task, fraught with ambiguity and illusion." I believe that most of us come to the same conclusion whenever we have the courage to stop teaching and try to discover what our students are actually learning.

Glenn Shafer has written a stimulating essay that uses the historical record to remind us of the subtlety, variety and importance of conceptual interpretations of uncertainty. He wonders if our institutional arrangements and our teaching and research practice adequately reflect this richness. Our teaching of beginners, I think, is weaker than our research or our institutional arrangements. Let us at least not forget this weak point in discussions of grander issues.