*Statistical Science*
1991, Vol. 6, No. 2, 141–148

# The Genoese Lottery

**D. R. Bellhouse**

*Abstract.* The number lotteries that are in abundance today derive from a lottery run in Genoa in the seventeenth century. The origins of the Genoese lottery are traced, and the earliest known probability calculations on it are given. What is probably the first American connection to the Genoese lottery is also given.

*Key words and phrases:* Combinations, history of lotteries, lottery patents, number lotteries.

## 1. INTRODUCTION

Several states in the United States and provinces in Canada, as well as many other countries in the world, run a number lottery in which each player may choose his or her own ticket number. A typical example is Canada's largest number lottery, Lotto 6/49. A player fills out a ticket in this lottery by choosing six numbers without replacement from the integers 1 through 49. On the draw nights, six numbered balls from 49 are chosen without replacement to determine the winning number combination. A seventh ball is drawn, without replacing the first six, as a bonus number. Typical of all lottery structures, prizes are awarded in decreasing amounts of money for some specific types of number combinations that have increasing probabilities. The first prize is to match all six of the regular balls chosen; the second prize is to match any five regular balls and the bonus ball; and the third, fourth and fifth prizes are to match any five, four and three regular balls, respectively.

The original idea behind Lotto 6/49, and the many others like it around the world, comes from a lottery run in Genoa from the seventeenth century. The basic structure of the Genoese lottery was to choose five numbers without replacement from 90. As the game came to be known in the eighteenth century, players had five bets to choose from: *extraits, ambes, ternes, quarternes* and *quines*, using the French terminology. These were bets on seeing one, two, three, four, and five particular numbers show respectively among the five selected. Additional bets were the *extraits déterminés* and *ambes déterminés*, in which the player had to guess the draw number on which the one or two numbers chosen were selected in the without replacement draw (Castres, 1785). There were other variations on the prize structure (*Enciclopedia Italiana*). This is slightly different from the structure of the modern number lottery. Consider, for example, the *ternes* bet. In this bet, a player chooses three numbers only and wins if all three are chosen among the five numbers drawn from 90. In a modern number lottery, a player would choose five numbers and would win the prize corresponding to the *ternes* bet if any three of his five numbers were chosen among the five drawn from 90.

The number lottery had one distinct advantage over other types of lottery games run prior to the modern era of lotteries. The mode of selection of the winners or winning numbers was quick and simple. Over a period of time, players recorded their bets with the lottery manager. On the day of the draw, five numbers from 90 were chosen without replacement. The winners were paid at prespecified odds after checking the amount of the bet in the lottery ledger where the bet was recorded. With other lotteries, the draw was preceded by the sale of printed lottery tickets, corresponding to the recording of bets in the number lottery. What took a great deal of time, several days or weeks, was the selection of the winning tickets. A typical lottery draw with tickets is described in Ashton (1893, pages 312–324). Winning tickets were drawn using two lottery wheels. With one wheel, lottery tickets were selected at random without replacement one by one. As a lottery ticket was selected from the first wheel, another ticket was selected from the second wheel that gave the value of the prize to be awarded or no prize if that ticket were blank. Drawings continued until all prizes were awarded.

In this article, the origins of the Genoese lottery are described, as well as the first attempts to calculate the probabilities of its prize structure. A variant of this lottery that ran in the United States

*D. R. Bellhouse is Professor of Statistics, Department of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario, Canada N6A 5B9.*

during the first half of the nineteenth century is also described.

## 2. EARLY HISTORY OF THE LOTTERY

A standard source for the origin of the Genoese lottery is Beck de Madaras (1912). There it is mentioned that the lottery "... took its origin from a political act at Genoa. At the elections to the great council five names out of ninety had to be drawn every year in that city, and betting began to be carried on on these names; this led to the formal 'lotto' or 'number' lottery in which names were replaced by numbers." Daston (1988, page 143), for example, gives some further detail. She states that the "Genoan-style" lottery was invented by Benedetto Gentile in 1610 and that the five numbers from 90 were drawn out of a rotating hopper by a child. The information related in Daston and Beck de Madaras is given together in much more detail in Beckmann (1846, pages 426–428), with the only exception being that the 1610 date is replaced by 1620; in fact, Beck de Madaras references Beckmann. Beckmann also states that the Genoese lottery was banned by Pope Benedict XIII, whose papacy ran from 1724 to 1730, and that the succeeding pope, Clement XII, set up a lottery at Rome. The *Enciclopedia Italiana*, under "Lotto," also refers to the story of Benedetto Gentile, but says that the five names were selected from 120 and incorrectly refers to the anti-lottery pope as Benedict XII.

This generally accepted story of the lottery's origin does not appear to be fully accurate. A detailed discussion of this story is given in the Appendix. What follows from the Appendix is that the Genoese number lottery was derived from elections by lot in Genoa, but, almost certainly, not by Benedetto Gentile. Based on the source material collected, the following scenario for the origins of the Genoese lottery is reasonable. At some time in the middle of the seventeenth century, elections by lot were carried out in Genoa and betting was allowed on the outcome. At each election two councillors or *Governatori* were chosen by lot from the 100 or 120 members who sat on the Petty Council of Genoa. Inspired by gamblers' interest in these elections, the state, or the state with a group of entrepreneurs and bankers, set up a parallel lottery that had more options to it. The added options would have made it more attractive to players. Instead of predicting two from 100 or 120, the lottery player could bet on one, two, three, four or five from 90 or 100 numbers. Other variations were later added. This, of course, is only conjecture. The true early history may be found one day in an early Italian travelogue or political tract that the current author has not seen. Or is may be buried somewhere in a Genoese archive, one day to be resurrected by someone knowledgeable in late Renaissance or early modern Italian language and history. Pasquier (see Euler, 1923) provides a hint that the lottery may date from the mid-seventeenth century, but he provides no references to his source material. In one paragraph, Pasquier (in Euler, 1923, pages xxiii–xxiv) mentions the Gentile story. In the next paragraph, he states that in 1643 the Genoese government let out the concession to an entrepreneur to run the lottery and this example was followed by other Italian cities. The history of the Genoese lottery, as its use spread across Europe in the eighteenth century, is well documented; see, for example, Bender (1832).

## 3. PROBABILITY CALCULATIONS ON THE LOTTERY

By the end of the eighteenth century several analyses, using probability, of the Genoese lottery had appeared. In terms of the game as played, the most comprehensive analyses are an article, entitled "Loterie," in *Encyclopédie Méthodique* (*Finances*), by Castres (1785), and a chapter in Parisot (1810) on the Genoese lottery. In both cases, all the possible variations in betting are given as well as the odds to win on each bet. A table containing similar information is given in the more modern *Enciclopedia Italiana*. Todhunter (1865, page 260) mentions that another set of volumes of the *Encyclopédie Méthodique* contains an article entitled "Loterie" by D'Alembert; see D'Alembert (1785, pages 337–338).

The earliest example, given by Todhunter (1865), of probability calculations on the Genoese lottery is from Euler (1765). Euler's involvement and interest in lotteries predates this paper. In September 1749, Frederick II of Prussia wrote to Euler asking him to work out the chances of winning each of the prizes in a Genoese-style lottery suggested to the king by an Italian named Roccolini. In this lottery, five numbers were chosen from 90. Players could bet on the outcome of any one, two or three of the numbers chosen. There were some variations to the game; for example, if one bet on a three number combination, a prize was given if two of the three numbers were selected. Euler replied with the correct probabilistic evaluation of the prize structure. Pasquier (in Euler, 1923, pages xxiii–xxiv) reports that one of Euler's mathematical notebooks, written between 1748 and 1750, contains several pages of notes on a number lottery. The letters are reprinted in Euler (1862, pages 550–552). More

than a decade later, in August 1763, Frederick again wrote to Euler about a lottery (Euler, 1862, pages 553-554). This was not a number lottery. Just prior to this letter, Euler had again become interested in lotteries. On March 10 of 1763, Euler read a paper on the Genoese lottery to *l'Académie de Berlin*. In this paper, published posthumously (Euler, 1862, pages 319-335; see also Euler, 1923, pages 466-494), Euler obtained general formulae for various prizes and prize structures in the number lottery. Suppose that there are $n$ numbers in a lottery from which $r$ are to be chosen at a draw. If a player bets on $x$ ($\leq r$) of these numbers to show, then Euler calculates that the probability of winning is $r(r-1)\cdots(r-x+1)/[n(n-1)\cdots(n-x+1)]$. He also goes on to calculate, for a number of cases, the probability that $y$ ($\leq x$) of the $x$ numbers chosen by the player show. The general solution is given by ${}_xC_y\,{}_{n-x}C_{r-y}/{}_nC_r$. This formula covers all the possibilities of play given under various names by Castres (1785) and Parisot (1810). For some of these variations in play, Euler makes numerical computations with $n = 90$ or 100 and $r = 5$. In the first article published in his lifetime on the Genoese lottery, Euler (1765, page 191) stated that the Genoese lottery was so well known that it did not need to be described; he gives no references. Euler's statement that the lottery was well known hints at the possible reason that the 1763 paper was not published in his lifetime. In the 1765 paper, Euler went beyond the calculation of the probabilities of the usual prizes by calculating the probabilities that simple sequences of numbers would be chosen in the Genoese lottery. This description of Euler's involvement in lottery analysis is given, with different detail and emphasis, by Maistrov, based on the work, in Russian, of Biermann (1957). Biermann (1957, page 656) makes reference to Bernoulli (1713) in making some combinatorial calculations; however, this Bernoulli makes no reference to the Genoese lottery. Euler (1785) published one other paper using the Genoese lottery as an example; the same result was obtained by Laplace (1774) who refers to the lottery as "*la loterie de l'École militaire.*" The problem is as follows: In a number lottery in which $r$ numbers are chosen from $n$ at any draw, find the probability that after $t$ draws of the lottery all the $n$ numbers will have been drawn. This problem is discussed in Todhunter (1865, pages 252-253 and 527).

Four mathematical works have come to light so far which predate Euler's work. These are Caramuel (1670), Frenicle (1729), Stampa (1700) and Bernoulli (1709). Frenicle de Bessy, a French mathematician, died in 1675, so his work is earlier than that the publication date of 1729. His unpublished mathematical work was first published from manuscript in 1693 and republished in 1729; see Busard (1970) and *Mémoires de l' Académie royale des sciences, Paris* (Anonymous, 1733) for a discussion of the sequence of publication that led to Frenicle (1729). A treatise on combinations, in which an analysis of the Genoese lottery appears, comprised only part of Frenicle's posthumously published work. Bernoulli, the last of these four to write on the Genoese lottery, was aware of Caramuel's work but not of that of the other two.

Stampa (1700) does not actually calculate the odds in the Genoese lottery. Rather, he mentions the lottery and then presents theorems leading to the calculation of the number of combinations of a set of dissimilar things taken one, two, three, four, five and more at a time (Propositions XII through XVI). He also provides some numerical tables at the end of the book. The calculations are done in what may be called the old style, using figurate numbers. Figurate numbers are described in Edwards (1987). Essentially Stampa (1700) shows that the combination ${}_nC_r$ is given by the figurate number $f_r^{n-r+1}$. The figurate number $f_r^{n-r+1}$ is the sum ${}_ra_1 + {}_ra_2 + \cdots {}_ra_{n-r+1}$, where the terms ${}_ra_1, {}_ra_2, \ldots, {}_ra_{n-r+1}$ can be obtained by noting that the $(r-1)$th finite difference with respect to $t$ of ${}_ra_t$ ($t > r$) is always unity for all $r$ and $t$, and that ${}_ra_1 = 1$ for all $r$.

The other three authors, Caramuel, Frenicle and Bernoulli, do calculate probabilities for the Genoese lottery, but not for the prize structures studied by Euler (1862). Like the modern number lotteries, these authors all assumed that a lottery player picked five numbers from 100. The prizes were based on correctly guessing any one, two, three, four or all five of the five numbers drawn from 100. For the *extraits*, *ambes*, *ternes*, *quarternes* and *quines* bets under Euler's description of the lottery, the probabilities of winning are ${}_{n-x}C_{r-x}/{}_nC_r$, for $x = 1, 2, 3, 4, 5$ respectively when $r$ numbers (in this case five) are chosen from $n$ (in this case 100). In this other structure, the probabilities of winning the prizes are ${}_rC_x\,{}_{n-r}C_{r-x}/{}_nC_r$, for $x = 1, 2, 3, 4, 5$ respectively, again when $r$ numbers are chosen from $n$.

Juan Caramuel y Lobkowitz was a Spaniard with Bohemian ancestry. By profession, he was a Cistercian monk who eventually rose to various bishoprics in the Roman Catholic Church. In addition to theology, he wrote extensively on architecture, mathematics and astronomy. See Vernet (1970) for a brief biography. Caramuel's treatment of the Genoese lottery, which is entitled, "*De Concertationibus Cosmopolitanis*" or "Of the cosmopolitan dispute," is found in a treatise on combinations that

comprises *Syntagma* (or Section) 6 of Caramual (1670); Bernoulli incorrectly refers to it as Section 7. The section begins with various combinatorial rules and calculations. Then results on probability, taken directly from Huygens (1657), are presented. The transcription of Huygens (1657) is so faithful that a misprint in Proposition III of Huygens's work, that was corrected in later editions, was included. Stigler (1990) notes that this typographical error was the first in the history of mathematical probability and that it did not seem to have caused any confusion. Following the reproduction of Huygens's work are analyses of some simple games of chance and then the discussion of the Genoese lottery based on a selection of five numbers from 100. Caramuel (1670, page 997) states that the payoffs used in this game on a bet of 10 gold pieces are 10, 100, 3000, 15000 and 100,000 gold pieces for correctly guessing any one, two, three, four or all fives chosen numbers, respectively. After a lengthy discussion and some calculations, he (Caramuel, 1670, page 1013) provides a table that shows what the payoffs on a bet of 10 gold pieces should be. On converting this to a bet of one gold piece, to compare to Bernoulli's (1709) results, the prizes are 0.8, 10.7, 334.1, 31,703.9 and 15,090,208 gold pieces, respectively. From his calculations, Caramuel noted that the error in the payoff structure is to the advantage of the lottery player in the lowest valued prize, but to his disadvantage in all other prizes. Bernoulli (1709) stated that Caramuel's calculations were incorrect and that it would take too long to explain why. Bernoulli argued that the value of each prize should be inversely proportional to the probability of winning it and, for the lottery to be a fair game, that the expected value of the prizes should be the same as the amount bet. Based on these assumptions, he found that the prizes for correctly guessing any one, two, three, four and all five numbers, from the five numbers chosen from chosen are 0.95, 10.9, 337.3, 31,700.0 and 15,057,504, respectively. He concluded that the Genoese merchants defraud each lottery player by about 42% of every gold piece bet. It is interesting to note that in the modern lotteries this "defrauding" is usually about 50% of the purchase price of the ticket. Bernoulli's treatment of the Genoese lottery is discussed in Hald (1990, page 377).

In his article, "*Abregé des combinaisons*," Frenicle (1729) presents rules for the calculation of ${}_nC_r$ and other combinatorial problems. He gives the more common formula for ${}_nC_r$ as $n(n-1)\cdots(n-r+1)/r(r-1)\cdots 2\cdot 1$. Near the end of the article, the Genoese lottery, based on a selection of five numbers from 100, is analyzed. Frenicle starts by noting that the lottery bookmakers paid 20,000 to 1 on the for guessing all five numbers correctly, 5000, or 6000, to 1 for guessing any four of the five and 500, or 600, to 1 for the any three of the five. For correctly guessing two of the five, or one of the five, generally nothing was paid. Frenicle carries out his analysis by assuming a pay-off of: 20,000 to 1 for five correct guesses; 5,000 to 1 for four correct; 300 to 1 the three; and 4 to 1 for two. On using the combinatorial rules he has devised, Frenicle correctly calculates the total number of chances, 75,287,520, and the number of chances for winning each prize: one for correctly guessing five; 475 for guessing four; 44,650 for three; and 1,384,150 for two. He finds the advantage to the bank in each case and shows that this advantage is greatest in the prize for correctly guessing all five and least in the prize for correctly guessing three of the five numbers.

## 4. AN AMERICAN CONNECTION

Prior to the modern lotteries of the twentieth century, lotteries were carried out in Britain and the United States using numbered tickets. The winners were chosen, possibly over several days or weeks, by selecting numbers using a wheel of fortune. The mode of drawing the winners in number lotteries such as the Genoese lottery is much simpler and faster. The number lottery also allowed the player to choose his own combination of numbers at stated odds. For some reason very few, if any, number lotteries were run in the British Empire or the United States. One plausible reason for this is that the number lotteries were run like regular bets with stated odds for the payout. In the other lotteries, provided that all the tickets were sold, the lottery manager was guaranteed a certain percentage of the take. One exception to the rule of no number lotteries in the English-speaking world was an American lottery that was a hybrid of the number lottery and wheel of fortune systems. In this lottery, a full set of tickets was printed, but the numbering of the tickets followed the principles of the number lottery. Information on this lottery was found among U.S. patent records.

During the first half of the nineteenth century, several American inventors patented lottery schemes or lottery numbering systems. See Leggett (1874, page 889) for a list of lottery patents. Of the seventeen lottery patents that were granted between 1815 and 1840, only two patent specifications survive. A fire at the U.S. Patent Office in December 1836 destroyed most of the records relating to patents. After the fire, there was an attempt to restore the lost patents. All persons having

patents, or persons possessing certified copies of patents, were requested to forward them to the Patent Office so that copies could be made. Copies of the patent specifications or descriptions that were submitted were made in large ledger books. These survive today in the U.S. National Archives and form almost the sole source of information on early U.S. patents. Sixteen of the seventeen lottery patents are pre-1836 and of this group only one survives. The surviving patent specification is a lottery numbering system; the patent was granted to John Rives on December 22, 1826. The numbering of U.S. patents began after the 1836 fire.

The other remaining lottery patent was granted to Joseph Vannini in 1840 (U.S. Patent #1400). This lottery numbering system is an "improvement" on an earlier lottery patent granted to Vannini on December 4, 1815. Although the specification for the earlier patent no longer exists, there is a description in Weiss and Weiss (1966, pages 20–23) of an 1824 lottery, the Queen's College Literature Lottery in New Jersey, which uses the earlier numbering system. Both of Vannini's patents are of interest since the patented numbering schemes use the principle of the number lottery and are based directly on the Genoese lottery. In the earlier patent, winning tickets are drawn by choosing five numbers without replacement from a larger set. The tickets are numbered using all ternary combinations, the combinations of three numbers from the larger set. In the specific example cited in Weiss and Weiss (1966) the draw selection was five numbers from the integers 1 through 35, with a total of 6,545 tickets numbered by all combinations of the numbers 1 through 35 taken three at a time. Specifically the ticket numbers are the elements of the set $\{(1,2,3),(1,2,4),\ldots,(1,2,35),\ldots,(33,34,35)\}$. The prizes were determined by the five numbers drawn and the order in which they were drawn. For example, the grand prize was determined by the ticket that matched the first three of the five numbers chosen. The second prize ticket was the one that matched the last three of the five numbers chosen. The complete prize structure is given in Weiss and Weiss (1966). In the 1840 patent, Vannini suggested a scheme based on choosing five numbers without replacement from 90. The ticket numbering is quite complex and appears to be unsuccessful in terms of the objectives laid out in the patent. A single ticket had 25 sets of combinations on it as its number: five of the 90 uniary combinations obtained by taking the 90 numbers one at a time, ten of the 4,005 binary combinations obtained by taking the 90 numbers two at a time, and ten of the 117,480 ternary combinations obtained by taking 90 numbers three at a time. Vannini suggested a total of 11,748 tickets comprising all the ternary combinations with ten of these combinations per ticket. The problem with this scheme comes when trying to balance the uniary and binary combinations over the 11,748 tickets. Vannini specified that, for the uniary combinations, the tickets are printed in sets of 18. In each set of 18 tickets, all 90 uniary combinations appear. However, he failed to realize that 18 does not divide into 11,748 evenly. Also, his explanation of how he was going to handle sets of ten from the 4,005 binary combinations is obscure. No evidence so far has come to light to show that this lottery was ever put into practice. Consequently, the main feature of this patent specification appears to be the use of the numbers five and 90 in the calculations.

## 5. DISCUSSION

As Daston (1988, page 144) has noted, "Although lottery problems were the stock-in-trade of the probabilists, mathematicians played a largely peripheral role in designing the lotteries." Daston goes on to note that Euler's work in the Genoese lottery was one of the notable exceptions. However, it is interesting to note that the work on the Genoese lottery published during Euler's lifetime had very little to do with the lottery game as it was played. Daston's observation is also generally true of the earlier mathematical work on the Genoese lottery. Both Frenicle (1729) and Laplace (1774) used the lottery as an example problem in combinatorics; Caramuel (1670) and Bernoulli (1709) both used it as an example of an unfair contract. Stampa's (1700) treatment is slightly different. His motivation was to make his readers aware of the issues, including the chances of winning, in games of chance. The Genoese lottery eventually found its way into recreational mathematics books. See, for example, Hutton's (1803, page 120–123) *Recreations, Vol. I.* The title page of this work says that is was originally composed by Jacques Ozanam (d. 1717) and later recomposed and enlarged by Jean-Etienne Montucla. The problem does not appear in Ozanam (1741), but it is discussed briefly in Montucla's (1802, page 389) *Histoire des Mathématiques, Tome Troisième.* Montucla's updating of Ozanam was published as early as 1778 (Montucla, 1778), but this book was not available to the present author.

In summary, although the probabilists had little impact on the Genoese lottery, since the eighteenth century this lottery game has had substantial impact on the the design of lottery games in both Europe and North America.

## APPENDIX: THE ORIGINS OF THE GENOESE LOTTERY

As mentioned in Section 2, a nineteenth century description of the origins of the Genoese lottery is given in Beckmann (1846). There are no source references on this lottery in Beckmann (1846). The original German edition, Beckmann (1805) gives two references, Labat (1730) and Volkmann (1771). Labat (1730) is one example, though slightly atypical, of the travel literature that flourished from the seventeenth to the nineteenth century. Generally, this literature described the Grand Tour that was followed through France and Italy by persons of rank or wealth. A comprehensive bibliography of the travel literature in English is given in Pine-Coffin (1974). A less comprehensive bibliography, but one that includes the Continental literature is found in Harder (1981). Unfortunately, upon checking many of the references in these bibliographies, no further description of the Genoese lottery was found. On using Labat (1730) and Volkmann (1771) as sources, as well as some other early source material on Italian politics and government, a slightly different picture emerges. Some evidence points to the conclusion that during the whole of the seventeenth century there was no council of 90 from which five were elected by lot, but that the lottery was indeed inspired by the Genoese election system.

In the late sixteenth and seventeenth century, a few Italian city states had governments determined, in part, by lot. A comprehensive description of the Venetian system appears in Contareno (1599). The mode of election of magistrates is described in detail. Citizens met in a room that contained three urns. One urn was placed in the middle of a special seat and the other two urns were placed at each side of the seat. The middle urn contained 36 gold balls and 24 silver balls. The other two urns each contained 30 gold balls and an infinite number, according to Contareno, of silver balls. The gold balls were all marked with special letters so that a citizen could not palm his own gold ball in the election. Citizens filed by the two outer urns choosing balls. Those 60 citizens who chose gold balls were eligible to choose from the middle urn. The 36 who selected the gold balls from the middle urn made up the group of electors who eventually chose the magistrates.

Grimstone (1615), which is an English translation of Avity (1614), has described some elections by lot in Venice, Genoa and Tuscany or Florence. According to Grimstone or Avity, the government of Genoa was in the hands of 28 ruling families. Members of these families made up a Great Council consisting of 400 individuals. From this Council, 100 were chosen to form a Petty Council or Senate. The Doge of Genoa and eight councillors or *Governatori*, a kind of executive committee, were elected for two-year terms by the two Councils. The *Governatori* were chosen two at a time every six months. The election process, as described by Grimstone (1615, pages 511–512) was fairly complicated and did not involve the use of lots. The only office that appears to have been determined, in part, by lot was a board of eight, called the Protectors of the Mount of St. George, which dealt with some financial matters and money lending (Grimstone, 1615, pages 513–514). An initial group of 80 was selected by lot, and then a subgroup of 34 was selected by lot from the 80, to comprise a selection committee for the Protectors. This is similar to the Venetian election previously described. Later in the seventeenth century, Gailhard (1668) described the government of Genoa. He also referred to the government as the Doge, the eight *Governatori*, the Great Council of 400 and the Petty or Little Council of 100, but he gave no information as to the mode of election. By the end of the seventeenth century, a change had been made in the election procedure to election by lot. In addition, a rudimentary form of the number lottery based on the election was in operation. Leti (1697, pages 140–141) has described both. Leti refers to the Petty Council or Senate as the Great Council of Nobility and to the Great Council as the Grand Council. The number of members comprising each at this time was 120 and 700, respectively. From the 120 who sat on the Petty Council two were chosen by lot every six months to serve two-year terms as *Governatori*. The names of the Petty Council members or senators were printed six months in advance of the election lottery. Betting, through banks in Genoa, was allowed on the outcome of the election. An individual who correctly guessed both selections in the election could get a return on the bet in excess of 100 times the amount bet. The Doge was also elected by lot, but no mention was made by Leti of any betting concerned with this election. Leti makes no mention of a number lottery in which five numbers are chosen from 90.

From the sources thus far, it appears that election by lot in Genoa probably did not occur until the latter half of the seventeenth century and that gambling on the outcome was part of this election. There is further evidence, although weak, that there was a number lottery separate from the election by lot. Stampa (1700, pages 19–21) gives a poem in Italian on the Genoese lottery. In the poem, there are three characters: a poet, a gambler and a cabalist. Throughout the poem the gambler complains

that he listened to the advice of the cabalist on how to bet and that the advice was bad. The gambler goes on to say that if he had not listened to the cabalist and had played his own numbers, listed as 1 (or A), 57 and 88, then he would have been a rich man. There is no mention of an election, merely a list of numbers.

To complicate matters, there is conflicting evidence in other source material. Both Caramuel (1670) and Frenicle (1729) say that the lottery came about as a result of betting on the random selection of five persons from 100 Genoese senators. It is impossible to say if Frenicle's original manuscript on combinations, written prior to 1675, had been published unaltered in 1693 or 1729. Since both Caramuel and Frenicle deal with the same prize structure and issues concerning the prize structure, the ideas in Frenicle (1729) on the Geonoese lottery may have been inspired by Caramuel (1670). It is interesting to note that neither Caramuel's nor Frenicle's work may not have been widely known among the mathematicians of the eighteenth century. Montmort (1713, page xxxv) mentions Prestet, Tacquet and Wallis as authors on the subject of combinatorics, but he makes no mention of Caramuel or Frenicle. In addition, Montmort (1713, pages 257-260) does analyze a lottery, but not the Genoese lottery.

Labat (1730) also provides conflicting evidence. It is important to note that, although Labat published his book in 1730, the description of the Genoese lottery was based on his recollections of a trip to Genoa in 1706. Labat (1730, pages 100-102) mentions, in very general terms, the election by lot and says that one could bet on two, three, four or more individuals. The gamester could also decide on the size of his bet. There is a brief description of the religious ceremony (a mass) surrounding the draw. Finally, there is a description of a gambler who bet on the selection of five particular numbers or names. When the first four matched his selection, the bankers or lottery managers offered to buy his ticket. The gambler refused. The fifth selection did not match. His ticket was now worthless, so the gambler went out and drowned himself.

Labat (1730) may not be totally reliable for the exact origins of this lottery. Labat was writing 20 to 25 years after his trip to Genoa, at a time when the lottery was well established and had been banned by the Pope. Moreover, part of Labat's report may be fictitious. As noted by Harder (1981, page 132), Labat was using his memoirs of Genoa to point out, in part, the avariciousness of the Genoese. Harder says further that Labat invented humorous anecdotes to make his case. The mass held for the election by lot with bookmakers hovering in the congregation, and the near winner, but eventual loser, who drowned himself, may be two of these anecdotes. Even if Labat is dismissed, it still leaves the evidence provided by Caramuel and Frenicle.

The story of Benedetto Gentile, the supposed inventor of the lottery who flourished circa 1620, appears in Volkmann (1771, pages 839-840), but not in Labat (1730). No source references are given by Volkmann. In view of the other collected information on this lottery, this source is too late to be reliable.

## ACKNOWLEDGMENTS

This work was supported by funds from the Social Sciences and Humanities Research Council of Canada. The author would like to thank Beate Gundert for her translation of excerpts from Stampa (1700). The author would also like to thank Stephen Stigler and William Kruskal for their comments on an earlier draft of this paper. Part of this research was carried out while the author was on leave at the University of Waterloo.

## REFERENCES

ANONYMOUS (1733). Mémoires de l'académie royale des sciences, depuis 1666 jusqu'à 1699. *Mémoires de l'Académie royale des sciences, Paris* **2** 195-197.

ASHTON, J. (1893). *A History of English Lotteries*. Leadenhall, London.

AVITY, P. D' (1614). *Les estats, empires, et principavtez dv monde*. Boscard, Paris.

BECK DE MADARAS, G. (1912). Lotteries, Continent of Europe. In *Dictionary of Political Economy* (R. H. I. Palgrave, ed.) **2** 640-643. Macmillan, London.

BECKMANN, J. (1805). *Beytåge zur Geschichte de Erfindugnen. Band 5*. Rummer, Leipzig.

BECKMANN, J. (1846). *A History of Inventions, Discoveries, and Origins* **2**, 4th ed. (Johnston, translator). Bohn, London.

BENDER, J. H. (1832). *Die Lotterie. Eine juristiche Abhandlung*. Mohr, Heidelberg.

BERNOULLI, J. (1713). *Ars Conjectandi, Opus Posthumum*. Thurnise, Basil. (Reprinted by Culture et Civilisation, 1968.)

BERNOULLI, N. (1709). *De Usu Artis Conjectandi in Jure*. Johannis Conradi, Basel. (Reprinted in *Die Werke von Jakob Bernoulli, Band 3*, 1975, pages 287-326, Birkhäuser Verlag, Basel. Translated into English by Thomas Drucker, 1976 (unpublished).)

BIERMANN, K. R. (1957). Zadachi genuezckogo loto v rabotakh klassikov teorii verohatnostej. *Istoriko-Matematicheskie Isseldovania* **10** 649-670. (In Russian.)

BUSARD, H. L. L. (1970). Bernard Frenicle De Bessy. In *Dictionary of Scientific Biography* (C. C. Gillispie, ed.) **5** 158-160. Scribner, New York.

CARAMUEL Y LOBKOWITZ, J. (1670). *Mathesis Biceps. Vetus, et Nova*. Laurentium Anisson, Campagna.

CASTRES, C. DE (1785). Loterie. *Encyclopédie Méthodique. Finances*. **2** 752-772. Panckouke, Paris.

CONTARENO G. (1599). *The Commonwealth and Government of Venice*. Mattes, London. (Reprinted by Da Capo Press, 1969.)

D'ALEMBERT, J. (1785). Loterie. *Encyclopédie Méthodiques Mathématiques*. **2** 337-338. Panckouke, Paris.

DASTON, L. (1988). *Classical Probability in the Enlightenment*. Princeton Univ. Press.

EDWARDS, A. W. F. (1987). *Pascal's Arithmetical Triangle*. Griffin, London.

*Enciclopedia Italiana di Scuienze, Lettere et Arti*. (1929/30). Instituto G. Treccani, Rome.

EULER, L. (1765). Sur la probabilité des séquences dans la lotterie Génoise. *Memoires de l'Académie Royale des Sciences et Belles-Lettres de Berlin* **21** 191-230.

EULER, L. (1785). Solutio quarundam quaestionum difficiliorum in calculo probabilium. *Opuscula Analytica* **2** 331-346.

EULER, L. (1862). *Opera Postuma: Mathematica et Physica* (P. H. Fuss and N. Fuss, eds.). St. Petersburg. (Reprinted by Kraus, 1969).

EULER, L. (1923). *Omnia Opera, Series Prima VII: Commentationes Algebraicae ad Theorem Combinationum et Probabilitatum Pertinentes* (L. G. du Pasquier, ed.). Teubner, Leipzig.

FRENICLE DE BESSY, B. (1729). Abregé des combinaisons. *Mémoires de l'Académie royale des sciences, Paris* **5** 87-125.

GAILHARD, J. (1668). *The Present State of the Princes and Republicks of Italy, With Observations on Them*. Starkey, London.

GRIMSTONE, E. (1615). *The Estates, Empires, and Principallities of the World*. Lownes, London.

HALD, A. (1990). *A History of Probability and Statistics and Their Applications before 1750*. Wiley, New York.

HARDER, H. (1981). *Le Président de Brosses et le voyage en Italie au dix-huitième siècle*. Slatkine, Geneva.

HUTTON, C. (1803). *Recreations in Mathematics and Natural Philosophy*. Kearsley, London.

HUYGENS, C. (1657). De Ratiociniis in Ludo Aleae. In *Francisci a Schooten Exercitationum Mathematicarum libri quinque* (F. van Schooten, ed.) 517-524. Elsevir, Leiden.

LABAT, J.-B. (1730). *Voyages en Espagne et en Italie dans les années 1705 et 1707* **2**. Delespine, Paris.

LAPLACE, P. S. DE (1774). Mémoire sur les suites récurro-récurrentes et sur leurs usages dans la théorie de hasards. *Mémoires de Mathématique et de Physique, présentés à l'Académie Royale des Sciences, par divers savants et lûs dans ses Assemblées* **6** 353. (Reprinted in *Oeuvres Complètes de Laplace* **8** (1891) 5-24. Gauthier-Villars et fils, Paris.)

LEGGETT, M. D. (1874). *Subject Matter Index of Patents for Inventions Issued by United States Patent Office from 1790 to 1873 Inclusive* **2**. Government Printing Office, Washington D.C. (Reprinted by Arno Press, 1976.)

LETI, G. (1697). *Critiques Historique, Politique, Morale, Economqiue, & Comique Sur Les Loteries, Anciennes, & Modernes, Spirituelles, & Temporelles, Des Etats, & Des Eglises*. Boeteman, Amsterdam.

MAISTROV, L. E. (1974). *Probability Theory: A Historical Sketch* (S. Kotz, translator). Academic, New York.

MONTMORT, P. D. DE (1713). *Essay d'Analyse sur les Jeux de Hazard*, 2nd ed. Quillau, Paris. (Reprinted by Chelsea, New York, 1980.)

MONTUCLA, J.-E. (1778). *Récréations Mathématiques et Physique. Nouvelle édition*. Paris.

MONTUCLA, J.-E. (1802). *Histoires des Mathématiques*. Agasse, Paris. (Reprinted by Blanchard, Paris, 1968.)

OZANAM, J. (1741). *Récréations Mathématiques. et Physique. Nouvelle édition*. Jombert, Paris.

PARISOT, S.-A. (1810). *Traité du Calcul Conjectural ou L'Art de Raisonner sur les Choses Futures et Inconnues*. Bernard, Paris.

PINE-COFFIN, R. S. (1974). *Bibliography of British and American Travel in Italy to 1860*. Olschki, Florence.

STAMPA, G. M. (1700). *Lvdvs Serio Expensus*. Malatest, Mediolani.

STIGLER, S. M. (1990). Apollo Mathematicus. Unpublished manuscript.

TODHUNTER, I. (1865). *A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace*. Macmillan, Cambridge. (Reprinted by Chelsea, 1949.)

VERNET, J. (1970). Juan Caramuel y Lobkowitz. In *Dictionary of Scientific Biography* (C. C. Gillispie, ed.) **3** 61. Scribner, New York.

VOLKMANN, J. J. (1771). *Historisch-kritische Nachrichten von Italien. Band 3*. Fritsch, Leipzig.

WEISS, H. B. and WEISS, G. M. (1966). *The Early Lotteries of New Jersey*. Past Times Press, Trenton, N.J.