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CORRECTION

HIGH DENSITY LIMIT THEOREMS FOR INFINITE SYSTEMS OF UNSCALED BRANCHING BROWNIAN MOTIONS

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BY LUIS G. GOROSTIZA

In the proof of Theorem 3 the derivation of $H(\phi, \psi; t)$ failed to take account of the fact that in general p_0 may be strictly positive, and is necessarily so in the critical and subcritical cases. The correct result is

$$H(\phi, \psi; t)$$

$$= e^{-Vp_0 t}\left\{\int \phi(x)\psi(x)\,dx + (\alpha + Vp_0)\int \phi(x)\int_0^t e^{(\alpha+Vp_0)r}\mathscr{T}_{2r}\psi(x)\,dr\,dx\right\}.$$

As a consequence, Theorem 3 should be replaced by

THEOREM 3. *$M^{I,T} \Rightarrow$ M^I and M$^{II,T} \Rightarrow M^{II}$ as $T \to \infty$, where $M = M^I + M^{II}$, and M^I and M^{II} are generalized centered Gaussian processes such that*

$$\operatorname{Cov}(\langle M_s^I, \phi\rangle, \langle M_t^I, \psi\rangle) = \int\int \phi(x)\psi(x)p_{t-s}(x-y)\,dx\,dy, \quad s \le t,$$

$$\operatorname{Cov}(\langle M_s^{II}, \phi\rangle, \langle M_t^{II}, \psi\rangle)$$

$$= (1 + e^{\alpha t} - e^{-Vp_0 s} - e^{\alpha(t-s)-Vp_0 s})\int\int \phi(x)\psi(y)p_{t-s}(x-y)\,dx\,dy$$

$$+ e^{\alpha t}m_2 V\int\int \phi(x)\psi(y)\int_0^s e^{\alpha r}p_{t-s+2r}(x-y)\,dr\,dx\,dy$$

$$-(\alpha + Vp_0)(1 + e^{\alpha(t-s)})e^{-Vp_0 s}$$

$$\cdot\int\int \phi(x)\psi(y)\int_0^s e^{(\alpha+Vp_0)r}p_{t-s+2r}(x-y)\,dr\,dx\,dy, \quad s \le t,$$

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and

$$\operatorname{Cov}(\langle M_s^I, \phi\rangle, \langle M_t^{II}, \psi\rangle)$$

$$= \begin{cases} (e^{\alpha(t-s)-Vp_0 s} - 1) \int\int \phi(x)\psi(y)p_{t-s}(x-y)\,dx\,dy \\ + (\alpha + Vp_0)e^{\alpha(t-s)-Vp_0 s} \\ \cdot \int\int \phi(x)\psi(y) \int_0^s e^{(\alpha+Vp_0)r} p_{t-s+2r}(x-y)\,dr\,dx\,dy, & s \le t, \\ (e^{-Vp_0 t} - 1) \int\int \phi(x)\psi(y)p_{s-t}(x-y)\,dx\,dy \\ + (\alpha + Vp_0)e^{-Vp_0 t} \\ \cdot \int\int \phi(x)\psi(y) \int_0^t e^{(\alpha+Vp_0)r} p_{s-t+2r}(x-y)\,dr\,dx\,dy, & s \ge t, \end{cases}$$

where $p_t(x) = e^{-\|x\|^2/2t}(2\pi t)^{-d/2}$.

It follows that:

1) M^I and M^{II} are not independent in the critical case, contrary to the statement in the paper.

2) In the critical case M^{II} is a more complicated generalized process than the generalized Ornstein-Uhlenbeck process claimed in the paper; and when $d \ge 3$, $M_t^{II} \Rightarrow M_\infty^{II}$ as $t \to \infty$, where M_∞^{II} is a generalized centered Gaussian random field with covariance kernel

$$2\delta(x-y) + m_2 V(4\pi)^{-1}\Gamma(d/2 - 1)\|x - y\|^{-d+2}.$$

3) In the subcritical case $M_t^{II} \Rightarrow W$ as $t \to \infty$, where W is the spatial standard Gaussian white noise.

4) In the special subcritical case $p_0 = 1$ the model represents an infinite system of independent killed Brownian motions, and $N_t^{II}(A)$ is the number of particles that have died before time t which would have been in the set A at time t if they had not died.

DEPARTAMENTO DE MATEMATICAS
CENTRO DE INVESTIGACION
Y DE ESTUDIOS AVANZADOS
APARTADO POSTAL 14-740
MEXICO 07000, D. F.