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BOOK REVIEW

ALAN F. KARR, *Point Processes and their Statistical Inference*. Dekker, New York, 1986, 504 pages, $89.75 (adoption price $39.75).

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A point process on a topological space E is a random, integer valued, measure on E (endowed with the Borel σ-algebra); it may be thought of as the measure which places unit mass at a random collection of points in E. The setting is sufficiently general to encompass a number of familiar paradigms (including, incidentally, independent and identically distributed sampling from a general space), clustering and marked processes, and rich enough to give rise to a deep and a substantial general theory. Both in general and in various specialised guises (for example as Poisson, Cox, renewal or stationary processes) point processes have proved enormously valuable as models for a wide range of phenomena. In this modelling context it is natural to ask two types of questions. The first concerns the probabilistic behaviour and related properties of the various types of processes, while the second involves making decisions, or inferences, about the specific process giving rise to a particular observed phenomenon. This book is designed to equip its reader with the machinery, results and insight to answer both types of questions.

The first two chapters of the book are devoted to probabilistic theory. Chapter One outlines what might be called the classical or distributional theory of point processes, including characterisations, convergence, transformations and approximations, marked and cluster processes and, finally, Palm distributions and stationary point processes. The second chapter deals with the more modern intensity theory of point processes on $\mathbb{R}^+$: with such a point process we can associate (effectively via the Doob–Meyer decomposition) a predictable process called a compensator (roughly analogous to a cumulative conditional hazard function except that, in general, it varies with the realisation of the point process and so is "random"). The difference between the point process and its compensator is a martingale, called the innovation process, and in many cases the compensator may be written as the integral of a positive predictable process called the stochastic intensity, the (random) hazard function of the above analogy. Having established this framework, the full weight of modern probability may be brought to bear, exploiting the martingale structure and predictability, to quite impressive effect. My only disappointment with the book, and then not an especially serious one, is that these chapters were not written in a slightly more leisurely style, at a somewhat lower level. All the bones of the subject are there, but very little of the flesh. The coverage is more than sufficient to make the book self-contained, but with a little more detail these chapters could happily stand alone as a very good introduction to point process theory.

Chapter Three sets the scene for a study of questions of inference for point processes, describing types of inference and the various possible forms of observa-

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tion. The next two chapters develop the two general strategies for inference, corresponding, respectively, to the distributional and intensity theories for the processes. Chapter Four deals with the empirical approach: the philosophy is to base inference for the law of the process on the empirical measure (the measure with mass n^{-1} at each of the n observed collections of points). The approach is robust and (if no conditions are placed on the class of possible laws) maximum likelihood. It rests heavily in spirit and in detail on existing results for empirical processes. The fifth chapter develops the martingale method of inference for point processes on $\mathbb{R}^+$ which admit a stochastic intensity. The idea is that to estimate an unobservable process one uses an observable process which differs from it by a martingale. When it applies, the method is very powerful. Its advantages are generality, tractability and the existence of a genuinely useful asymptotic theory.

The remaining five chapters, comprising the second half of the book, deal with more specialised situations: respectively, Poisson processes, Cox processes, renewal processes, stationary processes and point process sampling of stochastic processes. The general results are germane, but the key idea in each case is the development of ad hoc methods exploiting special structure. The book concludes with two appendices containing a brief treatment of spaces of measures and continuous-time martingales, and an extensive list of references. Each chapter ends with a large and very good collection of exercises (although some are possibly too central to the development) and an invaluable series of notes.

Depending on one's background (or one's prejudice) the word "statistical" in the title should perhaps not be taken too literally. The author is quick to point out that "this is not a book about data analysis." That is rather an understatement. The statistical sections of the book comprise a collection of elegant limit theorems in probability, deriving the asymptotic consistency of estimators or their convergence to normality (often in the form of a set or function indexed Gaussian process). Of course in view of the generality of the treatment such results represent a considerable achievement and it would be naive to hope for more. Furthermore, the infinite dimensional nature of almost everything of interest renders many familiar statistical ideas (for example interval estimation) meaningless, or at best difficult to interpret, in the point process context.

I suspect that some potential readers will not find the book easily accessible. The author may be a little optimistic in suggesting that a one semester graduate course in probability (and one in statistics) would provide an adequate background for the book, although with perseverance this might just suffice. With considerably more sophistication the reader would benefit enormously. The book is definitely better cast as a research monograph than a course text, and likely to be of more interest to probabilists than statisticians. In fact, for the well-equipped probabilist it is a very good book; an extensive, well written, and enriching account of an important field, fully warranting a place on their bookshelf and in their library. My only hope is that, for their sake, they can afford it.

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