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ONE DIMENSIONAL STOCHASTIC ISING MODELS WITH SMALL MIGRATION

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We consider one dimensional, attractive stochastic Ising models with finite range interactions in which the particles move according to the stirring process. We prove that the process is exponentially ergodic provided the migration rate is sufficiently small.

1. Introduction. The one dimensional stochastic Ising model is a continuous time Markov process on $\{-1, +1\}^{\mathbf{Z}}$: each site is occupied by a particle that either has state $+1$ or state -1. Holley (1985) proved that under certain conditions this process is exponentially ergodic: the semigroup acting on cylinder functions converges exponentially fast in the uniform norm. We will show that if, in addition, we allow the particles to exchange their sites at a sufficiently small rate, the system remains exponentially ergodic. A lower bound for the rate of convergence can be given. It depends on the migration intensity.

In order to state precisely what we prove, we need some notation. Let $\mathbf{Z}$ be the integers and $\{J_F\colon F \subset \mathbf{Z}\}$ be a finite range translation invariant potential, i.e., for each F, J_F is a real number with $J_F = 0$ if the diameter of F is larger than the range and $J_{F+x} = J_F$ for all $F \subset \mathbf{Z}$ and $x \in \mathbf{Z}$. The state space of our process is $E = \{-1, +1\}^{\mathbf{Z}}$. Give E the product topology and let $C(E)$ be the space of continuous real valued functions on E. Let D be the set of cylinder functions on E. That is,

$$D = \{ f \in C(E) \colon \text{there is a finite } \Lambda \subset \mathbf{Z} \text{ such that} \\ \text{if } \eta = \zeta \text{ on } \Lambda \text{ then } f(\eta) = f(\zeta)\}. \tag{1.1}$$

We define the generator of the stochastic Ising model by

$$\Omega_I f(\eta) = \sum_{x \in \mathbf{Z}} c(x, \eta)[f(\eta_x) - f(\eta)], \tag{1.2}$$

for $f \in D$ and $\eta \in E$, where

$$\eta_x(y) = \begin{cases} \eta(y) & \text{if } y \neq x, \\ -\eta(y) & \text{if } y = x \end{cases} \tag{1.3}$$

and the flip rates $c(x, \eta) > 0$ are such that

$$c(x, \eta)\exp\left[\sum_{F \ni x} J_F \prod_{y \in F} \eta(y)\right] \tag{1.4}$$

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does not depend on the coordinate $\eta(x)$. Two commonly used choices are

$$c(x,\eta) = \exp\Big[-\sum_{F\ni x} J_F \prod_{y\in F} \eta(y)\Big] \tag{1.5}$$

and

$$c(x,\eta) = \Big\{1 + \exp\Big[2\sum_{F\ni x} J_F \prod_{y\in F} \eta(y)\Big]\Big\}^{-1}. \tag{1.6}$$

We will assume that the rates are translation invariant and depend only on finitely many coordinates. This implies that the closure of Ω_I generates a unique Markovian semigroup $\{T_t\colon t \geq 0\}$ [see Liggett (1985)]. From this, together with (1.4), it follows that the unique Gibbs state for the potential $\{J_F\colon F \subset \mathbf{Z}\}$ is a reversible measure for the corresponding Markov process. We will also assume that $\eta \to c(x,\eta)$ is decreasing on $\eta(x) = 1$ and increasing on $\eta(x) = -1$. This implies that the system is attractive, that is, if $\eta \leq \zeta$ [i.e., $\eta(y) \leq \zeta(y)$ for all y], then copies of the process with these initial configurations can be constructed on the same space with $\eta_t \leq \zeta_t$ for all t.

Our next step is to define the stirring process. For $\eta \in E$ let

$$\eta_{xy}(u) = \begin{cases} \eta(y) & \text{if } u = x, \\ \eta(x) & \text{if } u = y, \\ \eta(u) & \text{otherwise.} \end{cases} \tag{1.7}$$

Let $p(x,y)$ be a symmetric, translation invariant transition function with finite range L and $p(x,x) = 0$. We define the generator of the stirring process by

$$\Omega_m f(\eta) = \sum_{\eta(x)\neq\eta(y)} p(x,y)\big[f(\eta_{xy}) - f(\eta)\big], \tag{1.8}$$

for $f \in D$ and $\eta \in E$. Since the process has finite range, a unique Markov process with this generator can be constructed [see Liggett (1985)].

Here we study a mixture of the Ising model and stirring process. For $\varepsilon > 0$ and $f \in D$ define its generator by

$$\Omega^{\varepsilon} f(\eta) = (\Omega_I + \varepsilon\Omega_m) f(\eta). \tag{1.9}$$

Again, the closure of Ω^{ε} generates a unique Markovian semigroup $\{T_t^{\varepsilon}\colon t \geq 0\}$. Our main result is the following theorem.

THEOREM 1.10. *There are constants ε_0, $\gamma > 0$ so that for all $\varepsilon \leq \varepsilon_0$,*

$$|T_t^{\varepsilon} f(\eta) - T_t^{\varepsilon} f(\zeta)| \leq A_f e^{-\gamma t}, \tag{1.11}$$

for all η, $\zeta \in E$ and $f \in D$, where γ depends on ε_0. $A_f < \infty$ and is only a function of f.

The theorem will be proved in Section 3. The proof is mainly based on results of Holley (1985), who proved that the process without migration is exponentially ergodic. The migration will be considered as a small perturbation which generates discrepancies in the coupled proces. We will show that if the migration rate is small, the discrepancies disappear faster than they are created. We want to note that the result is true for every finite range, translation invariant, attractive spin system with migration whose unperturbed system is exponentially ergodic. In Section 2 we state some results needed in the Proof of Theorem 1.10, which is carried out in Section 3.

2. Preparatory results. This section describes the basic coupling, states a known result and proves a key estimate that is needed for the Proof of Theorem 1.10.

Coupling. Let $\eta \geq \zeta$ be initial distributions. From the attractiveness of the Ising model it follows easily that the mixed process is monotone, that is, we can construct copies of the process with these initial distributions such that if $f \in C(E)$ satisfies $f(\eta) \geq f(\zeta)$ for $\eta \geq \zeta$, then for all $t \geq 0$, $T_t^\varepsilon(\eta) \geq T_t^\varepsilon f(\zeta)$ for $\eta \geq \zeta$. To show this we will use the basic coupling. It is designed to make the two processes agree as far as possible. It can be described as follows [see Liggett (1985)]: If $\eta_t(x) \neq \zeta_t(x)$ for $t \geq 0$ and $x \in \mathbf{Z}$, they flip independently of each other with the corresponding rates. If $\eta_t(x) = \zeta_t(x)$, they flip together with the highest possible rate. Particle exchanges always occur together at rate ε. The corresponding semigroup of the coupled process will be denoted by $(\tilde{T}_t^\varepsilon)_{t\geq 0}$ (if $\varepsilon = 0$, we drop the upper index).

Monotonicity implies that η_t^{+1} and η_t^{-1}, the processes starting from $\eta_0^{\pm 1} = \pm\vec{1}$ (the configurations with ± 1 at each site), are the largest and smallest possible states at time t. From this, together with translation invariance, it follows that it suffices to show that $T_t^\varepsilon f_0(+\vec{1}) - T_t^\varepsilon f_0(-\vec{1})$ converges exponentially fast to zero where $f_x(\eta) = \eta(x)$.

Holley's result. For the proof of the theorem we need the following result.

Proposition 2.1 [Holley (1985)]. *For every one dimensional, attractive stochastic Ising model of finite range potential there are constants $\delta_1 > 0$ and $0 < A_1 < \infty$ so that*

$$|T_t f_0(\eta) - T_t f_0(\zeta)| \leq A_1 e^{-\delta_1 t}, \tag{2.2}$$

for all $\eta, \zeta \in E$.

A stronger result without assuming attractiveness is now known [Holley and Stroock (1988)]. Our proof depends heavily on this assumption. Therefore we state the weaker result.

Estimates. Let $\eta, \zeta \in E$ with $\eta \geq \zeta$ and let $\xi := \eta - \zeta$ be the difference process with

$$\xi = \begin{cases} 2 & \text{if } \eta(x) > \zeta(x), \\ 0 & \text{if } \eta(x) = \zeta(x). \end{cases} \tag{2.3}$$

Because of the monotonicity, the process $(\xi_t)_{t\geq 0}$ is well defined for all times $t \geq 0$.

We need an estimate on how fast discrepancies can spread out. We use the same idea as in Holley and Stroock (1976), but we state the estimate in a form more convenient for our purpose. We define a right edge process similar to the one in the contact process [see Durrett (1980)]: We start the difference process with the negative half-line full of discrepancies, that is, $\xi^{(-\infty,0)}(x) = 2$ if $x \in (-\infty, 0)$, else 0, and define the right edge of discrepancies at time t as

$$r_t = \sup\{x : \xi_t^{(-\infty,0)}(x) = 2\}. \tag{2.4}$$

To estimate the location of r_t, we compare r_t with the right edge R_t of a process with no recovery. To define this process, we divide the line into boxes $I_n = [nK, (n+1)K)$, $n \in \mathbf{Z}$, where K is the range of the Ising model. We also start this process with the negative half-line full of discrepancies. As soon as there is one discrepancy in a box I_n, we fill the box completely with discrepancies. Let a_t be the largest n at time t for which there is a discrepancy in I_n. At rate $\sup_\eta c(0,\eta)$, a_t grows by 1. By construction, the discrepancies cannot generate new discrepancies in boxes other than neighboring ones. We do not allow a discrepancy, present at time t, to disappear at later times. Therefore, the right edge $R_t = Ka_t$ of this process grows linearly in time with speed $\alpha = K \sup_\eta c(0,\eta)$, i.e., R_t is a Poisson random variable with mean αt. Then by construction, $R_t \geq r_t$. We can now estimate the influence of discrepancies on 0 from sites that are far away from 0.

LEMMA 2.5. *Let $\Lambda_t = (-2\alpha t, 2\alpha t) \cap \mathbf{Z}$. Then there are positive constants A_2 and δ_2 so that for $t > 0$,*

$$\begin{aligned}
&\sum_{x \text{ off } \Lambda_t} P(\textit{discrepancy at } x \textit{ affects } 0 \textit{ by time } t) \leq A_2 e^{-\delta_2 t},\\
&\sum_{x \text{ off } \Lambda_t} |x| P(\textit{discrepancy at } x \textit{ affects } 0 \textit{ by time } t) \leq A_2 e^{-\delta_2 t}.
\end{aligned} \tag{2.6}$$

PROOF. P(discrepancy at $[2\alpha t] + k$ affects 0 by time t), $k \in \mathbf{N}$, can be estimated by a rate α Poisson process $N(t)$, where $[\cdot]$ denotes the integer part, and α is the rate defined above. Let $\theta > 0$. Then a common large deviation estimate shows that

$$\begin{aligned}
e^{\theta([2\alpha t]+k)} P(N(t) \geq [2\alpha t] + k) &\leq \mathbf{E} e^{\theta N(t)} = e^{-\alpha t} \sum_{l=0}^{\infty} e^{\theta l} \frac{(\alpha t)^l}{l!}\\
&= e^{-\alpha t(1-e^\theta)}.
\end{aligned} \tag{2.7}$$

Recall $[2\alpha t] \geq 2\alpha t - 1$. Hence,

$$P(N(t) \geq [2\alpha t] + k) \leq \exp\left[-t(2\alpha\theta + \alpha(1 - e^\theta)) - \theta(k-1)\right]. \tag{2.8}$$

Let $J(\theta) = 2\alpha\theta + \alpha(1 - e^{\theta})$. Then $J(1) = \alpha(3 - e) \equiv \delta_2 > 0$. Let $|x| = [2\alpha t] + k$. Then for $t > 0$,

$$\sum_{x \text{ off } \Lambda_t} P(\text{discrepancy at } x \text{ affects } 0 \text{ by time } t)$$

$$(2.9)\qquad \le \sum_{x \text{ off } \Lambda_t} P(N(t) \ge |x|) \le 2e^{-\delta_2 t} \sum_{k=0}^{\infty} e^{-(k-1)} = \frac{2e}{1 - e^{-1}} e^{-\delta_2 t}.$$

Likewise,

$$\sum_{x \text{ off } \Lambda_t} |x| P(\text{discrepancy at } x \text{ affects } 0 \text{ by time } t)$$

$$(2.10)\qquad \le \sum_{x \text{ off } \Lambda_t} |x| P(N(t) \ge |x|) \le 2e^{-\delta_2 t} \sum_{k=0}^{\infty} (2\alpha t + k) e^{-(k-1)}$$

$$= 2e\left(\frac{1}{(1 - e^{-1)})^2} + \frac{2\alpha t}{1 - e^{-1}}\right) e^{-\delta_2 t}.$$

Hence we can find a constant $0 < A_2 < \infty$ so that both (2.9) and (2.10) can be bounded by $A_2 e^{-\delta_2 t}$. □

The following lemma is crucial. It shows how to take into account an additional migration.

LEMMA 2.11. *Let $\eta \ge \zeta$ be translation invariant initial configurations of the coupled process $(\eta_t^\varepsilon, \zeta_t^\varepsilon)_{t \ge 0}$, that is, $\eta = +\vec{1}$ and $\zeta = -\vec{1}$. Let $f_0(\eta) = \eta(0)$. Then there exist positive constants A_3 and δ_3 such that*

$$(2.12)\qquad \left|\tilde{T}_s^\varepsilon \tilde{\Omega}_m \tilde{T}_{t-s}[f_0(\eta) - f_0(\zeta)]\right| \le P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0)) A_3 e^{-\delta_3(t-s)}.$$

PROOF. We denote by $\mathbf{P}$ expected values with respect to the mixed process, and by $\mathbf{Q}$ expected values with respect to the pure Ising dynamic. Until time s, the mixed dynamic acts on the process. At time s, we switch off the migration. We will estimate the influence each single discrepancy has on 0, by interpolating between $(\eta_s^\varepsilon, \zeta_s^\varepsilon)$, the configurations at time s. The interpolating pairs differ on exactly one site. We do this in the following way: Denote by $\psi_z \sigma_s$ the configuration σ_s flipped at site z. If $\eta_s^\varepsilon(z) > \zeta_s^\varepsilon$, define $\sigma_s^z = \psi_{z-k}\sigma_s^{z-k}$, where $k = \min\{l > 0 : \eta_s^\varepsilon(z - l) > \zeta_s^\varepsilon(z - l)\}$. Then $\sigma_s^z(z) = \eta_s^\varepsilon(z)$ and $\psi_z \sigma_s^z(z) = \zeta_s^\varepsilon(z)$. If $\eta_s^\varepsilon(z) = \zeta_s^\varepsilon(z)$, insert an arbitrary pair $(\sigma_s^z, \psi_z \sigma_s^z)$ with $\sigma_s^z > \psi_z \sigma_s^z$. The superscript z tells only at what site we looked. The operator π_{xy} exchanges the values at x and y. We can write the lhs of (2.12) as

$$(2.13)\qquad \Bigg|\mathbf{P}^{(\eta,\zeta)} \sum_{x,y} p(x,y) \sum_z \mathbf{1}_{\{\eta_s^\varepsilon(z) > \zeta_s^\varepsilon(z)\}} \Big\{\mathbf{Q}^{(\pi_{xy}\sigma_s^z, \pi_{xy}\psi_z\sigma_s^z)}[f_0(\eta_{t-s}) - f_0(\zeta_{t-s})]$$
$$- \mathbf{Q}^{(\sigma_s^z, \psi_z\sigma_s^z)}[f_0(\eta_{t-s}) - f_0(\zeta_{t-s})]\Big\}\Bigg|.$$

The proof will show that the sum is absolutely convergent.

We break the estimate for (2.13) up according to whether z or the pair (x, y) is either "close to 0" or "far away from 0." We will define this more precisely below. The key observation for estimating (2.13) is that the difference between the braces in (2.13) is zero if the information that spreads out either from the discrepancy at z or from the exchanges at (x, y) at time s has not reached zero by time t. The set $\mathbf{\Lambda}_{t-s}$ is the same as the one defined in Lemma 2.5.

(i) $z \in \mathbf{\Lambda}_{t-s}$, $(x, y) \in \mathbf{\Lambda}_{t-s}$: We estimate the terms in the difference separately by using Proposition 2.1. Each term is $\le A_1 e^{-\delta_1(t-s)}$. For fixed y, there are at most $4(t-s) + 2L$ x's and z's, respectively. The additional L takes boundary effects caused by migration into account. For fixed x and z there are $2L$ y's. We also use translation invariance to conclude that $\mathbf{P}^{(\eta,\zeta)}\mathbf{1}_{\{\eta_s^\varepsilon(z) > \zeta_s^\varepsilon(z)\}} \equiv P^{(\eta,\zeta)}(\eta_s^\varepsilon(z) > \zeta_s^\varepsilon(z)) = P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0))$. So,

$$\le P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0))(4(t-s) + 2L)^2 2L 2A_1 e^{-\delta_1(t-s)}. \tag{2.14}$$

(ii) $z \notin \Lambda_{t-s}$, $(x, y) \in \Lambda_{t-s}$: We use the first estimate of Lemma 2.5. The difference in the sum is always less than or equal to 2. There are at most $2L(4(t-s) + 2L)$ (x, y)-pairs. So,

$$\le P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0))2L(4(t-s) + 2L)2A_2 e^{-\delta_2(t-s)}. \tag{2.15}$$

(iii) $z \in \mathbf{\Lambda}_{t-s}$, $(x, y) \notin \mathbf{\Lambda}_{t-s}$: This estimate is basically the same as the one in (ii). We fix z and use the large deviation estimate for the flow of information of the (x, y)-pair. Actually, the estimate in Lemma 2.5 was only done for the spread of information that originated at a discrepancy. But since the speed of the flow of the information that originates at a pair (x, y) where we exchanged particles can be dominated by the same Poisson process, we can use Lemma 2.5 here. There are at most $2L(4(t-s) + 2L)$ terms. So again,

$$\le P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0))2L(4(t-s) + 2L)2A_2 e^{-\delta_2(t-s)}. \tag{2.16}$$

(iv) $z \notin \mathbf{\Lambda}_{t-s}$, $(x, y) \notin \mathbf{\Lambda}_{t-s}$: For this part we use the observation that the information originated at z or (x, y) must already have reached 0 to contribute a nonzero term in the estimate of (2.13). By $x \to_{t-s} 0$ we mean that information originated at x at time 0 reaches 0 by time $t-s$. So,

$$\le P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0))2L\sum_{x,z} 2P(x \to_{t-s} 0; z \to_{t-s} 0). \tag{2.17}$$

$P(x \to_{t-s} 0; z \to_{t-s} 0)$ can be split into four parts according to whether x and z (or their permutation) are on the same side of 0 or on different sides of 0,

$$\begin{aligned} &\le P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0))4L\Bigg\{4 \sum_{x \ge [2\alpha(t-s)]} \sum_{z \ge [2\alpha(t-s)]} P(N(t-s) \ge x; \\ &\qquad\qquad N(t-s) \ge z)\Bigg\}. \end{aligned} \tag{2.18}$$

The double sum can be broken up according to whether $x \ge [2\alpha(t-s)]$ and $[2\alpha(t-s)] \le z \le x$ or $z \ge [2\alpha(t-s)]$ and $[2\alpha(t-s)] \le x \le z$. Since the two cases are symmetric, we look only at one of them.

$$
\begin{aligned}
&\sum_{x \geq [2\alpha(t-s)]} \sum_{[2\alpha(t-s)] \leq z \leq x} P(N(t-s) \geq x;\, N(t-s) \geq z) \\
(2.19) \qquad &\leq \sum_{x \geq [2\alpha(t-s)]} \sum_{[2\alpha(t-s)] \leq z \leq x} P(N(t-s) \geq x) \\
&\leq \sum_{x \geq [2\alpha(t-s)]} x P(N(t-s) \geq x) \leq A_2 e^{-\delta_2(t-s)}.
\end{aligned}
$$

The last estimate follows from Lemma 2.5. Hence, (2.18) can be bounded by

$$
\leq P^{(\eta,\zeta)}(\eta_s^\varepsilon(0) > \zeta_s^\varepsilon(0)) 32 L A_2 e^{-\delta_2(t-s)}. \tag{2.20}
$$

Finally, by adding up all four estimates we can find constants $0 < A_3 < \infty$ and $0 < \delta_3 < \infty$ so that (2.12) holds. □

3. Proof of Theorem 1.10. Recall that because of attractiveness and translation invariance, it suffices to show that there exist constants $\varepsilon_0 > 0$ and $\gamma > 0$ (which depends on ε_0) so that for all $\varepsilon \leq \varepsilon_0$,

$$
\left| T_t^\varepsilon f_0(+\vec{1}) - T_t^\varepsilon f_0(-\vec{1}) \right| \leq A e^{-\gamma t}, \tag{3.1}
$$

where $0 < A < \infty$. The proof will be done by using the formula of partial integration to extend the result from the Ising dynamic to the mixed dynamic.

We have to bound $T_t^\varepsilon f_0(+\vec{1}) - T_t^\varepsilon f_0(-\vec{1})$. The influence of the inside discrepancies will be estimated by using the formula of partial integration and by Lemma 2.11. The formula of integration by parts says

$$
\tilde{T}_t^\varepsilon f(\eta,\zeta) = \tilde{T}_t f(\eta,\zeta) + \varepsilon \int_0^t \tilde{T}_s^\varepsilon \tilde{\Omega}_m \tilde{T}_{t-s} f(\eta,\zeta)\, ds. \tag{3.2}
$$

Define by $g_t^\varepsilon T_t^\varepsilon f_0(\eta) - T_t^\varepsilon f_0(\zeta)$. With Proposition 2.1 and Lemma 2.11 we can write (3.2) as

$$
g^\varepsilon(t) \leq A_1 e^{-\delta_1 t} + \varepsilon \int_0^t A_3 e^{-\delta_3(t-s)} g^\varepsilon(s)\, ds. \tag{3.3}
$$

The goal is to show that $g^\varepsilon(t)$ converges exponentially fast to zero. It is trivial that $g^\varepsilon(t) \leq 2$. Plugging this bound into the right-hand side of (3.3) gives

$$
\begin{aligned}
g^\varepsilon(t) &\leq A_4 e^{-\delta_4 t} + 2 \cdot \varepsilon \int_0^t A_4 e^{-\delta_4(t-s)}\, ds \\
&\leq A_4 e^{-\delta_4 t} + 2 \frac{\varepsilon A_4}{\delta_4},
\end{aligned} \tag{3.4}
$$

where $A_4 = \max(A_1, A_3)$ and $\delta_4 = \min(\delta_1, \delta_3)$. Continuing this and using induction, we get

$$
g^\varepsilon(t) \leq A_4 e^{-\delta_4 t} \sum_{k=0}^{n} \frac{(\varepsilon A_4 t)^k}{k!} + 2 \left(\frac{\varepsilon A_4}{\delta_4} \right)^{n+1} \tag{3.5}
$$

for $t \geq 0$ and $n \in \mathbf{N}$. Let $n \to \infty$. Then, if $\varepsilon A_4 < \delta_4$, we get

$$g^{\varepsilon}(t) \leq A_4 e^{-(\delta_4 - \varepsilon A_4)t}. \tag{3.6}$$

Hence, if $\varepsilon_0 < \delta_4 / A_4$, we obtain

$$\begin{aligned} T_t^{\varepsilon} f_0(+\vec{1}) - T_t^{\varepsilon} f_0(-\vec{1}) &\leq A_4 e^{-(\delta_4 - \varepsilon A_4)t} \\ &\leq A e^{-\gamma t}, \end{aligned} \tag{3.7}$$

for all $\varepsilon \leq \varepsilon_0$, where γ and A are arbitrarily positive constants. This proves Theorem 1.10. □

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