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ELLSBERG REVISITED: A NEW LOOK AT COMPARATIVE PROBABILITY

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Suppose you are given the opportunity of guessing whether it will snow or not in Chicago next Christmas. If you guess correctly, you win $1,000; if not, you win nothing. Which event, *snow* or *no snow*, would you bet on?

It is widely accepted among decision theorists that your answer reveals which of the two events you deem more probable. Furthermore, if your choices over a field of events obey certain postulates of coherency and consistency, then there is a probability measure P on the field that reflects your choices: you regard A as more probable than B if, and only if, $P(A) > P(B)$.

Numerous experiments have shown that people often violate those postulates, so they lack full descriptive validity. Moreover, because of systematic and persistent violations of one of the postulates—an independence axiom—the theory has been questioned on its normative adequacy as a guide to well-reasoned judgments and choices.

The purpose of the present paper is to examine a weaker set of postulates that avoids the independence axiom as well as the usual assumption of fully transitive preferences. Despite this weakening, the assumptions imply that there is a unique normalized functional ρ on pairs of events that preserves choices in the sense that A is more probable than B if, and only if, $\rho(A, B) > 0$. The functional ρ has several nice mathematical properties, including "conditional additivity," that reflect vestiges of numerical probability, and it is related to the conventional measure P by $\rho(A, B) = P(A) - P(B)$ when the omitted independence axiom is coupled to the other postulates.

1. Introduction. The aim of this paper is to present an axiomatization of comparative subjective probability that is based on comparisons between simple decisions, in the manner of Ramsey (1931), de Finetti (1937) and Savage (1954); uses calibration by canonical lotteries or extraneous scaling probabilities, as in Anscombe and Aumann (1963), Pratt, Raiffa and Schlaifer (1964), and Fishburn (1967); resolves Ellsberg's (1961) paradoxes; employs only axioms that are defensible postulates of consistency and coherence, but does not presume that binary comparisons are transitive or that they satisfy conventional additivity (linearity, independence) axioms; accounts for the possibility that comparisons between events may not be separable in the events; and yields an exact numerical representation of comparative probability that is "conditionally additive." The axioms might be viewed as relaxations of those used previously by Anscombe and Aumann, or Pratt, Raiffa and Schlaifer that, because they invoke neither transitivity nor additivity, accommodate judgmental behaviors that violate these traditional postulates, yet nevertheless admit a precise numerical measure that preserves the binary relation of comparative probability.

In contrast to the conventional representation

$$A \succ^* B \Leftrightarrow P(A) > P(B), \tag{1}$$

where $\succ^*$ (" is more probable than") is a binary relation on an algebra $\mathscr{A}$ of events A, B, $\cdots$ and P is a finitely-additive probability measure on $\mathscr{A}$, the representation for the new

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axiomatization yields

$$A \succ^* B \Leftrightarrow \rho(A, B) > 0, \tag{2}$$

where ρ is a real-valued function on $\mathscr{A} \times \mathscr{A}$ that is

$$\textit{monotonic}: A \supseteq B \Rightarrow \rho(A, B) \geq 0,$$

$$\textit{skew-symmetric}: \rho(A, B) = -\rho(B, A),$$

$$\textit{conditionally additive}: A \cap B = \varnothing \Rightarrow$$

$$\rho(A \cup B, C) + \rho(\varnothing, C) = \rho(A, C) + \rho(B, C), \tag{3}$$

and is normalized against the universal event Ω and the empty event $\varnothing$ by

$$\rho(\Omega, \varnothing) = 1.$$

When $C = \varnothing$ in (3), $\rho(\varnothing, C)$ vanishes and $\rho(\cdot, \varnothing)$ on $\mathscr{A}$ is a conventional probability measure.

Note also that the new representation implies $\rho(A, C) \geq \rho(B, C)$ when $A \supset B \supset C$ since, with $D = A \backslash B$,

$$\begin{aligned}\rho(A, C) - \rho(B, C) &= \rho(D \cup B, C) - \rho(B, C)\\ &= \rho(D, C) + \rho(B, C) - \rho(\varnothing, C) - \rho(B, C)\\ &= \rho(D \cup C, C) \geq 0\end{aligned}$$

by (3), skew-symmetry, and monotonicity. Other implications, including the fact that the absolute value of $\rho(A, B)$ never exceeds 1, are discussed in Fishburn (1983).

The essential difference between the new representation and (1) is the nonseparability of A and B in (2). This reflects the position that a comparison between two events may involve an unbreakable linkage or association between them. Or, to put it differently, the ways one views A within the comparisons of A versus B and A versus C may be quite different, especially if B and C are qualitatively unlike. This will be illustrated shortly.

The rest of the paper is organized as follows. The next section reviews Ellsberg's two main examples, observes how (1) fails there, then shows how (3) offers a plausible model for the predominant judgments in the examples. Section 3 develops the structures used in our theory and summarizes its axioms. Section 4 presents the main representation-uniqueness theorem, which includes the specialized aspects of (2) and (3). The final section shows how the representation is affected when the assumptions it avoided, namely transitivity and linearity, are restored. For example, the addition of a simple linearity axiom implies that ρ can be decomposed as

$$\rho(A, B) = P(A) - P(B),$$

so that (1) then follows from (2).

It should be noted that (1) and similar representations of subjective probability have been axiomatized directly in terms of $\succ^*$ on $\mathscr{A}$ without recourse to the calibration device used here. Kraft, Pratt and Seidenberg (1959) give necessary and sufficient conditions for (1) when $\mathscr{A}$ is finite (see also Scott, 1964), and Suppes and Zanotti (1976) do the same thing for arbitrary $\mathscr{A}$ under an extended structure. Other axiom systems sufficient for (1) and close relatives are provided by Koopman (1940, 1941), Savage (1954), Villegas (1964, 1967), Luce (1967, 1968), Fishburn (1969, 1975), Fine (1971, 1973), Niiniluoto (1972), Roberts (1973), Narens (1974) and Wakker (1981). The lottery-based theories of Pratt-Raiffa-Schlaifer and others are reviewed within the setting of subjective expected utility in Fishburn (1981).

Finally, since the conception of personal probability set forth by Ramsey, de Finetti, and Savage was designed in part for a normative theory of decision making in the face of uncertainty, it is natural to wonder how the latter would be changed by the revisions proposed here. I regard this as a subject for further investigation and shall defer comment. However, recent analyses of nonlinear utility theories for risky decision making by Chew and MacCrimmon (1979), Chew (1983), Machina (1982) and Fishburn (1982c) indicate possible directions this work might take.

2. Ellsberg's examples. Ellsberg's classic paper (1961) on what can go wrong with the conception of subjective probability set forth by Ramsey, de Finetti, and Savage used two main examples to illustrate the difficulties. I shall present these in slightly modified form, then show how the new conception accommodates their supposedly contradictory judgments. I shall not try to summarize Ellsberg's own analysis of the matter since it is far-ranging and deserves to be read in the original, as does Raiffa's comment (1961).

EXAMPLE 1. An urn contains 100 balls: 25 are marked $R1$, 25 are marked $B1$, and the other 50 are marked $R2$ and $B2$ in an unknown proportion. An individual is to compare options concerning the identity of a ball to be chosen at random. We suppose that he has no reason to believe that the $R2/B2$ proportion is more in favor of one designation than the other. Four options that correspond directly to the four designations are

r_1: win \$1000 if chosen ball is $R1$, nothing otherwise;
b_1: win \$1000 if chosen ball is $B1$, nothing otherwise;
r_2: win \$1000 if chosen ball is $R2$, nothing otherwise;
b_2: win \$1000 if chosen ball is $B2$, nothing otherwise.

Following the usual decision-oriented conception of subjective probability, we assume that a definite preference for one option over another means that the individual regards the event of the first that returns \$1000 as more probable than the event of the second that returns \$1000. Similarly, if he is indifferent between two options, he regards their paying events as equally probable.

As before, $\succ^*$ indicates "is more probable than," and we shall let $\sim^*$ denote "is equally probable as." It is not unusual for a person to be indifferent between r_1 and b_1, and between r_2 and b_2, and also to prefer r_1 to r_2 and b_1 to b_2. Suppose this is true of our individual, so that

$$R1 \sim^* B1, \quad R2 \sim^* B2, \quad R1 \succ^* R2, \quad B1 \succ^* B2.$$

The qualitative differences suggested earlier arise here from the specificity of $R1$ and $B1$ (25 balls each) versus the ambiguity of $R2$ and $B2$ (50 balls total, in unknown proportion), as noted by Ellsberg (1961) and Sherman (1974), among others. The comparison of $R1$ and $B1$ involves two quite specific events, whereas the comparison of $R1$ and $R2$ pits a specific against an ambiguous event.

Since the preceding list of $\sim^*$ and $\succ^*$ statements is consistent with (1), consider a further comparison between

1: win \$1000 if chosen ball is $R1$ or $B1$, nothing otherwise;
2: win \$1000 if chosen ball is $R2$ or $B2$, nothing otherwise.

Since these involve the same number of balls, we presume they are indifferent, so that

$$R1 \cup B1 \sim^* R2 \cup B2.$$

Although there is no inherent contradiction when this is added to the preceding list, the final three statements violate (1), which would require $P(R1) > P(R2)$, $P(B1) > P(B2)$ and $P(R2) + P(B2) = P(R1) + P(B1)$. Addition and cancellations leave $0 > 0$. Consequently, if P exists at all, it cannot be additive.

EXAMPLE 2. In this example the urn contains 90 balls: 30 are red; the other 60 are black and yellow in unknown proportion. The relevant options are

r: win \$1000 if Red is chosen, nothing otherwise;
b: win \$1000 if Black is chosen, nothing otherwise;
ry: win \$1000 if Red or Yellow is chosen, nothing otherwise;
by: win \$1000 if Black or Yellow is chosen, nothing otherwise.

Many individuals prefer *r* to *b* and *by* to *ry*, both of which suggest a preference for greater specificity. These give $R \succ^* B$ and $B \cup Y \succ^* R \cup Y$, which are inconsistent with (1) since it would require $P(R) > P(B)$ and $P(B) + P(Y) > P(R) + P(Y)$. Ellsberg notes that this example provides a direct violation of Savage's independence axiom or so-called sure-thing principle.

Other examples of a related nature have been discussed by Allais (1953), MacCrimmon (1968), and Slovic and Tversky (1974), among others. Collectively, they demonstrate persistent and systematic violations of the type of independence and linearity axioms needed for (1) and related expressions in expected utility theory. It is also possible to construct situations in which transitivity is often violated, as shown by Tversky (1969).

To show how the representation of (2) and (3) provides a resolution of Ellsberg's examples, we note the following corollary of that representation: If $A_1, \cdots, A_n$ are pairwise disjoint events, and similarly for $B_1, \cdots, B_m$, then

$$\rho(\cup_{i=1}^n A_i, \cup_{j=1}^m B_j) = \sum_{i=1}^n \sum_{j=1}^m \rho(A_i, B_j) - (m-1) \sum_{i=1}^n \rho(A_i, \varnothing) + (n-1) \sum_{j=1}^m \rho(B_j, \varnothing). \tag{4}$$

This will be proved during the proof of Theorem 1 in Section 4. Because of (4), we need only specify ρ for the most elementary pairs of events at issue.

Possible specifications for Examples 1 and 2 are shown in Table 1. The entry in the row labeled A and the column labeled B is $\rho(A, B)$. By prior specification, the matrices are skew-symmetric (we omit the last row, for $\varnothing$), and the entries in the $\varnothing$ column are nonnegative and sum to unity.

According to (2) and (4), the matrix for Example 1 gives $R1 \sim^* B1$, $R2 \sim^* B2$, $R1 \succ^* R2$, $R1 \succ^* B2$, $B1 \succ^* R2$, $B1 \succ^* B2$, $R1 \cup B1 \sim^* R2 \cup B2$, $R1 \cup B2 \sim^* B1 \cup R2$ and $R1 \cup R2 \sim^* B1 \cup B2$. These include the given statements listed earlier in the example. The computations involving (4) are

$$\begin{aligned}\rho(R1 \cup B1, R2 \cup B2) &= \rho(R1, R2) + \rho(R1, B2) + \rho(B1, R2) + \rho(B1, B2) \\ &\quad - \rho(R1, \varnothing) - \rho(B1, \varnothing) + \rho(R2, \varnothing) + \rho(B2, \varnothing) \\ &= 4(0.03) - 2(0.28) + 2(0.22) = 0,\end{aligned}$$

$$\begin{aligned}\rho(R1 \cup B2, B1 \cup R2) &= \rho(R1, B1) + \rho(R1, R2) + \rho(B2, B1) + \rho(B2, R2) \\ &\quad - \rho(R1, \varnothing) - \rho(B2, \varnothing) + \rho(B1, \varnothing) + \rho(R2, \varnothing) \\ &= 0 + 0.03 - 0.03 + 0 - 0.28 - 0.22 + 0.28 + 0.22 = 0,\end{aligned}$$

and similarly for $\rho(R1 \cup R2, B1 \cup B2)$.

The ρ matrix for Example 2 gives $R \succ^* B$, $R \succ^* Y$, $B \sim^* Y$, $R \cup B \sim^* R \cup Y$, $B \cup Y \succ^* R \cup Y$, and $B \cup Y \succ^* R \cup B$, which are consistent with the judgments in the example.

We conclude this section with a further remark on Example 2 that is suggested by Raiffa's analysis (1961). Suppose that, instead of the simple options of the example, namely *r*, *b*, *ry* and *by*, we consider a comparison between the following even-chance gambles on pairs of options:

I. *r* with probability ½ or *by* with probability ½;
II. *b* with probability ½ or *ry* with probability ½.

TABLE 1
Values of $\rho(A, B)$.

		R1	**B1**	**R2**	**B2**	∅
EXAMPLE 1	**R1**	0	0	.03	.03	.28
	B1	0	0	.03	.03	.28
	R2	−.03	−.03	0	0	.22
	B2	−.03	−.03	0	0	.22
						1.00

		R	**B**	**Y**	∅
EXAMPLE 2	**R**	0	.02	.02	.38
	B	−.02	0	0	.31
	Y	−.02	0	0	.31
					1.00

Since r is preferred to b, and by is preferred to ry, a conventional dominance axiom says that gamble I ought to be preferred to gamble II. However, the gambles are equivalent in the sense that, regardless of which ball might be drawn, each has probability ½ of yielding the $1000 prize. Therefore, another axiom, for equivalence, suggests that the only sensible thing is indifference between gambles I and II.

Consequently, so long as we stick with the initial preference of r over b and by over ry, either the conventional dominance axiom or the equivalence-indifference axiom must be avoided. In fact, the axioms proposed in the next section include equivalence-indifference but not conventional dominance—which is closely related to typical linearity axioms—and thus our theory will require indifference between gambles I and II in the preceding illustration.

3. Axioms. Let Ω be a nonempty set, $\mathscr{A}$ a Boolean algebra of subsets of Ω, and G the set of gambles on $\mathscr{A}$:

$$G = \{f : \mathscr{A} \rightarrow [0, 1] : f(A) > 0 \text{ for no more than a finite number of } A \in \mathscr{A}, \ \Sigma_{\mathscr{A}} f(A) = 1\}.$$

We interpret states $\omega \in \Omega$ and events $A \in \mathscr{A}$ in the sense of Savage (1954). We interpret gamble $f \in G$ as an option that yields a prize or valued object V with probability $\Sigma\{f(A) : \omega \in \mathscr{A}\}$ when state ω obtains (is the true state) and yields nothing with complementary probability $1 - \Sigma\{f(A) : \omega \in A\}$ when ω obtains, for each $\omega \in \Omega$. Alternatively, f can be viewed as follows: first, an $A \in \mathscr{A}$ is "chosen" according to the probabilities $f(A)$; given the chosen A, f yields V if the true state is in A and yields nothing otherwise. Readers who are disturbed by the implied use of numerical probability so directly when our objective is to axiomatize a version of subjective probability, might wish to view the numerical values as areas of the unit square—as in the canonical lotteries of Pratt, Raiffa and Schlaifer (1964), or as probabilities for events associated with a random mechanism (roulette wheel) about which there would be no disagreement.

The set of gambles is convex since $\lambda f + (1 - \lambda)g \in G$ when $f, g \in G$ and $\lambda \in [0, 1]$. The degenerate gamble that assigns probability 1 to $A \in \mathscr{A}$ will be denoted f_A. According to previous discussion, we shall interpret A as subjectively more probable than B when f_A is preferred to f_B. The following convex combinations of degenerate gambles have the indicated interpretations according to the preceding paragraph:

$\frac{1}{2} f_A + \frac{1}{2} f_\Omega$: if A obtains, get V with pr. 1,

if A does not obtain, get V with pr. ½;

$$\lambda f_A + (1-\lambda) f_B\text{: if } A\backslash B \text{ obtains, get } V \text{ with pr. } \lambda,$$
$$\text{if } B\backslash A \text{ obtains, get } V \text{ with pr. } 1-\lambda,$$
$$\text{if } A \cap B \text{ obtains, get } V \text{ with pr. } 1,$$
$$\text{if } \Omega\backslash(A \cup B) \text{ obtains, get } V \text{ with pr. } 0.$$

Our axioms concern the behavior of a binary relation $\succ$ ("is preferred to") and its induced relations $\backsim$ ("is indifferent to") and $\succsim$ ("is preferred or indifferent to") on G, where

$$f \backsim g \quad \text{if not} \quad (f \succ g) \quad \text{and not} \quad (g \succ f)$$
$$f \succsim g \quad \text{if} \quad f \succ g \quad \text{or} \quad f \backsim g.$$

There are six axioms. The first three, which are the same as the basic axioms for nonlinear measurable utility in Fishburn (1982a), take no special regard of $\mathscr{A}$ as a set of events. The next two axioms, which are universally adopted in the events context, lead to the subjective probability interpretation. The final axiom is the equivalence-indifference axiom. For convenience, we omit the universal quantifiers, noting here that the axioms apply to all $f, g, h \in G$, all λ strictly between 0 and 1, and all $A, B \in \mathscr{A}$.

AXIOM 1. *If $f \succ g$ and $g \succ h$ then $g \backsim \alpha f + (1-\alpha) h$ for at least one $\alpha \in (0, 1)$.*

AXIOM 2. *If $f \succ g$ and $f \succsim h$, then $f \succ \lambda g + (1-\lambda) h$; if $g \succ f$ and $h \succsim f$, then $\lambda g + (1-\lambda) h \succ f$; if $f \backsim g$ and $f \backsim h$, then $f \backsim \lambda g + (1-\lambda) h$.*

AXIOM 3. *If $f \succ g$, $g \succ h$, $f \succ h$ and $g \backsim \frac{1}{2} f + \frac{1}{2} h$, then $\lambda f + (1-\lambda) h \backsim \frac{1}{2} f + \frac{1}{2} g$ if and only if $\lambda h + (1-\lambda) f \backsim \frac{1}{2} h + \frac{1}{2} g$.*

AXIOM 4. $f_\Omega \succ f_\varnothing$.

AXIOM 5. *If $A \supseteq B$ then $f_A \succsim f_B$.*

AXIOM 6. *If $\Sigma\{f(C) : \omega \in C\} = \Sigma\{g(C) : \omega \in C\}$ for all $\omega \in \Omega$, then $f \backsim g$.*

Axioms 4 (nontriviality) and 5 (monotonicity) require no comment. Axiom 6 asserts that if f and g have the same probability of yielding prize V in every possible state, then f and g will be indifferent. It is not quite transparent since equivalent f and g could look different on the surface, but it is a compelling consistency principle.

The first three axioms are, respectively, conditions of continuity, dominance, and symmetry. Axiom 1, familiar from expected utility, says that some nontrivial convex combination of f and h will be indifferent to any third gamble that lies between them by $\succ$. Axiom 2 retains some of the flavor of the conventional dominance axiom mentioned at the end of the preceding section, but it is limited by having one gamble, f, on the same side of the relational statements. For example, if you prefer f to both g and h, you will prefer f to any convex combination of g and h.

Axiom 3 is a special case of the following symmetry principle. Suppose $\succ$ is transitive on $\{f, g, h\}$ with $f \succ g \succ h$, and that g is midway in preference between f and h, so that $g \backsim \frac{1}{2} f + \frac{1}{2} h$. Then an indifference between any two convex combinations of f, g and h will remain an indifference when f and h are interchanged throughout. When we make this interchange in $\lambda f + (1-\lambda) h \backsim \frac{1}{2} f + \frac{1}{2} g$, we get $\lambda h + (1-\lambda) f \backsim \frac{1}{2} h + \frac{1}{2} g$, as in the statement of the axiom. In geometric language, one can visualize f and h as "equally distant" from but on opposite sides of g: the interchange of f and h in a "balance equation" for $\backsim$ preserves the balance of $\backsim$.

Potential axioms not included among our six fall into two sets according to whether they are implied by the six. The following *are* implied by Axioms 1 through 6:

asymmetry: if $f \succ g$ then not $(g \succ f)$;
monotonicity: if $f \succ g$ and $1 \geq \alpha > \beta \geq 0$, then $\alpha f + (1-\alpha)g \succ \beta f + (1-\beta)g$;
unique continuity: if $f \succ g \succ h$ then $g \sim \alpha f + (1-\alpha)h$ for exactly one $\alpha \in (0, 1)$;
statewise dominance: if $\Sigma\{f(A) : \omega \in A\} \geq \Sigma\{g(A) : \omega \in A\}$ for all $\omega \in \Omega$, then $f \succsim g$.

The first three are implied by Axioms 1 and 2; the fourth is a special case of Savage's sure-thing principle (1954, pages 21–22).

The following *are not* implied by Axioms 1 through 6:

transitive indifference: if $f \sim g$ and $g \sim h$, then $f \sim h$;
transitive preference: if $f \succ g$ and $g \succ h$, then $f \succ h$;
linearity: if $f \sim g$ then $\frac{1}{2}f + \frac{1}{2}h \sim \frac{1}{2}g + \frac{1}{2}h$;
linearity: if $f \succ g$ and $0 < \lambda < 1$, then $\lambda f + (1-\lambda)h \succ \lambda g + (1-\lambda)h$;
additivity: if $f_A \succ f_B$ and $C \cap (A \cup B) = \varnothing$ then $f_{A\cup C} \succ f_{B\cup C}$.

We shall return to these in Section 5.

Of our six basic axioms, the one most vulnerable to criticism seems to be Axiom 2. Its problems are not unlike the difficulties in Ellsberg's examples caused by specificity versus ambiguity. For a simple illustration, suppose an urn contains 100 Black and Red balls in unknown proportion. Let $f = \frac{1}{2}f_\varnothing + \frac{1}{2}f_\Omega$, so f yields prize V with probability $\frac{1}{2}$ regardless of which ball is drawn. Also let f_B (f_R) yield V if the drawn ball is Black (Red), and nothing otherwise. The preceding section suggests that some people will have $f \succ f_B$ and $f \succ f_R$. These require $f \succ \frac{1}{2}f_B + \frac{1}{2}f_R$ by Axiom 2. But, since $\frac{1}{2}f_B + \frac{1}{2}f_R$ yields V with probability $\frac{1}{2}$ regardless of which ball is drawn, Axiom 6 requires $f \sim \frac{1}{2}f_B + \frac{1}{2}f_R$. Consequently, if both Axioms 2 and 6 are to hold, the only sensible comparisons for f versus f_B and f_R are $f \sim f_B$ and $f \sim f_R$. But we might wish to admit the triple $\{f \succ f_B, f \succ f_R, f \sim \frac{1}{2}f_B + \frac{1}{2}f_R\}$ as a reasonable set of judgments and retain Axiom 6 while dropping Axiom 2.

The main casualty of the new representation caused by omission of Axiom 2 would be conditional additivity (3), which can be viewed as a first-order generalization of regular additivity. I presently regard the problem of how best to proceed in the absence of Axiom 2 as an interesting open problem.

4. Main theorem. Our main representation theorem involves a functional ρ on $G \times G$. By definition, ρ is *skew-symmetric* if $\rho(f, g) = -\rho(g, f)$ for all $f, g \in G$, and *bilinear* if it is linear separately in each argument:

$$\rho(\lambda f + (1-\lambda)g, h) = \lambda\rho(f, h) + (1-\lambda)\rho(g, h)$$
$$\rho(h, \lambda f + (1-\lambda)g) = \lambda\rho(h, f) + (1-\lambda)\rho(h, g).$$

If ρ is skew-symmetric and linear in its first argument, then it is obviously linear in its second argument. Because of our interest in comparative probability, we extend ρ to $\mathscr{A} \times \mathscr{A}$ by the definition

$$\rho(A, B) = \rho(f_A, f_B).$$

In addition, we define $\succ^*$ on $\mathscr{A}$ by

$$A \succ^* B \Leftrightarrow f_A \succ f_B.$$

THEOREM 1. *Axioms* 1 *through* 6 *hold if, and only if, there is a unique skew-symmetric bilinear functional* ρ *on* $G \times G$ *such that, for all* $f, g \in G$,

$$f \succ g \Leftrightarrow \rho(f, g) > 0, \tag{5}$$

and such that ρ *on* $\mathscr{A} \times \mathscr{A}$ *is monotonic, conditionally additive as in* (3), *has* $\rho(\Omega, \varnothing) = 1$, *and for all A, B, ∈ $\mathscr{A}$ satisfies*

$$A \succ^* B \Leftrightarrow \rho(A, B) > 0. \tag{2}$$

PROOF. By Theorem 1 in Fishburn (1982a), Axioms 1, 2 and 3 hold if and only if there is a skew-symmetric bilinear functional ρ on $G \times G$ that satisfies (5) for all $f, g \in G$, and such a ρ is unique up to a similarity transformation $\rho \to \alpha\rho$ ($\alpha > 0$). By Axiom 4, $\rho(\Omega, \varnothing) = \rho(f_\Omega, f_\varnothing) > 0$. We fix the scale unit of ρ by setting $\rho(\Omega, \varnothing) = 1$, so that ρ is uniquely determined by this normalization. Axiom 5 implies that ρ is monotonic on $\mathscr{A} \times \mathscr{A}$ and (2) is immediate from (5). Axioms 4 and 5 are clearly necessary for the representation of the theorem.

Suppose Axiom 6 holds, $A, B, C \in \mathscr{A}$ and $A \cap B = \varnothing$. Let

$$f = 1/3\, f_{A\cup B} + 1/3\, f_\varnothing + 1/3\, f_C, \quad g = 1/3\, f_A + 1/3\, f_B + 1/3\, f_C.$$

Then f and g satisfy the hypothesis of Axiom 6, so that $f \sim g$. Therefore $\rho(f, g) = 0$ by (5) and the definition of $\sim$. Bilinearity applied to $\rho(f, g)$ gives

$$\rho(A \cup B, A) + \rho(A \cup B, B) + \rho(A \cup B, C) + \rho(\varnothing, A) + \rho(\varnothing, B) + \rho(\varnothing, C) + \rho(C, A) + \rho(C, B) + \rho(C, C) = 0, \tag{6}$$

with $\rho(C, C) = 0$ by skew-symmetry. In addition, when

$$f' = 1/2\, f_{A\cup B} + 1/2\, f_\varnothing, \quad g' = 1/2\, f_A + 1/2\, f_B,$$

Axiom 6 implies $f' \sim g'$, so $\rho(f', g') = 0$ and

$$\rho(A \cup B, A) + \rho(A \cup B, B) + \rho(\varnothing, A) + \rho(\varnothing, B) = 0.$$

When applied to (6), this leaves (3) in view of skew-symmetry. Thus ρ on $\mathscr{A} \times \mathscr{A}$ is conditionally additive, and the sufficiency proof is complete.

Assume henceforth that the representation of Theorem 1 holds. We prove (4) before verifying Axiom 6. Let $\{A_1, \cdots, A_n\}$ and $\{B_1, \cdots, B_m\}$ be nonempty sets of pairwise disjoint events in $\mathscr{A}$. Then repeated applications of (3) and skew-symmetry give (4) as follows:

$$\begin{aligned}
\rho(\cup A_i, \cup B_j) &= \rho(A_1, \cup B_j) + \rho(\cup_{i\geq 2} A_i, \cup B_j) + \rho(\cup B_j, \varnothing) \\
&\;\;\vdots \\
&= \textstyle\sum_i \rho(A_i, \cup B_j) + (n-1)\rho(\cup B_j, \varnothing) \\
&= \textstyle\sum_i [\sum_j \rho(A_i, B_j) - (m-1)\rho(A_i, \varnothing)] + (n-1) \sum_j \rho(B_j, \varnothing).
\end{aligned}$$

To verify Axiom 6, assume that f and g satisfy its hypotheses. If either f or g is $f_\varnothing$, then so is the other, and the conclusion of Axiom 6 holds since $\rho(f_\varnothing, f_\varnothing) = 0$. Assume henceforth that neither f nor g is $f_\varnothing$. Let $A_0 = B_0 = \varnothing$, let A_i's be the nonempty events where $f(A_i) > 0$, for $i = 1, \cdots, n$, and let B_j's be the nonempty events where $g(B_j) > 0$, for $j = 1, \cdots, m$. Also let

$$\alpha_i = f(A_i), \quad i = 0, 1, \cdots, n, \quad \text{so} \quad \textstyle\sum_i \alpha_i = 1,$$

$$\beta_i = g(B_j), \quad j = 0, 1, \cdots, m, \quad \text{so} \quad \textstyle\sum_j \beta_j = 1.$$

All α_i and β_j are positive, except perhaps α_0 and β_0.

Since the A_i and B_j need not be pairwise disjoint, we shall let $\{C_1, \cdots, C_N\}$ be the smallest family of nonempty and pairwise disjoint subsets of Ω in $\mathscr{A}$ such that each A_i for

$i \geq 1$ and each B_j for $j \geq 1$ is the union of one or more C_k. Let

$$a_i = |\{k: C_k \subseteq A_i\}|, \quad i \geq 1,$$

$$b_j = |\{k: C_k \subseteq B_j\}|, \quad j \geq 1,$$

and, by the hypotheses of Axiom 6, for each C_k and $\omega \in C_k$ let

$$\pi_k = \sum\{f(A_i) : \omega \in A_i\} = \sum_{i:A_i \supseteq C_k} \alpha_i = \sum\{g(B_j) : \omega \in B_j\} = \sum_{j:B_j \supseteq C_k} \beta_j.$$

By prior interpretation, π_k is the probability of winning prize V if either f or g is used, when the true state is in C_k. Using bilinearity, skew-symmetry, and (4), we get

$$\begin{aligned}
\rho(f, g) &= \textstyle\sum_{i=0}^{n} \sum_{j=0}^{m} \alpha_i \beta_j \rho(A_i, B_j) \\
&= \beta_0 \textstyle\sum_{i=1}^{n} \alpha_i \rho(A_i, \varnothing) - \alpha_0 \sum_{j=1}^{m} \beta_j \rho(B_j, \varnothing) + \sum_{i\geq 1} \sum_{j \geq 1} \alpha_i \beta_j \\
&\quad \cdot [\textstyle\sum_{C_k \subseteq A_i} \sum_{C_h \subseteq B_j} \rho(C_k, C_h) - (b_j - 1) \sum_{C_k \subseteq A_i} \rho(C_k, \varnothing) + (a_i - 1) \sum_{C_k \subseteq B_j} \rho(C_k, \varnothing)] \\
&= \beta_0 \textstyle\sum_{k=1}^{N} \pi_k \rho(C_k, \varnothing) - \alpha_0 \sum_{k=1}^{N} \pi_k \rho(C_k, \varnothing) \\
&\quad - \textstyle\sum_{j \geq 1} \beta_j (b_j - 1) \sum_{i \geq 1} \sum_{C_k \subseteq A_i} \alpha_i \rho(C_k, \varnothing) \\
&\quad + \textstyle\sum_{i \geq 1} \alpha_i (a_i - 1) \sum_{j \geq 1} \sum_{C_k \subseteq B_j} \beta_j \rho(C_k, \varnothing) \\
&\quad + \textstyle\sum_{1 \leq k < h \leq N} \rho(C_k, C_h) [\sum_{i: C_k \subseteq A_i} \alpha_i \sum_{j: C_h \subseteq B_j} \beta_j - \sum_{i: C_h \subseteq A_i} \alpha_i \sum_{j: C_k \subseteq B_j} \beta_j] \\
&= \textstyle\sum_k \rho(C_k, \varnothing) [\beta_0 \pi_k - \alpha_0 \pi_k - \sum_{j \geq 1} \beta_j (b_j - 1) \pi_k + \sum_{i \geq 1} \alpha_i (a_i - 1) \pi_k] \\
&\quad + \textstyle\sum_{k<h} \rho(C_k, C_h) [\pi_k \pi_h - \pi_h \pi_k] \\
&= \textstyle\sum_k \pi_k \rho(C_k, \varnothing) [\sum_{j \geq 0} \beta_j - \sum_{i \geq 0} \alpha_i + \sum_{i \geq 1} \alpha_i a_i - \sum_{j \geq 1} \beta_j b_j] \\
&= \textstyle\sum_k \pi_k \rho(C_k, \varnothing) [0 + \sum_{h=1}^{N} \pi_h - \sum_{h=1}^{N} \pi_h] \\
&= 0,
\end{aligned}$$

and therefore $f \sim g$ by (5). This verifies Axiom 6, and the proof is complete. □

The preceding proof reveals that Axiom 5, the monotonicity axiom, plays no role in Theorem 1 except to ensure that ρ on $\mathscr{A} \times \mathscr{A}$ is monotonic. The theorem remains valid if Axiom 5 and monotonicity for ρ on $\mathscr{A} \times \mathscr{A}$ are simultaneously deleted.

Suppose $\Omega = \{\omega_1, \cdots, \omega_n\}$ and $\mathscr{A}$ is the set of all subsets of Ω. Let

$$\rho_i = \rho(\omega_i, \varnothing), \quad i = 1, \cdots, n$$

and

$$\rho_{ij} = \rho(\omega_i, \omega_j), \quad i, j \in \{1, \cdots, n\},$$

with $\rho_i \geq 0$ and $\sum \rho_i = 1$ by normalization. Then, given Theorem 1, ρ on $\mathscr{A} \times \mathscr{A}$ can be computed from (4):

$$\rho(A, B) = \textstyle\sum_{i \in A} \sum_{j \in B} \rho_{ij} - (|B| - 1) \sum_{i \in A} \rho_i + (|A| - 1) \sum_{j \in B} \rho_j,$$

and ρ on $G \times G$ can be computed from ρ on $\mathscr{A} \times \mathscr{A}$ with the use of bilinearity:

$$\rho(\textstyle\sum \alpha_i f_{A_i}, \sum \beta_j f_{B_j}) = \sum_i \sum_j \alpha_i \beta_j \rho(A_i, B_j).$$

This yields a "valid" representation so long as monotonicity holds $[A \supseteq B \Rightarrow \rho(A, B) \geq 0]$ since conditional additivity is implied by (4). To prove the latter claim, recall that conditional additivity (3) requires

$$\rho(A \cup B, C) + \rho(\varnothing, C) = \rho(A, C) + \rho(B, C)$$

when $A \cap B = \varnothing$. By the preceding form of (4), and $A \cap B = \varnothing$,

$$\begin{aligned}
&\rho(A \cup B, C) + \rho(\varnothing, C) \\
&\quad = \textstyle\sum_{i \in A \cup B} \sum_{k \in C} \rho_{ik} - (|C| - 1) \sum_{i \in A \cup B} \rho_i + (|A \cup B| - 1) \sum_{k \in C} \rho_k - \sum_{k \in C} \rho_k \\
&\quad = \textstyle\sum_{i \in A} \sum_{k \in C} \rho_{ik} + \sum_{j \in B} \sum_{k \in C} \rho_{jk} - (|C| - 1) \sum_{i \in A} \rho_i - (|C| - 1) \sum_{j \in B} \rho_j \\
&\qquad + \textstyle\sum_{k \in C} \rho_k (|A| - 1 + |B| - 1) \\
&\quad = \rho(A, C) + \rho(B, C).
\end{aligned}$$

Consequently, any ρ_i that are nonnegative and sum to 1, along with any ρ_{ij} that are skew-symmetric, are suitable for the representation if they do not violate monotonicity.

Determination of the ρ_i and ρ_{ij} for the finite-states case can be made with appropriate indifference comparisons. In the simple two-state situation, $\Omega = \{\omega_1, \omega_2\}$, if $\omega_1 \succ^* \omega_2$ then, when α and β satisfy

$$\alpha f_{\omega_1} + (1 - \alpha) f_{\varnothing} \sim f_{\omega_2}$$

and

$$\beta f_{\omega_1} + (1 - \beta) f_{\varnothing} \sim 1/2\, f_{\omega_2} + 1/2\, f_{\varnothing}, \quad (\beta \leq \alpha)$$

we have $\alpha \rho_{12} = (1 - \alpha)\rho_2$ and $\beta \rho_{12} + \beta \rho_1 = (1 - \beta)\rho_2$. These two equations and $\rho_1 + \rho_2 = 1$ yield

$$\rho_1 = \frac{\alpha - \beta}{\alpha - \beta + \alpha\beta}, \quad \rho_2 = \frac{\alpha\beta}{\alpha - \beta + \alpha\beta}, \quad \rho_{12} = \frac{(1 - \alpha)\beta}{\alpha - \beta + \alpha\beta}.$$

If $\omega_1 \sim^* \omega_2$ then $\rho_{12} = 0$ and, with β as specified above,

$$\rho_1 = 1 - \beta \text{ and } \rho_2 = \beta.$$

Determination of the ρ_i and ρ_{ij} for larger states cases follows a similar procedure.

5. Transitivity and linearity. The theory sketched in the preceding sections has both descriptive and normative relevance. Its main descriptive weakness is surely the implication of precise measurement since real judgments are often vague or imprecise in ways not captured by the axioms. But the axioms seem likely to hold to a fair approximation in many situations. If systematic violations can be demonstrated, as with independence or linearity in the examples of Ellsberg and others, the most likely form they will take is probably the form illustrated at the end of Section 3.

The normative sense of the axioms should be clear from previous comments here and from discussions by Ramsey, Savage, and others. Each axiom has a defensible claim as a guide for well-reasoned judgment or choice. The theory's main normative weakness for some will be that it does not go far enough, especially since it embraces neither transitivity nor additivity as a normative principle. Of the two, I regard additivity or linearity as less suited for normative status than transitivity, even though I no longer understand why transitivity deserves the place often accorded to it as an unassailable rule for rationality.

In any event, I shall conclude with two theorems that show what happens when transitivity and then linearity are coupled to Axioms 1 through 6.

THEOREM 2. *Suppose the representation of Theorem* 1 *holds and* $\sim$ *on* G *is transitive. Then* $\succ$ *on* G *is transitive and, for all* $A, B \in \mathscr{A}$,

$$\rho(A, B) = \rho(A, \varnothing)\rho(\Omega, B) - \rho(B, \varnothing)\rho(\Omega, A) \tag{7}$$

and

$$A \succ^* B \Leftrightarrow \frac{\rho(A, \varnothing)}{\rho(A, \varnothing) + \rho(\Omega, A)} > \frac{\rho(B, \varnothing)}{\rho(B, \varnothing) + \rho(\Omega, B)}. \tag{8}$$

PROOF. Transitivity of $\succ$ follows from transitivity of $\backsim$, and Axioms 1 and 2, as in the proof of Proposition 1 in Fishburn (1982a). The rest of the proof follows the approach of the proofs of Lemma 3 and Theorem 3 in Fishburn (1982b) for "closed" $\succ$. When a, b, r, q and x in those proofs are respectively set at 1, 1, Ω, $\varnothing$ and $\varnothing$, and w and u are defined on G by

$$w(f) = \rho(f_\Omega, f) + \rho(f, f_\varnothing), \quad u(f) = \rho(f, f_\varnothing),$$

both w and u are linear and $\rho(f, g)$ decomposes as

$$\begin{aligned}\rho(f, g) &= u(f)w(g) - u(g)w(f)\\ &= \rho(f, f_\varnothing)[\rho(f_\Omega, g) + \rho(g, f_\varnothing)] - \rho(g, f_\varnothing)[\rho(f_\Omega, f) + \rho(f, f_\varnothing)]\\ &= \rho(f, f_\varnothing)\rho(f_\Omega, g) - \rho(g, f_\varnothing)\rho(f_\Omega, f).\end{aligned}$$

When $f = f_A$ and $g = f_B$, this and (2) give (7) and (8). □

Although $\rho(f, g)$ is not displayed in Theorem 2, bilinearity and (7), or the preceding equality in the proof, give

$$\rho(\textstyle\sum \alpha_i f_{A_i}, \sum \beta_j f_{B_j}) = [\sum \alpha_i \rho(A_i, \varnothing)][\sum \beta_j \rho(\Omega, B_j)] - [\sum \beta_j \rho(B_j, \varnothing)][\sum \alpha_i \rho(\Omega, A_i)].$$

Moreover, the ratios in (8) are the λ's in indifference statements such as $f_A \backsim \lambda f_\Omega + (1 - \lambda) f_\varnothing$ since this is tantamount to $\lambda\rho(\Omega, A) = (1 - \lambda)\rho(A, \varnothing)$, or to $\lambda = \rho(A, \varnothing)/[\rho(A, \varnothing) + \rho(\Omega, A)]$.

The linearity condition of Herstein and Milnor (1953) will be used in our final theorem:

$$\forall f, g, h \in G: \text{if } f \sim g \text{ then } \tfrac{1}{2} f + \tfrac{1}{2} h \sim \tfrac{1}{2} g + \tfrac{1}{2} h. \tag{9}$$

THEOREM 3. *Suppose the representation of Theorem 1 holds along with (9). Then $\backsim$ and $\succ$ on G are transitive and, for all $A, B \in \mathscr{A}$,*

$$\rho(A, B) = \rho(A, \varnothing) - \rho(B, \varnothing),$$

and (1) holds with $P(A) = \rho(A, \varnothing)$.

PROOF. The proof of Proposition 1 in Fishburn (1982a) shows that full transitivity is implied by (9) and Axioms 1 and 2. Since $\Omega \succ^* \varnothing$ and $\Omega \succsim^* A \succsim^* \varnothing$, there is a unique λ for $A \in \mathscr{A}$ such that

$$f_A \backsim \lambda f_\Omega + (1 - \lambda) f_\varnothing.$$

Then, by two applications of (9),

$$\tfrac{1}{2} f_A + \tfrac{1}{2} f_\varnothing \backsim \tfrac{1}{2}[\lambda f_\Omega + (1 - \lambda) f_\varnothing] + \tfrac{1}{2} f_\varnothing,$$

$$\tfrac{1}{2} f_A + \tfrac{1}{2} f_\Omega \backsim \tfrac{1}{2}[\lambda f_\Omega + (1 - \lambda) f_\varnothing] + \tfrac{1}{2} f_\Omega.$$

The representation of Theorem 1 yields the following from the three preceding indifference statements:

$$\lambda\rho(\Omega, A) + (1 - \lambda)\rho(\varnothing, A) = 0$$

$$\lambda\rho(\Omega, A) + (2 - \lambda)\rho(\varnothing, A) + \lambda = 0$$

$$(1 + \lambda)\rho(\Omega, A) + (1 - \lambda)\rho(\varnothing, A) - (1 - \lambda) = 0.$$

The first two of these imply that $\rho(A, \varnothing) = \lambda$, and the first and third imply that

$\rho(\Omega, A) = 1 - \lambda$. Therefore

$$\rho(A, \varnothing) + \rho(\Omega, A) = 1 \quad \text{for all} \quad A \in \mathscr{A}.$$

The use of this in (7) and (8) completes the proof of Theorem 3. □

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