The Annals of Statistics
1990, Vol. 18, No. 1, 172–190

ON THE INADMISSIBILITY OF STEP-DOWN PROCEDURES FOR THE HOTELLING T^2 PROBLEM[1]

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Step-down testing procedures for the Hotelling T^2 problem are considered. It is shown that the classical step-down procedure proposed by S. N. Roy, R. E. Bargmann and J. Roy is inadmissible among invariant procedures (for a suitable invariance group) in many situations, including all those for which more than two steps are contemplated. It is also noted that in most cases, the power of the step-down procedure decreases in at least one of the noncentrality parameters over part of the parameter space. Finally, several alternative admissible step-down procedures are proposed.

1. Introduction. Step-down procedures in multivariate testing problems, formally introduced by Roy and Bargmann (1958) and Roy (1958), studied further by Dempster (1963) and also implicit in work of Rao (1948) and others, have been advocated as competitors to multivariate likelihood ratio tests (LRT). If a null hypothesis $\mu = 0$ is to be tested against $\mu \neq 0$, where $\mu \equiv (\mu_1, \dots, \mu_p)$ is a vector parameter, the Roy–Bargmann–Roy (RBR) step-down procedure sequentially tests $\mu_1 = 0$ vs. $\mu_1 \neq 0$; then $\mu_2 = 0$ vs. $\mu_2 \neq 0$ given $\mu_1 = 0$; then $\mu_3 = 0$ vs. $\mu_3 \neq 0$ given $\mu_1 = 0$, $\mu_2 = 0$, etc., applying a conditionally level α_i test at step i. The procedure stops if any one of these conditional tests rejects its null hypothesis, in which case the overall null hypothesis $\mu = 0$ is rejected, or if each conditional test accepts its null hypothesis, in which case $\mu = 0$ is accepted.

In the Hotelling T^2 problem, where μ is the mean vector of a multivariate normal population with unknown covariance matrix Σ, at step i the RBR step-down procedure employs the level α_i LRT for testing $\mu_i = 0$ vs. $\mu_i \neq 0$, given $\mu_1 = \cdots = \mu_{i-1} = 0$. These LRT statistics are mutually independent when $\mu = 0$, permitting easy determination of the overall significance level α from the α_i [see (2.12)]. Furthermore, each of these stepwise LRT's has a *conditional optimality property* (see Section 2) and each is based on a univariate F-statistic which is easier to compute than the multivariate T^2 statistic which

Received March 1988; revised March 1989.

[1]An earlier version of this paper was delivered at a Statistical Research Conference dedicated to the memory of Jack Kiefer and Jacob Wolfowitz. The conference was held at Cornell University, July 6–9, 1983, and was a special meeting of the Institute of Mathematical Statistics, co-sponsored by the American Statistical Association. Financial support was provided by the National Science Foundation, the Office of Naval Research, the Army Research Office–Durham, and units of Cornell University.

[2]Research supported in part by National Science Foundation Grant No. MCS-82-01771.

[3]Research supported in part by National Science Foundation Grants No. MCS-83-01807 and DMS-86-03489.

AMS 1980 *subject classifications*. Primary 62H15; secondary 62C07, 62F03.

Key words and phrases. Step-down testing procedures, admissibility, invariant procedures, Hotelling T^2 problem, minimal complete class.

determines the overall LRT for testing $\mu = 0$ vs. $\mu \neq 0$. (Of course, the importance of this computational advantage has diminished considerably since 1958.) Like the overall LRT, the RBR step-down test is unconditionally unbiased, similar and consistent (provided no step is omitted, i.e., no $\alpha_i = 0$). These properties remain valid under the more general assumption, which we adopt henceforce, that each μ_i represents a *subvector* of μ, i.e., each μ_i consists of one or more components of μ. See Mudholkar and Subbaiah (1980a, 1987) for a comprehensive review.

Although the RBR step-down test employs a conditionally optimal level α_i test at each step [see the comment following (2.12)], until now very little has been learned about its unconditional optimality, but three isolated facts suggest caution. Subbaiah and Mudholkar (1978a, b) pointed out that in the Hotelling T^2 problem, the unconditional power of a two-step procedure need not be monotonically increasing in the first noncentrality parameter Δ_1 [see (2.10)] if the significance level α_1 at the first step lies in an (unspecified) sufficiently small neighborhood of 0. The results of Marden and Perlman (1980) show that the level α_i LRT at step $i \geq 2$ is unconditionally inadmissible if $\alpha_i > \alpha_i^*$ [cf. (2.13)]. In a more general setting, Jensen and Foutz (1980) suggested heuristically that the asymptotic power at the second step may be relatively low against certain alternatives if the first- and second-stage test statistics are not independent. To our knowledge, however, there has been no comprehensive study of unconditional optimality properties of step-down procedures.

This paper contains a decision-theoretic investigation of the RBR step-down procedure for the Hotelling T^2 problem. This procedure is described in detail in Section 2, where our main result (Theorem 2.1) is stated. This result implies that in most cases, including any for which more than two steps are contemplated, the RBR step-down procedure is inadmissible. The proof, presented in Section 4, is based on Theorem 3.1, which gives a necessary and sufficient condition for admissibility in a related invariance-reduced testing problem.

In Section 5 the result of Subbaiah and Mudholkar (1978a, b) is extended by a different argument to show that for a specifiable range of significance levels, the power function of the RBR step-down procedure is nonmonotone in certain portions of the parameter space. In Section 6, several alternative admissible step-down procedures are proposed.

The proofs of our main results, Theorems 2.1 and 3.1, rely heavily on the results in Marden and Perlman (1980) and Marden (1982), abbreviated throughout as MP (1980) and M (1982), respectively.

2. The Roy–Bargmann–Roy (RBR) step-down procedure. We treat the Hotelling T^2 problem in the following canonical form, allowing covariates [cf. MP (1980)]. One observes X and S, independent, where

$$X \sim \mathscr{N}_p(\mu, \Sigma), \qquad S \sim \mathscr{W}_p(\Sigma, n), \tag{2.1}$$

that is, X is a p-dimensional normal random (column) vector with mean vector μ and nonsingular covariance matrix Σ, and S is a $p \times p$ random Wishart matrix with mean $n\Sigma$ and n degrees of freedom, $n \geq p$. Partition μ into $q + 1$

subvectors $\mu_0, \mu_1, \dots, \mu_q$ where μ_i is $p_i \times 1$, with $p_0 + p_1 + \cdots + p_q = p$, $p_0 \geq 0$ and $p_i \geq 1$ for $1 \leq i \leq q$, and partition X, S and Σ accordingly. The Hotelling T^2 problem (with covariates) is that of testing

$$H_0 : \mu = 0 \quad \text{vs.} \quad H_A : \mu_0 = 0 \tag{2.2}$$

based on (X, S) with Σ unknown. (The usual T^2 problem has $p_0 = 0$; when $p_0 > 0$, μ_0 represents the vector of known covariate means: note that $\mu_0 = 0$ under both H_0 and H_A.)

Let T_i^2 be the Hotelling T^2 statistic based on the first $p_0 + p_1 + \cdots + p_i$ variates:

$$T_i^2 = \begin{pmatrix} X_0 \\ X_1 \\ \vdots \\ X_i \end{pmatrix}' \begin{pmatrix} S_{00} & S_{01} & \cdots & S_{0i} \\ S_{10} & S_{11} & \cdots & S_{1i} \\ \vdots & \vdots & & \vdots \\ S_{i0} & S_{i1} & \cdots & S_{ii} \end{pmatrix}^{-1} \begin{pmatrix} X_0 \\ X_1 \\ \vdots \\ X_i \end{pmatrix}, \qquad 0 \leq i \leq q. \tag{2.3}$$

($T_0^2 \equiv 0$ when $p_0 = 0$.) The likelihood ratio test (LRT) for problem (2.2) is based on $(T_q^2 - T_0^2)/(1 + T_0^2)$ (which reduces to T_q^2, the usual T^2 statistic, when $p_0 = 0$). For $i = 1, \dots, q$, the level α_i LRT for the component problem of testing

$$H_0^{(i)} : \mu_j = 0, \quad 0 \leq j \leq i \quad \text{vs.} \quad H_A^{(i)} : \mu_j = 0, \quad 0 \leq j \leq i - 1 \tag{2.4}$$

accepts $H_0^{(i)}$ when

$$Y_i \leq a_i \equiv a_i(\alpha_i), \tag{2.5}$$

where

$$Y_i = (T_i^2 - T_{i-1}^2)/(1 + T_{i-1}^2), \qquad 1 \leq i \leq q. \tag{2.6}$$

(Note that $H_0^{(1)} \supset \cdots \supset H_0^{(q)} = H_0$.) The RBR step-down procedure combines these component tests in the following way. First, test $H_0^{(1)}$ vs. $H_A^{(1)}$ based on Y_1. If $H_0^{(1)}$ is rejected, stop and reject H_0. If $H_0^{(1)}$ is accepted, proceed to test $H_0^{(2)}$ vs. $H_A^{(2)}$ based on Y_2. Continue until either (a) some $H_0^{(i)}$ is rejected, $1 \leq i \leq q$, in which case H_0 is rejected, or (b) $H_0^{(1)}, \dots, H_0^{(q)}$ are all accepted, in which case H_0 is accepted. Summarizing the RBR step-down procedure:

$$\text{accept } H_0 \text{ iff } Y_i \leq a_i(\alpha_i), \qquad 1 \leq i \leq q. \tag{2.7}$$

Let $Y_0 = T_0^2$. The joint distribution of $(Y_0, Y_1, \dots, Y_q)$ can be described as follows [cf. (1.2) of MP (1980)]:

$$\begin{aligned} Y_i \mid Y_0, Y_1, \dots, Y_{i-1} &\sim \chi^2_{p_i}\left(\frac{\Delta_i}{1 + T_{i-1}^2}\right) \Big/ \chi^2_{n_i}, \qquad 1 \leq i \leq q, \\ Y_0 &\sim \chi^2_{p_0}/\chi^2_{n_0}, \end{aligned} \tag{2.8}$$

where $n_i = n - (\Sigma_{j=0}^{i} p_j) + 1$. Note that

$$(2.9) \qquad 1 + T_{i-1}^2 = \prod_{j=0}^{i-1} (1 + Y_j).$$

The χ^2 variates in each ratio in (2.8) are independent and $\chi_\nu^2(\lambda)$ (χ_ν^2) denotes a noncentral (central) chi square random variable with ν degrees of freedom and noncentrality parameter $\lambda \geq 0$. The parameters $\Delta_1, \dots, \Delta_q$ in (2.8) are given by

$$(2.10) \qquad \Delta_i = \tau_i^2 - \tau_{i-1}^2 \geq 0, \qquad 1 \leq i \leq q,$$

where τ_i^2 is defined as T_i^2 in (2.3) but with (X, S) replaced by (μ, Σ). Note that $\tau_0^2 = 0$ and

$$\sum_{i=1}^{q} \Delta_i = \tau_q^2 = \mu' \Sigma^{-1} \mu,$$

so that under H_0, $\Delta \equiv (\Delta_1, \dots, \Delta_q) = (0, \dots, 0)$ and $Y_0, Y_1, \dots, Y_q$ are independent (nonnormalized) central F random variables. Thus, if we choose

$$(2.11) \qquad a_i \equiv a_i(\alpha_i) = (p_i/n_i) F(p_i, n_i, \alpha_i),$$

where $F(p_i, n_i; \alpha_i)$ denotes the upper α_ith point of the (normalized) F distribution with p_i and n_i degrees of freedom, then α_i is the level of the ith component test (2.5) and the overall level of the step-down test (2.7) is

$$(2.12) \qquad \alpha \equiv 1 - \prod_{i=1}^{q} (1 - \alpha_i).$$

[From (2.8) and the monotone likelihood ratio of the noncentral F density, it is seen that test (2.5) is the conditionally uniformly most powerful level α_i test based on Y_i for the component problem (2.4).]

In order to investigate the decision-theoretic properties of the step-down tests (2.7), we shall use the fact that $(Y_0, Y_1, \dots, Y_q)$ is the maximal invariant statistic under a group G of block triangular matrices acting on (X, S) defined in Section 3. In that section we shall apply Theorem 2.1 of M (1982) to characterize the minimal complete class of G-invariant tests for problem (2.2) (Theorem 3.1). In Section 4 we use this characterization to prove the following main result:

THEOREM 2.1. *For* $0 < \alpha < 1$, *the step-down test* (2.7) *is admissible among* G-*invariant tests for problem* (2.2) *if and only if*

(i) $p_0 > 0$, *at most one of* $\alpha_1, \dots, \alpha_q$ *is positive and that one* (*say* α_i) *satisfies* $0 < \alpha_i \leq \alpha_i^*$

or

(ii) $p_0 = 0$, *at most one of* $\alpha_2, \dots, \alpha_q$ *is positive and that one* (*say* α_i) *satisfies* $0 < \alpha_i \leq \alpha_i^*$, *where* α_i^* *is defined by*

$$(2.13) \qquad F(p_i, n_i; \alpha_i^*) = 1.$$

The case $p_0 > 0$, $q = 1$ was treated in MP (1980), page 45, where typical values of α_i^* are tabulated.

If $\alpha_i = 0$ for some i (i.e., $a_i = \infty$), then the step-down test (2.7) skips step i, so cannot be consistent. To see this, set $\Delta^{(\gamma)} = (0, \dots, 0, \gamma, 0, \dots, 0)$, where $\gamma > 0$ occurs in the ith position. Clearly $\sum \Delta_i^{(\gamma)} \to \infty$ as $\gamma \to \infty$, but the statistics $Y_0, Y_1, \dots, Y_{i-1}, Y_{i+1}, \dots, Y_q$ remain independent central F variates, hence the power of the test (2.7) remains at α.

3. The minimal complete class of G-invariant tests. Let G be the group of nonsingular $p \times p$ matrices A of the lower block-triangular form

$$A = \begin{pmatrix} A_{00} & 0 & \cdots & 0 \\ A_{10} & A_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ A_{q0} & A_{q1} & \cdots & A_{qq} \end{pmatrix},$$

where A_{ij} is $p_i \times p_j$. The testing problem (2.2) is invariant under G acting on (X, S) via $A: (X, S) \to (AX, ASA')$. The maximal invariant statistic and maximal invariant parameter can be represented by

$$Y \equiv \left(Y_0, Y_1, \dots, Y_q\right) \quad \text{and} \quad \Delta \equiv \left(\Delta_1, \dots, \Delta_q\right),$$

respectively [Giri, Kiefer and Stein (1963); Giri (1968)]. The invariance-reduced problem (2.2) can be described as that of testing

$$H_0: \Delta = 0 \quad \text{vs.} \quad H_A: \Delta_1 \geq 0, \dots, \Delta_q \geq 0, \qquad \Delta \neq 0, \tag{3.1}$$

based on Y.

Let $y = (y_0, y_1, \dots, y_q)$, let $f_\Delta(y)$ denote the density (with respect to Lebesgue measure) of Y under Δ and let

$$R_\Delta \equiv f_\Delta / f_0 \tag{3.2}$$

denote the likelihood ratio of Y. From (2.8) [also see Giri, Kiefer and Stein (1963)] it follows that

$$R_\Delta = \prod_{i=1}^{q} R_{\Delta_i}^{(i)}(1 - u_{i-1}, v_i), \tag{3.3}$$

where

$$v_i = y_i / \left((1 + y_i)\left(1 + t_{i-1}^2\right)\right), \qquad 0 \leq i \leq q, \tag{3.4a}$$

$$1 + t_i^2 = \prod_{j=0}^{i} (1 + y_j), \qquad 0 \leq i \leq q, \tag{3.4b}$$

$$u_i = \sum_{j=0}^{i} v_j \left(= t_i^2 / \left(1 + t_i^2\right)\right), \qquad 0 \leq i \leq q, \tag{3.4c}$$

$$R_{\Delta_i}^{(i)}(1 - u_{i-1}, v_i) = \exp(-\Delta_i(1 - u_{i-1})/2) G_i(\Delta_i v_i / 2), \qquad 1 \leq i \leq q, \tag{3.4d}$$

$$G_i(z) = {}_1F_1((p_i + n_i)/2, p_i/2; z), \qquad 1 \leq i \leq q, \tag{3.4e}$$

and where

$$\text{(3.5)} \qquad {}_1F_1(a, b; z) = \sum_{k=0}^{\infty} \frac{\Gamma(a+k)}{\Gamma(a)} \frac{\Gamma(b)}{\Gamma(b+k)} \frac{z^k}{k!}$$

denotes the confluent hypergeometric function for $a, b > 0$. [We set $t_{-1}^2 = 0$, so $v_0 = y_0/(1 + y_0) = u_0$; if $p_0 = 0$, then t_0^2, y_0, u_0 and v_0 are absent.] Note that R_Δ can be expressed solely as a function of v, i.e., $R_\Delta(v)$, by (3.4c). Also note that

$$\text{(3.6)} \qquad R_{\Delta_i}^{(i)}(1 - u_{i-1}, v_i) = R_\Delta(v_i, u_i),$$

where $R_\Delta(v, u)$ is given by (2.11) of MP (1980).

Define $u = (u_1, \dots, u_q)$ (regardless of whether $p_0 = 0$ or $p_0 > 0$), $v = (v_0, v_1, \dots, v_q)$ and set

$$\mathscr{U} = \{u | 0 < u_1 < \cdots < u_q < 1\},$$

$$\mathscr{V} = \left\{ v | 0 < \sum_{i=0}^{q} v_i < 1, \quad 0 < v_0, v_1, \dots, v_q \right\}.$$

Let $\mathscr{C}_u$ denote the set of all relatively closed, convex and nonincreasing subsets of $\mathscr{U}$. [$\tilde{C} \subseteq \mathscr{U}$ is nonincreasing if ($u \in \tilde{C}$, $u' \in \mathscr{U}$, $u_i' \le u_i$ for $1 \le i \le q$) $\Rightarrow u' \in \tilde{C}$]. Define $\mathscr{C}$ to be the set of all pre-images in $\mathscr{V}$ of sets $\tilde{C} \in \mathscr{C}_u$ under the mapping $v \to u$ given by (3.4c). Note that

$$\text{(3.7)} \qquad C \in \mathscr{C} \Leftrightarrow \begin{cases} C \text{ is relatively closed and convex in } \mathscr{V}, \text{ and} \\ \left(v \in C,\ v' \in \mathscr{V},\ \sum_0^i v_j' \le \sum_0^i v_j \text{ for } 1 \le i \le q \right) \Rightarrow v' \in C \end{cases}.$$

Thus all vectors ν normal to the boundary of C in $\mathscr{V}$ are of the form

$$\text{(3.8)} \qquad \nu = (\nu_0, \nu_1, \dots, \nu_q) \quad \text{with } \nu_0 = \nu_1 \text{ (or } \nu_0 \text{ absent if } p_0 = 0) \text{ and } \nu_1 \ge \nu_2 \ge \cdots \ge \nu_q \ge 0.$$

Now define

$$\text{(3.9)} \qquad \begin{aligned} l_i \equiv l_i(v) &= \frac{\partial}{\partial \Delta_i} R_\Delta \bigg|_{\Delta = 0} \\ &= \frac{1}{2} \left[\frac{(p_i + n_i)}{p_i} v_i - (1 - u_{i-1}) \right], \qquad i = 1, \dots, q, \end{aligned}$$

$$\text{(3.10)} \qquad \begin{aligned} d(v; \lambda, \pi_0, \pi_1) = \sum \lambda_i l_i &+ \int_{\{0 < \Sigma \Delta_i \le 1\}} \left[\frac{R_\Delta - 1}{\Sigma \Delta_i} \right] \pi_0(d\Delta) \\ &+ \int_{\{1 \le \Sigma \Delta_i\}} R_\Delta \pi_1(d\Delta), \end{aligned}$$

where π_0 is a finite measure on $\{\Delta|0 < \Sigma\Delta_i \leq 1\}$, π_1 is a locally finite measure on $\{\Delta|1 \leq \Sigma\Delta_i\}$ and $\lambda \equiv (\lambda_1, \ldots, \lambda_q)$ is a vector in $\mathbb{R}^q$ with $\lambda_i \geq 0$, $1 \leq i \leq q$. The following theorem describes the minimal complete class of G-invariant tests for problem (2.2). [A test is described in terms of its test function $\phi \equiv \phi(v) = P(\text{reject } H_0|v)$. Note that the mapping $y \to v$ determined by (3.4a, b) is 1–1, so that v is an equivalent representation of the maximal invariant statistic. Also, I_B denotes the indicator function of a set B.]

THEOREM 3.1. *A test* $\phi(v)$ *is admissible for problem* (3.1) [i.e., *is admissible among G-invariant tests for problem* (2.2)] *if and only if it is of the form*

$$\phi(v) = 1 - I_{C \cap A'} \quad a.e.\ [Lebesgue], \tag{3.11}$$

where $C \in \mathscr{C}$,

$$A' = \{v | d(v; \lambda, \pi_0, \pi_1) \leq c\} \tag{3.12}$$

for some (λ, π_0, π_1) *as above*, $|c| < \infty$ *and*

$$|d(v; \lambda, \pi_0, \pi_1)| < \infty \quad for\ v \in \text{interior}(C).$$

PROOF. We shall show that Theorem 2.1 of M (1982) can be applied to problem (3.1) under the following correspondences [quantities appearing in M (1982) are written first]: $\theta \leftrightarrow \Delta$, $p \leftrightarrow q$, $\Theta \leftrightarrow \mathbb{R}_q^+$, $V \leftrightarrow \mathbb{R}_q^+$, $x \leftrightarrow v$, $w \leftrightarrow u$, $X \leftrightarrow \mathscr{V}$, $\mathscr{W} \leftrightarrow \mathscr{U}$, $\mathscr{C}_w \leftrightarrow \mathscr{C}_u$ and $\mathscr{C} \leftrightarrow \mathscr{C}$, where $\mathbb{R}_q^+$ denotes the closed nonnegative orthant in $\mathbb{R}_q$.

First, condition (2.2) of the Local Assumption of M (1982), page 963, is satisfied here with $l_i \leftrightarrow l_i(v)$ and $\sigma \leftrightarrow \Sigma\Delta_i$, while condition (2.3), page 964, is verified by showing that there exists $K < \infty$ such that

$$\sup_{0 < \Sigma\Delta_i \leq 1} \frac{|R_\Delta - 1|}{\Sigma\Delta_i} \leq K \tag{3.13}$$

for all $v \in \mathscr{V}$. To show the latter, simply note that

$$\frac{|R_\Delta - 1|}{\Sigma\Delta_i} \leq \sum_{i=1}^{q} \left\{ \frac{|R_{\Delta_i}^{(i)} - 1|}{\Delta_i} \prod_{j=i+1}^{q} R_{\Delta_j}^{(j)} \right\}$$

and apply (d) of MP (1980), page 55.

Next, we show that the problem (3.1) is covered by Case B of M (1982), page 964, and is in fact "almost exponential" [M (1982), page 969, equation (4.1)]. That is, R_Δ in (3.3) can be written as

$$R_\Delta(v) = a(\Delta) b(v; \Delta) \exp\left(\sum_{i=1}^{q} \Delta_i u_i / 2 \right), \tag{3.14}$$

where $u_i \equiv u_i(v)$ is given in (3.4) and

$$a(\Delta) = \exp\left(-\sum_{i=1}^{q} \Delta_i/2\right) \prod_{i=1}^{q} (1+\Delta_i)^{n_i/2},$$

$$b(v;\Delta) = \prod_{i=1}^{q} b^{(i)}(v_i;\Delta_i),$$

where

$$b^{(i)}(v_i;\Delta_i) = (1+\Delta_i)^{-n_i/2} R_{\Delta_i}^{(i)}(v_i, v_i). \tag{3.15}$$

Condition (4.2) of M (1982) is written here as

$$0 < i(v) \equiv \inf_{\Delta \in \mathbb{R}_q^+} b(v;\Delta) \le \sup_{\Delta \in \mathbb{R}_q^+} b(v;\Delta) \equiv s(v) < \infty. \tag{3.16}$$

To verify (3.16), note that for fixed $v_i > 0$, $b^{(i)}(v_i;\Delta_i)$ is continuous in Δ_i, $b^{(i)}(v_i;0) = 1$ and

$$\lim_{\Delta_i \to \infty} b^{(i)}(v_i;\Delta_i) = \kappa_i v_i^{n_i/2} \tag{3.17}$$

for some $\kappa_i > 0$ [cf. 13.1.4 of Abramowitz and Stegun (1964)]. Thus the first and last inequalities in (3.16) hold. Equations (2.21), (2.26) and (2.27) of MP (1980) show that

(3.18) $\quad R_{\Delta_i}^{(i)}(v_i, v_i)$ $\left[\text{hence } b^{(i)}(v_i;\Delta_i)\right]$ is strictly increasing in v_i when $\Delta_i > 0$,

so that for all v_i, $b^{(i)}(v_i;\Delta_i) \le b^{(i)}(1;\Delta_i)$, which by the above is bounded in Δ_i by a constant $s_i < \infty$. Hence we also have that

$$s(v) \le \prod_{i=1}^{q} s_i < \infty. \tag{3.19}$$

By the discussion in Section 4 of M (1982), therefore, Case B of that paper covers problem (3.1), since (2.7) and (2.8) of M (1982) hold trivially, while $\int s(v) f_0(v)\,dv < \infty$ follows from (3.19).

Finally, to verify that the hypothesis (2.17) of Theorem 2.1 of M (1982) holds, it must be shown that if $(\lambda, \pi_0, \pi_1, c) \ne (0,0,0,0)$, then

(3.20) $\quad \{v | d(v;\lambda,\pi_0,\pi_1) = c\}$ has Lebesgue measure 0.

From (3.3), however, R_Δ can be rewritten as

$$R_\Delta = \exp\left(-(1-v_0)\sum_{i=1}^{q} \Delta_i/2\right) \prod_{i=1}^{q-1} \exp\left(v_i \sum_{j=i+1}^{q} \Delta_j/2\right) \prod_{i=1}^{q} G_i(\Delta_i v_i/2), \tag{3.21}$$

so that R_Δ is strictly increasing in v_i if $\Delta_i > 0$, $1 \le i \le q$. Since $l_i(v)$ also is strictly increasing in v_i [cf. (3.9)], it follows that if $(\lambda, \pi_0, \pi_1) \ne (0,0,0)$, then

$d(v;\lambda,\pi_0,\pi_1)$ is strictly increasing in some v_i, implying (3.20). If $(\lambda,\pi_0,\pi_1) = (0,0,0)$, then $c \neq 0$ but $d \equiv 0$ for a.e. v, so (3.20) holds in this case as well. □

4. Proof of Theorem 2.1. The acceptance region A of the step-down procedure (2.7) can be expressed as

$$A = \left\{v \in \mathscr{V} \mid v_1 \le c_1(1-u_0),\dots,v_q \le c_q(1-u_{q-1})\right\}, \tag{4.1}$$

where $c_i = a_i/(a_i+1)$. For later convenience, we note here that by (2.11) and (2.13),

$$\alpha_i\left\{\begin{matrix}\le\\=\\>\end{matrix}\right\}\alpha_i^* \Leftrightarrow a_i\left\{\begin{matrix}\ge\\=\\<\end{matrix}\right\}\frac{p_i}{n_i} \Leftrightarrow c_i\left\{\begin{matrix}\ge\\=\\<\end{matrix}\right\}\frac{p_i}{p_i+n_i}. \tag{4.2}$$

If any $c_i = 0$, then A is empty, and if all c_i's are 1, then $A = \mathscr{V}$. Thus to avoid trivialities, we will assume $c_i > 0$ for all i and some $c_i < 1$. We break the proof of Theorem 2.1 into two parts: the necessary condition for procedure (2.7) to be admissible and the sufficient condition.

Part I: *The necessary condition.* Suppose that procedure (2.7) is admissible for problem (3.1), so that

$$A = A' \cap C \tag{4.3a}$$

for some A' and C as in the statement of Theorem 3.1. For $i \ge 1$, let

$$P_i \equiv \left\{v \mid v_i = c_i(1-u_{i-1}),\ v_j < c_j(1-u_{j-1}) \text{ for } j \neq i\right\} \tag{4.3b}$$

denote the ith "face" of the set A, so that

$$\partial A = \operatorname{closure}\left(\bigcup_{i=1}^{q} P_i\right). \tag{4.4}$$

[For $B \subseteq \mathscr{V}$, ∂B denotes the relative boundary of B in $\mathscr{V}$, and closure(B) denotes the relative closure of B in $\mathscr{V}$.] Figures 1, 2, 3 and 4 represent A when $(p_0, q) = (0,2), (1,1), (0,3)$ and $(1,2)$, respectively.

The proof proceeds in steps. We first show that the only time a face P_i can arise from set C in (4.3a) is when $p_0 = 0$ and $i = 1$. Step 2 involves showing that at most one face can arise from the set A'. Finally, in Step 3 we show that for any face P_i to arise from A' it must be that $c_i \ge p_i/(p_i+n_i)$, hence by (4.2), $\alpha_i \le \alpha_i^*$. The technical statements of the results to be verified in these three steps are as follows:

$$P_i \cap \partial C \text{ is empty if either } p_0 > 0 \text{ or } p_0 = 0 \text{ and } i > 1, \tag{4.5}$$

$$P_i \subseteq \partial A' \text{ for at most one } i \text{ with } c_i < 1, \tag{4.6}$$

$$P_i \subseteq \partial A' \text{ implies that } c_i \ge p_i/(p_i+n_i). \tag{4.7}$$

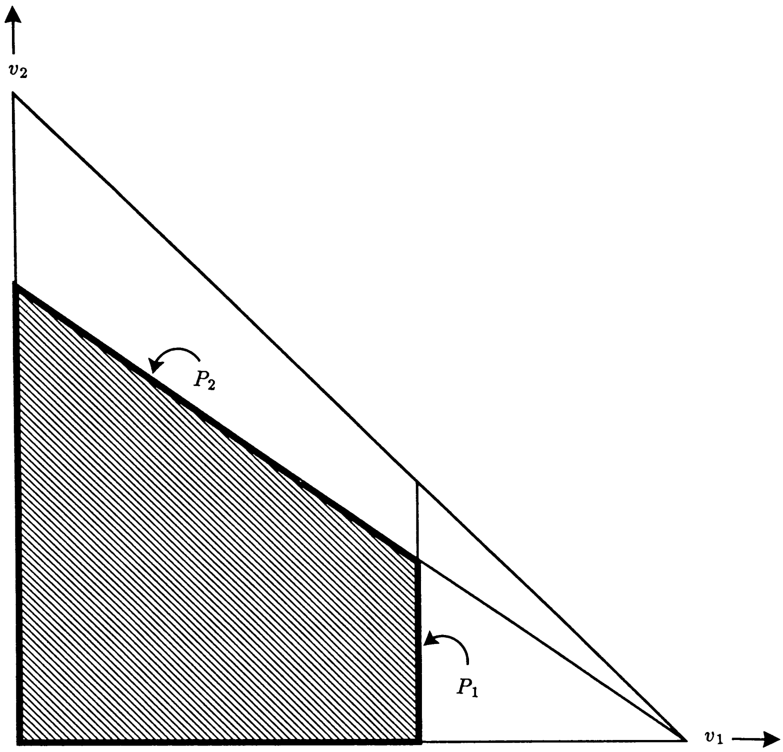

FIG. 1. *The acceptance region A [shaded area; cf.* (4.1)] *for the step-down procedure* (2.7) *when* $q = 2$, $p_0 = 0$. *The hyperplanes* P_i [*cf.* (4.3*b*)] *are indicated.*

Equations (4.5), (4.6) and (4.7) prove that (i) or (ii) of Theorem 2.1 holds, as follows. By (4.3a), $\partial A \subseteq \partial A' \cup \partial C$, hence by (4.4),

$$\bigcup_{i=1}^{q} P_i \subseteq \partial A' \cup \partial C.$$

By (4.5),

$$\bigcup_{i=1}^{q} P_i \subseteq \partial A' \quad \text{if } p_0 > 0; \qquad \bigcup_{i=2}^{q} P_i \subseteq \partial A' \quad \text{if } p_0 = 0.$$

By (4.6), only one P_i in the above unions can be nonempty ($c_i = 1$ implies that P_i is empty), so there exists a k such that

$$(4.8) \quad \bigcup_{i=1}^{q} P_i = P_k, \quad k \geq 1, \text{if } p_0 > 0; \qquad \bigcup_{i=2}^{q} P_i = P_k, \quad k \geq 2, \text{if } p_0 = 0.$$

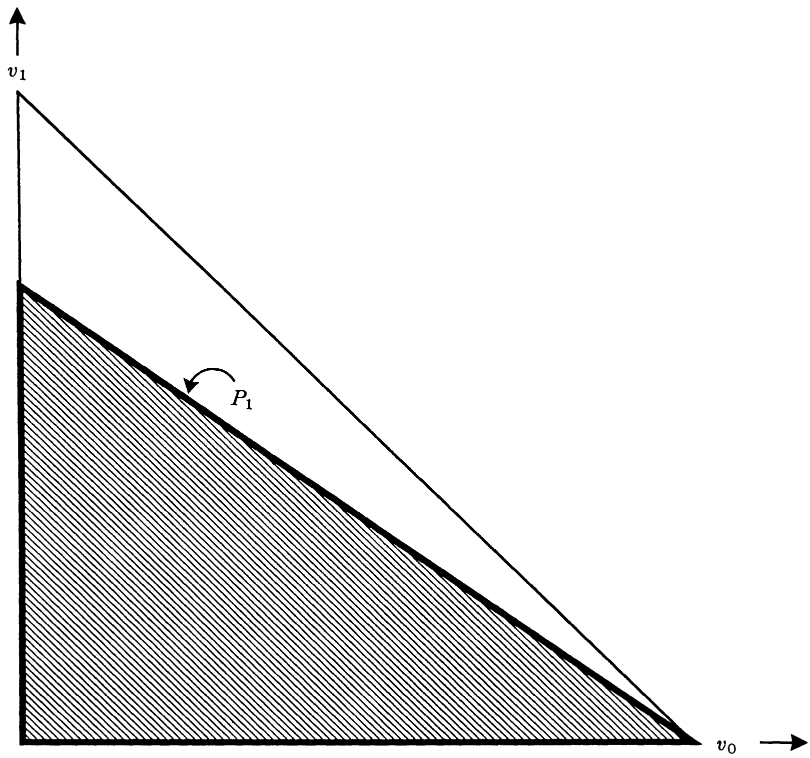

FIG. 2. *The acceptance region A [shaded area; cf.* (4.1)] *for the step-down procedure* (2.7) *when* $q = 1$, $p_0 > 0$. *The hyperplanes* P_i [*cf.* (4.3*b*)] *are indicated.*

Finally, (4.7) shows that for the k in (4.8),

$$c_k \geq p_k/(p_k + n_k) \tag{4.9}$$

and (4.2) shows that (4.9) holds if and only if $\alpha_k \leq \alpha_k^*$. Thus, recalling (4.4) and (4.8), we have that

$$\begin{aligned} \partial A &= \text{closure}(P_k), & \alpha_k \leq \alpha_k^*, \quad \text{if } p_0 > 0, \\ \partial A &= \text{closure}(P_1 \cup P_k), & \alpha_k \leq \alpha_k^*, \quad \text{if } p_0 = 0. \end{aligned} \tag{4.10}$$

Since the boundary of A in (4.1) determines A, condition (i) or (ii) of Theorem 2.1 follows from (4.10).

Now we verify (4.5), (4.6) and (4.7).

STEP 1: PROOF OF (4.5). Suppose that P_i intersects ∂C. Since P_i is relatively open in the plane which contains it, ∂C must intersect P_i at an inner point

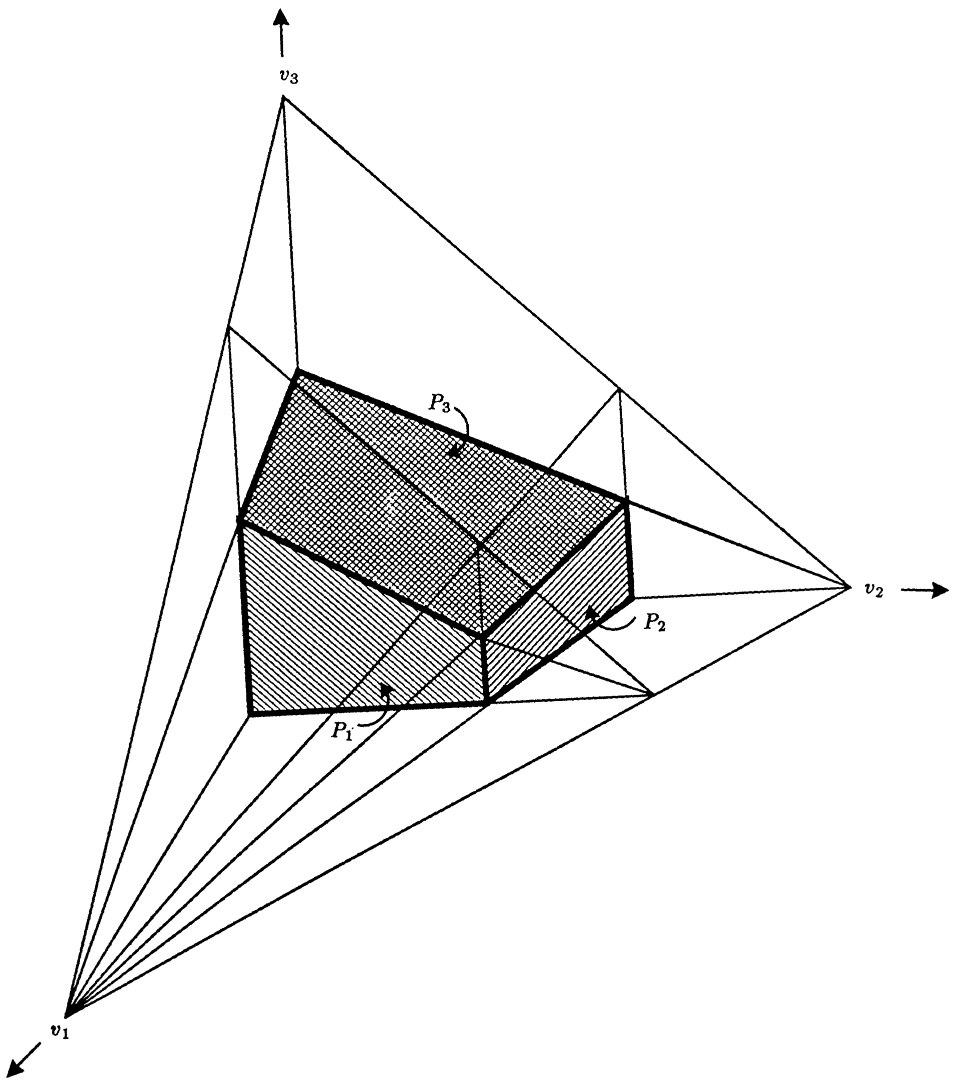

FIG. 3. *The acceptance region A [shaded area; cf.* (4.1)] *for the step-down procedure* (2.7) *when* $q = 3$, $p_0 = 0$. *The hyperplanes* P_i [*cf.* (4.3*b*)] *are indicated.*

of P_i. But since $P_i \subseteq C$ and C is convex, it must be that $P_i \subseteq \partial C$. Thus P_i is in a plane tangent to C. Vectors normal to P_i are proportional to $\nu \equiv (1, 1, \ldots, 1, 1/c_i, 0, \ldots, 0)$, where there are i (resp. $i - 1$) 1's if $p_0 > 0$ ($p_0 = 0$), since P_i is contained in the hyperplane

$$\left\{ v | v_0 + v_1 + \cdots + v_{i-1} + \frac{1}{c_i} v_i = 1 \right\}.$$

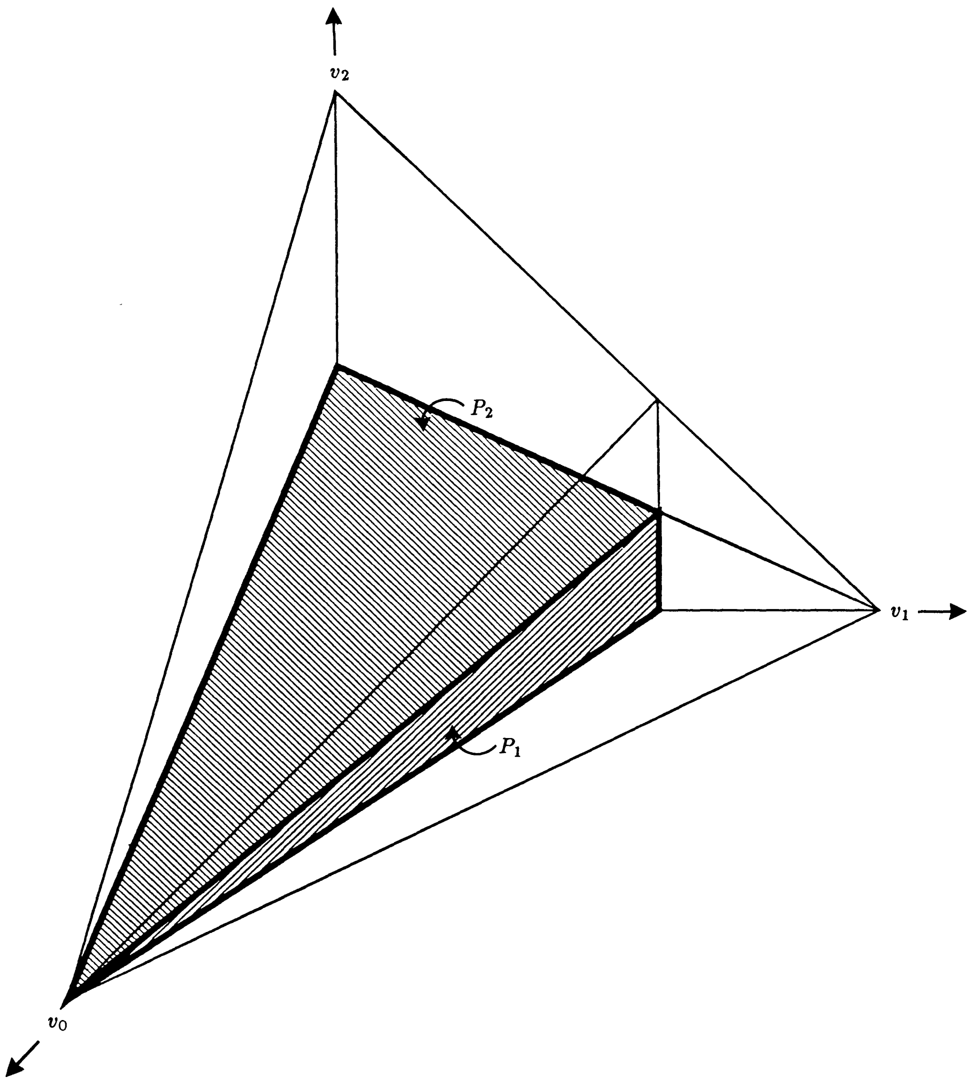

FIG. 4. *The acceptance region A [shaded area; cf.* (4.1)] *of the step-down procedure* (2.7) *when* $q = 2$, $p_0 > 0$. *The hyperplanes* P_i [*cf.* (4.3*b*)] *are indicated.*

The only way that ν can satisfy condition (3.8) is for p_0 to be 0 and i to be 1, so that $\nu = (1/c_1, 0, \ldots, 0)$. Hence (4.5) holds.

STEP 2: PROOF OF (4.6). Let i be the smallest index for which $c_i < 1$ and $P_i \subseteq \partial A'$. Since d is continuous in v, (3.12) shows that

$$d(v; \lambda, \pi_0, \pi_1) = c \quad \text{for } v \in P_i. \tag{4.11}$$

Fix v_k for $k \neq i+1$. From (3.4e) and (3.5), it is clear that $G_i(z)$ is strictly increasing in $z > 0$. Thus from (3.21), R_Δ is strictly increasing in v_{i+1} unless $\Delta_{i+1} = \cdots = \Delta_q = 0$. Also, by (3.9), $\Sigma_1^q \gamma_j l_j$ is strictly increasing in v_{i+1} unless $\gamma_{i+1} = \cdots = \gamma_q = 0$. Thus d in (3.10) is strictly increasing in v_{i+1} unless

$$\begin{aligned} &\Delta_{i+1} = \cdots = \Delta_q = 0 \quad \text{a.e. } [\pi_0] \text{ and } [\pi_1], \text{ and} \\ &\gamma_{i+1} = \cdots = \gamma_q = 0. \end{aligned} \tag{4.12}$$

Since P_i depends only on $(v_0, \dots, v_i)$ in some neighborhood N of a point $v^0 \in P_i$, (4.11) shows that d must be constant in v_{i+1} over $N \cap P_i$. Thus (4.12) must hold. But then d does not depend on $(v_{i+1}, \dots, v_q)$. Thus $P_k \not\subseteq \partial A'$, or P_k is empty, for $k > i$. Hence (4.6) holds.

STEP 3: PROOF OF (4.7). Suppose that $P_i \subseteq \partial A'$ and $c_i < 1$. As in Step 2, (4.11) and (4.12) hold. From (3.4c, d) and (3.9), it is clear that for $j < i$, $R_{\Delta_j}^{(j)}(1 - u_{j-1}, v_j)$ and l_j are nondecreasing in each of $v_0, \dots, v_{j-1}$. Suppose that

$$c_i < p_i/(p_i + n_i). \tag{4.13}$$

Since $v_i = c_i(1 - u_{i-1})$ if $v \in P_i$, from (3.9) and (3.4c) we see that l_i is *strictly* increasing in each of $v_0, \dots, v_{i-1}$ for $v \in P_i$. Also, by Lemma 2.6(a) of MP (1980), $\exp(-z)G_i(z)$ is strictly increasing and $\exp(-z)G_i(zp_i/(p_i + n_i))$ is strictly decreasing in z, hence

$$R_{\Delta_i}^{(i)}(1 - u_{i-1}, c_i(1 - u_{i-1}))$$

is *strictly* increasing in each of $v_0, \dots, v_{i-1}$ if $\Delta_i > 0$. Since (4.12) holds, we see that for $v \in P_i$, d is strictly increasing in each of $v_0, \dots, v_{i-1}$ if (4.13) holds and if

$$(\lambda_i, \pi_0(\{\Delta_i > 0\}), \pi_1(\{\Delta_i > 0\})) \neq (0,0,0). \tag{4.14}$$

By (4.11), we know that d cannot be strictly increasing in $v_0, \dots, v_{i-1}$. Hence (4.13) or (4.14) must be false. Since (4.14) being false would imply that d does not depend on v_i, an impossibility given that $P_i \subseteq \partial A'$ and P_i is nonempty, we have that (4.13) is false, proving (4.8).

Part II: *The sufficient condition.* For any $c_i \geq p_i/(n_i + p_i)$, MP (1980), Section 3.4, exhibit $(\gamma_i, \bar{\pi}_0, \bar{\pi}_1, c)$, where $\gamma_i \geq 0$, $\bar{\pi}_0$ is a finite measure on $(0,1]$ and $\bar{\pi}_1$ is a locally finite measure on $[1, \infty)$, such that

$$\gamma_i l_i + \int_0^1 \left[\left(R_{\Delta_i}^{(i)} - 1 \right) / \Delta_i \right] \bar{\pi}_0(d\Delta_i) + \int_1^\infty R_{\Delta_i}^{(i)} \bar{\pi}_1(d\Delta_i) \leq c \tag{4.15}$$

$$\text{if and only if } v_i \leq c_i(1 - u_{i-1}).$$

Thus taking π_0 and π_1 to be $\bar{\pi}_0$ and $\bar{\pi}_1$, respectively, on $\{\Delta | \Delta_i \geq 0,\ \Delta_k = 0,$

$k \neq i\}$ and zero elsewhere, and λ to be $(0,\dots,0,\gamma_i,0,\dots,0)$, we obtain from (3.10), (3.12) and (4.15) that the corresponding set A' is given by

$$A' = \{v | v_i \le c_i(1 - u_{i-1})\}. \tag{4.16}$$

Suppose that (i) of Theorem 2.1 holds. Then the test function φ determined by the step-down procedure (2.7) is of the form (3.11) with A' as in (4.16) and $C = \mathscr{V}$, hence is admissible by Theorem 3.1. If (ii) holds, then φ is of the form (3.11) with A' as in (4.16) and $C = \{v | v \le c_1\}$. But $C \in \mathscr{C}$ since $v_1 = u_1$ if $p_0 = 0$, so again φ is admissible. □

5. Nonmonotonicity of the power function of the RBR procedure. To motivate the result of this section, we first extend the argument of Subbaiah and Mudholkar (1978a, b) regarding nonmonotonicity of power from a two-step RBR procedure with $p_0 = 0$ to the general RBR procedure (2.7). In the procedure (2.7), first assume that $\alpha_i = 0$ (i.e., $a_i = \infty$) for $i \neq l$ where $1 < l \le q$, i.e., every step except step l is omitted. Then, by (2.8) and (2.8a), the power of this procedure is given by

$$\begin{aligned}\pi(\Delta_1,\dots,\Delta_q) &\equiv P_{\Delta_1,\dots,\Delta_l}\{Y_l > a_l\}\\ &= E_{\Delta_1,\dots,\Delta_{l-1}}\left[P_{\Delta_l}\{Y_l > a_l | Y_0, Y_1,\dots,Y_{l-1}\}\right]\\ &= E_{\Delta_1,\dots,\Delta_{l-1}}\left[P_{\Delta_l}\{Y_l > a_l | T^2_{l-1}\}\right].\end{aligned}$$

By the strict monotone likelihood ratio property of the (conditional) noncentral F density of Y_l given T^2_{l-1}, for fixed $\Delta_l > 0$ the conditional probability is strictly decreasing in T^2_{l-1}. Since (unconditionally),

$$T^2_{l-1} \sim \chi^2_{p_0 + \cdots + p_{l-1}}(\Delta_1 + \cdots + \Delta_{l-1})/\chi^2_{n_{l-1}},$$

$\pi(\Delta_1,\dots,\Delta_q)$ is strictly decreasing in $\Delta_1,\dots,\Delta_{l-1}$ for fixed $\Delta_l,\dots,\Delta_q$. Thus, by the continuity of the power function of the general RBR procedure as $\alpha_i \to 0$ for $i \neq l$, this power function cannot be monotonically increasing in $\Delta_1,\dots,\Delta_{l-1}$ for sufficiently small α_i, $i \neq l$. Note that this argument does not specify how small the α_i, $i \neq l$, must be for nonmonotonicity to occur.

Here we present a more specific result for the case $p_0 = 0$. Namely, assume that $0 < \alpha < 1$ [cf. (2.12)], $\alpha_i > 0$ for $1 \le i \le l$ and $\alpha_k < \alpha_k^*$, where $1 \le k < l \le q$ and α_k^* is defined by (2.13). Set $\Delta_i = 0$ for $i \neq k, l$ and define

$$\begin{aligned}\beta(\Delta_k, \Delta_l) &= P_{\Delta_k,\Delta_l}\{\text{test (2.7) accepts } H_0\}\\ &= P_{\Delta_k,\Delta_l}\{Y_1 \le a_1,\dots,Y_q \le a_q\}.\end{aligned} \tag{5.1}$$

Then it may be shown [cf. Marden and Perlman (1988)] that

$$\frac{\partial}{\partial \Delta_k} \beta(\Delta_k, \Delta_l)\bigg|_{\Delta_k = 0} > 0 \quad \text{for sufficiently large } \Delta_l. \tag{5.2}$$

This shows that for sufficiently small Δ_k and large Δ_l, $\beta(\Delta_k, \Delta_l)$ ($\equiv 1 -$ power) is strictly increasing, hence the power is strictly decreasing, as a function of Δ_k.

In particular, for the case $l = q$, this result implies that if $0 < \alpha_i < \alpha_i^*$ for $1 \le i < q$ and $0 < \alpha_q < 1$, then for sufficiently large Δ_q and sufficiently small Δ_i, $1 \le i < q$, the power of the RBR procedure (2.7) (with $p_0 = 0$) is strictly decreasing in each Δ_i, $1 \le i < q$.

6. Admissible step-down procedures. We use the general term step-down procedure to refer to any G-invariant test for (2.2), i.e., any test depending on (X, S) only through $(Y_0, Y_1, \dots, Y_q)$. Consider the following alternative step-down procedure for the testing problem (2.2):

$$\text{accept } H_0 \text{ iff } T_i^2 \le b_i, \qquad 1 \le i \le q, \tag{6.1}$$

or, equivalently,

$$\text{accept } H_0 \text{ iff } 1 + T_i^2 \equiv \prod_{j=0}^{i} (1 + Y_j) \le 1 + b_i, \qquad 1 \le i \le q, \tag{6.1$'$}$$

where $b_1, \dots, b_q$ are (nonunique) constants determined to achieve overall significance level α. [Compare (6.1) and (6.1′) to the RBR procedure (2.7).] Because t_i^2 is an increasing function of u_i for $i = 1, \dots, q$ [see (3.4c)], the acceptance region (6.1) is convex and nonincreasing in $(u_1, \dots, u_q)$, hence belongs to the class $\mathscr{C}$ defined in Section 3. By Theorem 3.1, therefore, the acceptance region (6.1) determines an admissible test for problem (3.1), hence is admissible among G-invariant tests for problem (2.2).

In fact, the test described by (6.1) is admissible among *all* tests for problem (2.2). To see this, slightly extend the argument on page 50 of MP (1980) to show that for each $i = 1, \dots, q$, the acceptance region $\{(X, S) | T_i^2 \le b_i\}$ satisfies the condition of the main theorem of Stein (1956), hence so does the intersection of these regions, which therefore determines an admissible test for problem (2.2). [For the case $p_0 = 0$, this result is a special case of a theorem of Schwartz (1967).]

Whether in fact (6.1) determines a test that is practical as well as admissible depends on the ease with which the critical values b_i can be determined or approximated—this question remains to be investigated. Of course, by setting $b_1 = \cdots = b_{q-1} = \infty$, the test (6.1) reduces to that based on the overall T^2 statistic T_q^2, which has an F distribution. See Section 4 of MP (1980) for a comparison of the power function of this test with that of the test based on

$$\frac{T_q^2 - T_0^2}{1 + T_0^2} \equiv \prod_{i=1}^{q} (1 + Y_i) - 1, \tag{6.2}$$

which is the likelihood ratio test for problem (2.2).

Any (proper or improper) Bayes test for problem (3.1) has the following form:

$$\text{accept } H_0 \text{ iff } \int_{H_A} R_\Delta \pi(d\Delta) \le c, \tag{6.3}$$

where R_Δ is the likelihood ratio defined in (3.2), π is a (finite or locally finite) measure on the orthant H_A in (3.1) and c is a nonnegative constant. Let ν_i denote the gamma measure on $(0, \infty)$ given by

$$\nu_i(d\Delta_i) = \Delta^{p_i/2-1} e^{-s_i \Delta_i} \, d\Delta_i, \tag{6.4}$$

where $s_i \ge 0$. When

$$\pi(d\Delta) = \prod_{i=1}^{q} \nu_i(d\Delta_i) \tag{6.5}$$

it follows from (3.3), (3.4c) and (3.6) above and from (3.19) of MP (1980) that

$$\begin{aligned} \int_{H_A} R_\Delta \pi(d\Delta) &= (\text{const}) \cdot \prod_{i=1}^{q} (2s_i + 1 - u_{i-1})^{n_i/2} (2s_i + 1 - u_i)^{-(p_i+n_i)/2} \\ &= (\text{const}) \cdot \prod_{i=1}^{q} \left(2s_i + \frac{1}{1 + t_{i-1}^2}\right)^{n_i/2} \left(2s_i + \frac{1}{1 + t_i^2}\right)^{-(p_i+n_i)/2}. \end{aligned} \tag{6.6}$$

When

$$\pi(d\Delta) = \sum_{i=1}^{q} \eta_i \delta_i(\Delta) \nu_i(d\Delta_i), \tag{6.7}$$

where

$$\delta_i(\Delta) = \begin{cases} 1, & \Delta_i > 0 \text{ and } \Delta_j = 0 \text{ for } j \ne i, \\ 0, & \text{otherwise}, \end{cases}$$

and $\eta_1, \dots, \eta_q$ are nonnegative constants, then

$$\int_{H_A} R_\Delta \pi(d\Delta) = \sum_{i=1}^{q} h_i \eta_i \left(2s_i + \frac{1}{1 + t_{i-1}^2}\right)^{n_i/2} \left(2s_i + \frac{1}{1 + t_i^2}\right)^{-(p_i+n_i)/2} \tag{6.8}$$

for some positive constants $h_1, \dots, h_q$. By Theorem 3.1, the (proper or improper) Bayes tests based on the statistics in (6.6) and (6.8) with t_i^2 replaced by T_i^2 are admissible for problem (3.1), hence are admissible among G-invariant tests for problem (2.2). In particular, when $s_1 = \cdots = s_q = 0$, the statistics in (6.6) and (6.8) assume the simplified forms [apply (3.4b)]

$$\prod_{i=1}^{q} (1 + Y_i)^{n_i/2}(1 + T_i^2)^{p_i/2} = (1 + Y_0)^{\tilde{p}_1/2} \prod_{i=1}^{q} (1 + Y_i)^{(n_i+\tilde{p}_i)/2}, \tag{6.9}$$

$$\begin{aligned} &\sum_{i=1}^{q} h_i \eta_i (1 + Y_i)^{n_i/2}(1 - T_i^2)^{p_i/2} \\ &\quad = \sum_{i=1}^{q} h_i \eta_i (1 + Y_i)^{n_i/2} \prod_{j=0}^{i} (1 + Y_j)^{p_i/2}, \end{aligned} \tag{6.10}$$

where $\tilde{p}_i = p_i + \cdots + p_q$. Since $n_i + \tilde{p}_i$ decreases with i, the statistic (6.9) assigns decreasing weights to $1 + Y_1, \ldots, 1 + Y_q$, but otherwise resembles $1 + T_q^2 \equiv \prod_{i=0}^{q}(1 + Y_i)$. [When $p_0 = 0$, Y_0 is absent in (6.9) and (6.10).]

Mudholkar and Subbaiah (1980b) proposed a modified step-down procedure for the testing problem (2.2). They suggested that Fisher's method for combining independent p-values be applied to the p-values attained by each of the component tests in (2.5), $i = 1, \ldots, q$. (Recall that $Y_1, \ldots, Y_q$ are independent under H_0.) Marden and Perlman (1989) have shown that this procedure is admissible when $p_0 = 0$ and $p_1 = \cdots = p_q = 2$, but inadmissible when $p_0 \geq 0$ and $\min\{p_1, \ldots, p_q\} = 1$. □

REFERENCES

ABRAMOWITZ, M. and STEGUN, I. A. (1964). *Handbook of Mathematical Functions.* Dover, New York.

DEMPSTER, A. P. (1963). Multivariate theory for general stepwise methods. *Ann. Math. Statist.* **34** 873–883.

GIRI, N. (1968). Locally and asymptotically minimax tests of a multivariate problem. *Ann. Math. Statist.* **39** 171–178.

GIRI, N., KIEFER, J. and STEIN, C. (1963). Minimax character of Hotelling's T^2 test in the simplest case. *Ann. Math. Statist.* **34** 1524–1535.

JENSEN, D. R. and FOUTZ, R. V. (1980). On resolving statistical hypotheses. *J. Statist. Comput. Simulation* **10** 309–312.

MARDEN, J. I. (1982). Minimal complete classes of tests of hypotheses with multivariate one-sided alternatives. *Ann. Statist.* **10** 962–970.

MARDEN, J. I. and PERLMAN, M. D. (1980). Invariant tests for means with covariates. *Ann. Statist.* **8** 25–63.

MARDEN, J. I. and PERLMAN, M. D. (1988). On the inadmissibility of step-down procedures for the Hotelling T^2 problem. Technical Report No. 122, Dept. Statist., Univ. of Washington.

MARDEN, J. I. and PERLMAN, M. D. (1989). On the inadmissibility of the modified step-down test based on Fisher's method for combining independent p-values. In *Contributions to Probability and Statistics: Essays in Honor of Ingram Olkin* (L. Gleser, M. Perlman, S. Press, A. Sampson, eds.) 472–485. Springer, New York.

MUDHOLKAR, G. S. and SUBBAIAH, P. (1980a). A review of step-down procedures for multivariate analysis of variance. In *Multivariate Statistical Analysis* (R. P. Gupta, ed.) 161–178. North-Holland, Amsterdam. (The final 22 references in the original manuscript were omitted inadvertently from the published paper.)

MUDHOLKAR, G. S. and SUBBAIAH, P. (1980b). Testing significance of a mean vector—a possible alternative to Hotelling's T^2. *Ann. Inst. Statist. Math.* **32A** 43–52.

MUDHOLKAR, G. S. and SUBBAIAH, P. (1987). Some simple optimal tests in multivariate analysis. In *Advances in Multivariate Statistical Analysis* (A. K. Gupta, ed.) 253–275. Reidel, Dordrecht.

RAO, C. R. (1948). Tests of significance in multivariate analysis. *Biometrika* **35** 58–79.

ROY, J. (1958). Step-down procedure in multivariate analysis. *Ann. Math. Statist.* **29** 1177–1187.

ROY, S. N. and BARGMANN, R. E. (1958). Tests of multiple independence and the associated confidence bounds. *Ann. Math. Statist.* **29** 491–503.

SCHWARTZ, R. E. (1967). Admissible tests in multivariate analysis of variance. *Ann. Math. Statist.* **38** 698–710.

STEIN, C. (1956). The admissibility of Hotelling's T^2-test. *Ann. Math. Statist.* **27** 616–623.

SUBBAIAH, P. and MUDHOLKAR, G. S. (1978a). Inferences concerning a mean vector when the variables are grouped into subsets. *Biometrical J.* **20** 15–24.

SUBBAIAH, P. and MUDHOLKAR, G. S. (1978b). A comparison of two tests for the significance of a mean vector. *J. Amer. Statist. Assoc.* **73** 414–418.

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