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Comment

David R. Brillinger

INTRODUCTION

This Report presents such a wealth of scientifically important, well-defined technical problems that all future submitters of nonnovel grant proposals will have some explaining to do. The Report is well-written, stimulating and full of sentences and phrases worth highlighting. I commend the Panel.

CURSORY REMARKS

The Report, Section 5, comments on the problem of noticing "interesting events" in the presence of masses of data. Perhaps some of the many existing procedures for detecting outliers might be of use.

The Report comments often on the problems of "aliasing" in many places. This seems to be an appropriate point in time to rethink the whole topic in both its active (selection of the measurement locations) and passive (working with those at hand) modes for all the various types of processes.

The Panel is concerned with how to move things forward. Having data sets and readme's conveniently available for anonymous ftp-ing seems an elementary way of quickly involving computer-literate statisticians. Email means that statisticians can collaborate with researchers around the world.

I was surprised to read in Section 7: "Even the El Niño phenomenon that affects weather patterns on a global scale can be initiated by an SST anomaly in the eastern tropical Pacific of only a degree or two." Is the genesis of El Niño really so well known?

TIDES

There was minimal mention of the study of tides in the Report. I do not doubt that the Panel felt that they were important, and since I was asked to provide a data analysis, I will report on one for some tidal data.

Tidal prediction has been important for many years, surely predating the report's stated 1769 birth date of oceanography. Novel numerical techniques and computing machines have been developed, for example, the harmonic analyser of Lord Kelvin (Kelvin, 1911), and used routinely for preparing tables for harbours around the world.

The Bay of Fundy lies between New Brunswick and Nova Scotia. It is renowned for having the highest tides in the world, reaching 17 meters at times and places. Some data for it have have been studied. They were a sequence of $T = 2160$ hourly observations made in the Bay, near St. John, for the interval 1 January to 31 March 1991. A simple graph of the series shows a dominating periodicity of just over 12 hours. The top display of Figure 1 is the log-periodogram of the data flattened by subtracting the result of a robust smoothing; specifically, what is plotted is

$$(1)\quad \log\left(\left|\sum_{t=0}^{T-1} Y(t)\exp\{-i\lambda t\}\right|^2\right) - \operatorname*{loess}_{\omega}\left\{\log\left(\left|\sum_{t=0}^{T-1} Y(t)\exp\{-i\omega t\}\right|^2\right)\right\}$$

where loess_ω is the robust smoother of Cleveland (1979). The robust character of this smoother "removes" the peaks. The dashed horizontal line gives an approximate upper 95% marginal significance

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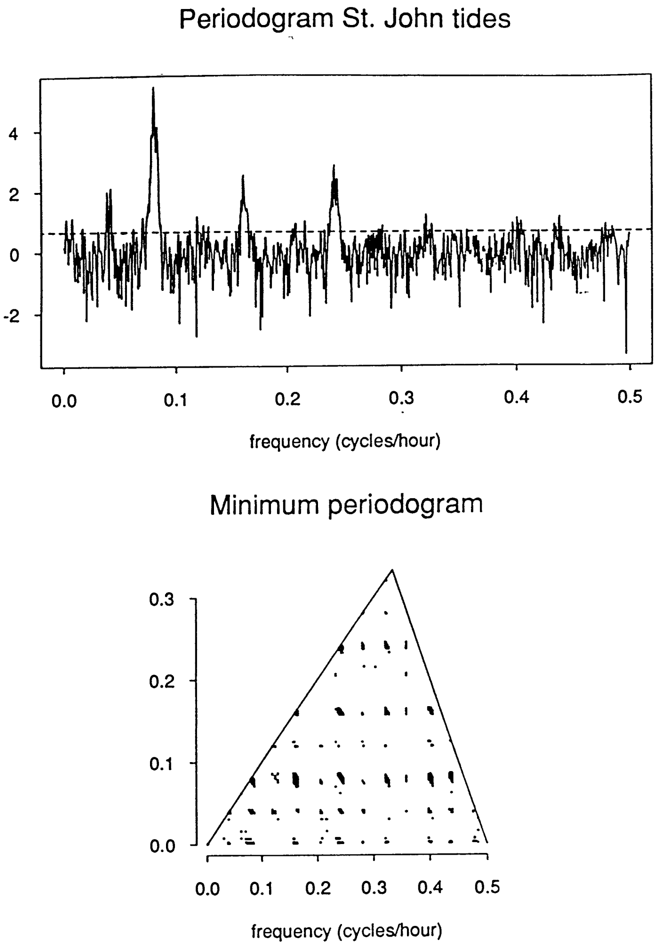

FIG. 1. *The top display is the flattened log-periodogram* (1). *The bottom display provides the locations of the frequency pairs where the statistic* (3) *exceeds the* 95% *null level.*

level. Various peaks are apparent, with the dominant one near $\frac{1}{12}$ cycles per hour.

A pertinent model for this data is

$$Y(t) = \mu + \sum_{k=1}^{K} \rho_k \cos(\omega_k t + \phi_k) + \varepsilon(t), \tag{2}$$

with $\varepsilon(\cdot)$ stationary noise and with the ω_k corresponding to periods related to the motions of the moon and sun. In practice, the preparers of tables may take K to be around 25.

The basic phenomenon of tides is nonlinear (see Cartwright, 1969), so certain of the frequency components of (2) may be anticipated to interact (and this is referred to several times in the Report). Consider in particular the possibility that the frequencies ω_1, ω_2 and $\omega_3 = \omega_1 + \omega_2$ are all three present in (2). To examine this possibility, consider the statistic

$$\min\left\{J^T(\omega_1),\ J^T(\omega_2),\ J^T(\omega_1 + \omega_2)\right\}, \tag{3}$$

where $J^T(\lambda)$ is the flattened periodogram whose logarithm is given by (1). Under elementary moment and mixing conditions the large sample distribution of the statistic (3) is that of e_1 if one $\rho = 0$, of $\min\{e_1, e_2\}$ if two ρ's are 0, of $\min\{e_1, e_2, e_3\}$ if all three of the ρ's are 0, where e_1, e_2 and e_3 are independent exponentials with mean 1. The conservative procedure is to use the distribution of e_1. The locations of values, significant at the 5% level, are plotted in the bottom display of Figure 1. Substantial structure is present. Much of this structure remains if one repeats the computations having removed the principal tidal frequencies by least squares. One has strong evidence for the presence of interactions.

The statistic (3) appears preferable to the more commonly employed third-order periodogram. This last, for example, will be large if any of the frequency components $\omega_1, \omega_2, \omega_1 + \omega_2$ has a large amplitude.

A statistic like (3) was suggested in Brillinger (1980). More details on this analysis may be found in Brillinger (1994). A problem that remains is to understand the character of breadths of the peaks in the figure.

CONCLUSIONS

There are novel problems at every turn. One of the jobs of statistics is to transfer quantitative technology between substantive fields. This Report highlights many opportunities for doing so.

Among the suggestions of the Report for ways to proceed, I strongly support that of (small) workshops of statisticians with substantive scientists. These are highly productive in my experience.

ACKNOWLEDGMENTS

Keith Thompson of Dalhousie University provided the Bay of Fundy data. Partial support was given from NSF Grant DMS-93-00002 and ONR Grant N00014-94-1-0042.