REGINA Y. LIU AND KESAR SINGH

Rutgers University

Many different types of bootstrap confidence intervals have appeared in the literature. There is now an urgent need for a unified account of the developments and for comparative studies which can provide some recommendations. This article is very timely in this sense. We do agree that the so-called percentile method is currently the most popular one although it is probably the worst one. The article should also serve as an excellent reference for explicit terms (especially the n^{-1} terms) in some commonly needed Edgeworth expansions.

We would like to concentrate our discussion on two topics: the shortest length bootstrap confidence intervals and Efron's bias correction. The idea of shortest length bootstrap confidence intervals is an important one, which up to this point has not received the attention it deserves. Hopefully, the present paper will help in giving it fair treatment. The increased accuracy due to the phenomenon of one-term Edgeworth correction by the bootstrap is well understood in the case of one-sided intervals which can be used for one-sided testing of hypotheses. However, the situation becomes quite complex in the case of two-sided intervals. The asymmetric intervals based on a bootstrap histogram of Studentized statistics do reflect the asymmetric shape of the true sampling distribution by making correction for skewness. The impact on the total coverage or the length of the intervals depends on population parameters and it seems no definite conclusions can be reached. Hall's Table 1 does shed light on this important issue. Therefore, the shortest bootstrap confidence intervals (which need no analytic work) seem to be attractive alternatives in this case. The present article also reports a very surprising finding on these intervals, which is the simultaneous gain in the coverage probability and the reduction in the length. Some intuitive explanations would be desirable.

For those who are used to looking at bootstrap through Edgeworth expansions, the article presents a very clear view of Efron's bias correction and of the accelerated bias correction (abc). The interpretation of the acceleration constant in terms of skewness brings out the connection between the abc and the one-term Edgeworth correction. If second-order correctness is the objective, then it is not clear why one would want to use the abc (which does seem to need some analytic work) instead of simply bootstrapping a Studentized statistic. Generally, an estimator of the standard error is available for the purpose of Studentizing. Furthermore, if we consider third-order correctness, the bootstrap approximation for the Studentized statistics is better than the abc, due to a phenomenon referred to as a partial correction in Liu and Singh (1987) (cf. Remark 2). In this context, we would like to bring to the reader's attention one-step modified pivots such as the modified t-statistics

$$t_1 = t + \frac{(2x^2 + 1)}{6\sqrt{n}} \frac{\hat{\mu}_3}{s_n^3}$$

discussed in Abramovitch and Singh (1985), whose Edgeworth expansion starts with an n^{-1} term instead of an $n^{-1/2}$ term. As a result, the bootstrap approximation of the distribution of such a modified pivot is automatically third-order correct.

We believe that this extensive theoretical study will prove to be very helpful to the users of the bootstrap technique in making a "right choice" of a bootstrap confidence interval.

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DEPARTMENT OF STATISTICS
HILL CENTER
RUTGERS UNIVERSITY
NEW BRUNSWICK, NEW JERSEY 08903

MICHAEL R. VEALL

McMaster University

As a bootstrap practitioner and applied econometrician/statistician, my comments will focus on the implications this fine paper in bootstrap theory has for applied research. From my perspective, it seems that two branches of research into bootstrap confidence intervals have developed. What I shall call the "asymptotic" approach follows the initial motivation of Efron and emphasizes finding confidence interval estimates where no feasible analytic alternatives exist. Much of Peter Hall's paper is instead in the "finite sample" branch in that, as he notes, it largely assumes that there are suitable analytic standard error estimates and hence analytic confidence intervals are available; the bootstrap is used to improve on the large sample approximations required for most confidence interval construction. Hall's research therefore provides a simulation alternative to analytic Edgeworth expansion and inversion. I find the asymptotic and finite sample approaches complementary and, interestingly, that they each correspond to growing needs in applied econometric research.

Applied econometricians, like other kinds of statistical practitioners, often diverge sharply from standard statistics textbook approaches. One such divergence has been the estimation of very complex models in which the formulae for standard errors are too complicated to calculate analytically. For these situations the bootstrap was an important innovation and found rapid acceptance among econometricians, many of whom were already using some kind of simulation/resample technique to aid inference in this context.

To take an example along this line from my own work, I used the bootstrap to estimate confidence intervals in a complicated forecasting problem involving future electricity demand [Veall (1987a)]. As have most econometricians, I used the percentile method. Hall's "looking-up-the-tables-backwards" argument has