

the theory of a certain class of stochastic processes motivated by models for learning. It will certainly be well received and appreciated.

UNIVERSITE LAVAL  
CITE UNIVERSITE  
QUEBEC, CANADA

K. B. ATHREYA AND P. E. NEY, *Branching Processes*. Springer-Verlag, Berlin, 1972, 287 pp.

Review by JOHN A. WILLIAMSON  
*University of Colorado, Boulder*

The authors in their preface state clearly their purpose in writing the book, "to give a unified treatment of the limit theory of branching processes." They hold to their purpose and generally accomplish their end.

The authors concentrate on the decade between the publication of Harris' book (T. E. Harris, *The Theory of Branching Processes* (1963)) and their own. Referring to Harris, the authors write, "only enough material is repeated to make the treatment essentially self-contained. For example, certain foundational questions on the construction of processes to which we have nothing new to add are not developed." The reader can find in this book the classical limit laws, most appearing in their sharpest form, as well as recent new results, and all are presented in a way that can only be called elegant. The emphasis in the book is on single-type processes with the first two chapters devoted to the Galton-Watson process and the next two to the continuous time Markov and Bellman-Harris age-dependent branching processes. Multi-type processes are discussed only in the fifth chapter where, appropriately, the reader is referred to Harris for several proofs.

The basic techniques of functional iteration, of martingales, of convex function bounding, of Taylor's Series remainder estimates, of renewal equation analysis, and of comparison lemma manipulation, together with refinements, are all there for the reader to master. The authors have an interest in, and great facility with, technique. The technical high point, not surprisingly, is their Chapter IV on the age-dependent process.

Having expressed a general enthusiasm for the book, I want next to attempt to identify that group of readers for whom the book will be of most interest, and then finally to discuss what I feel is one serious omission.

This book is definitely required reading for any person intending to attempt research in the branching processes area as well as for anyone who plans only to keep abreast of the literature in this field. Not only do the contents of the various chapters equip the reader for these undertakings, but the problem sets at the end of each chapter provide several open questions and many challenges that test whether the reader has grasped the techniques of the chapter. (Since

the writing of the book the following questions have been settled: Chapter I, No. 9, extensions; N. H. Bingham, London and R. A. Doney, Manchester; Chapter IV, No. 1; D. P. Grey, Sheffield—a partial result; Chapter IV, No. 17, T. Savits, Pittsburgh; Chapter IV, No. 18; W. Esty, Wisconsin; Chapter V, page 227, last sentence before complements and problems, M. Goldstein, Montreal, and P. Ney, Wisconsin—independently.)

Chapter VI with its surveys of such topics as random environments, immigration, cascades, and branching random walks and diffusions, provides the applied probabilist with a precise mathematical description of several models of practical interest. However, nowhere in the book will the reader find any discussion of the estimation of extinction probabilities nor of the estimation of the offspring distribution so that the reader with applied interests in this area will have to look elsewhere.

Those with little background in branching processes who wish to be eased into the subject with heuristic discussions and lots of examples would be well-advised to study certain sections in Harris' book and the relevant parts of Karlin's book, (S. Karlin, *A First Course in Stochastic Processes* (1969)) either before, or concurrently with, their reading of the Athreya and Ney book.

It is this reviewer's opinion that the most serious omission in the book is the lack of any mention of the general age-dependent branching process. If not a chapter, then at least a section on the subject in Chapter 6, I feel, should have been included. The authors have, in a sense, allowed themselves an out by stating with regard to the topics included in Chapter 6 that "... these are a reflection of our own interests and there is no attempt to catalog or even dent the great variety of special models . . ." However, the whole approach in the book—a detailed development of the basic Galton–Watson process in the first two chapters; then in the Markov and age-dependent cases reducing where possible "analogous questions to their Galton–Watson counterparts; and then concentrating on the genuinely new or different aspects of these processes"—would seem to dictate the inclusion of the most general single-type branching process with the property that its generation sizes form a Galton–Watson process. The existence of another book that contains some material on the subject (C. J. Mode, *Multitype Branching Processes* (1971)) together with the fact that the limit theorem for the convergence in distribution of the normalized population size in the supercritical case by R. Doney (*J. Appl. Probability* (1972)) had not yet appeared at the time of the writing of the book, argue that the authors should be excused from criticism on this point. But the papers by Crump and Mode (*J. Math. Anal. Appl.* (1968 and 1969)), by Ryan (Ph. D. dissertation, Chapter 2, Cornell, (1968)), and by Durham (*J. Appl. Probability* (1971)) had appeared and this along with the obvious usefulness of a model where the offspring are allowed to arrive during the life time of the parent and where past births are allowed to have an effect on the number of future births and on length of life would, in my opinion, tip the balance in the direction of inclusion.

In summary, this well-written and well-organized book is primarily for the technical specialist and near-specialist, though Chapter 6 with its description of some of the variations on the branching theme should be of interest to the applied probabilist as well. Those wishing to read on the general age-dependent branching process will have to turn to the journals and to another book.

DEPARTMENT OF STATISTICS  
THE UNIVERSITY OF ROCHESTER  
ROCHESTER, NEW YORK 14627