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CORRECTION

AN INVARIANCE PRINCIPLE FOR DIFFUSION IN TURBULENCE

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The use of the Poincaré inequality in (44), page 768, is in error. Instead, we should have used the Poincaré–Wirtinger inequality; see [1]. The estimation of the first term on the right-hand side of (42), page 768, thus needs to be reworked.

By the Poincaré–Wirtinger inequality and the fact that $|u_{k,\varepsilon}| \le |y_{k,\varepsilon}| + 1$ we have, for a certain positive constant c,

$$\begin{aligned}\iint_{\Omega_{2T,2R}} |u_{k,\varepsilon}\phi_t|\,dx\,dt &\le \iint_{\Omega_{2T,2R}} |y_{k,\varepsilon}|\,dx\,dt + |\Omega_{2T,2R}| \\ &\le \int_0^{2T} \left|\int_{B_{2R}} y_{k,\varepsilon}(t,x)\,dx\right| dt \\ &\quad + c\iint_{\Omega_{2T,2R}} |(\nabla y_k)(t/\varepsilon^2, x/\varepsilon)|\,dx\,dt + |\Omega_{2T,2R}|.\end{aligned} \tag{E1}$$

Since

$$\partial_t y_{k,\varepsilon}(t,x) = \sum_{i,j=1}^d \partial_{x_i}\big(a_{i,j,\varepsilon}(t,x)\partial_{x_j} y_{k,\varepsilon}(t,x)\big)$$

we have

$$\begin{aligned}\left|\int_{B_{2R}} y_{k,\varepsilon}(t,x)\,dx\right| &\le \left|\int_{B_{2R}} y^0_{k,\varepsilon}(x)\,dx\right| \\ &\quad + \sum_{i,j=1}^d \int_0^t \int_{\partial B_{2R}} |a_{i,j,\varepsilon}(s,x)||\partial_{x_j} y_{k,\varepsilon}(s,x)|\,ds\,S(dx).\end{aligned} \tag{E2}$$

Integrating both ends of (E1) over R from R_0 to $2R_0$ and using (E2) we obtain

$$\begin{aligned}&R_0 \iint_{\Omega_{2T,2R_0}} |u_{k,\varepsilon}\phi_t|\,dx\,dt \\ &\quad \le \int_{R_0}^{2R_0} dR \iint_{\Omega_{2T,2R}} |u_{k,\varepsilon}\phi_t|\,dx\,dt \\ &\quad \le 2T\int_{R_0}^{2R_0} dR \left|\int_{B_{2R}} y^0_{k,\varepsilon}(x)\,dx\right|\end{aligned}$$

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$$+\int_{R_0}^{2R_0} dR \int_0^{2T} dt \int_0^t \int_{\partial B_{2R}} |a_{i,j,\varepsilon}(s,x)||\partial_{x_j} y_{k,\varepsilon}(s,x)|\, ds\, S(dx)$$

$$+ cR_0 \iint_{\Omega_{2T,4R_0}} |(\nabla y_k)(t/\varepsilon^2, x/\varepsilon)|\, dx\, dt + R_0|\Omega_{2T,4R_0}|$$

$$\text{(E3)} \qquad \le 2T \int_{R_0}^{2R_0} dR \left| \int_{B_{2R}} y^0_{k,\varepsilon}(x)\, dx \right|$$

$$+ \int_0^{2T} dt \int_0^t \int_{A_{2R_0,4R_0}} |a_{i,j,\varepsilon}(s,x)||\partial_{x_j} y_{k,\varepsilon}(s,x)|\, ds\, dx$$

$$+ cR_0 \iint_{\Omega_{2T,4R_0}} |(\nabla y_k)(t/\varepsilon^2, x/\varepsilon)|\, dx\, dt + R_0|\Omega_{2T,4R_0}|$$

where $A_{2R_0,4R_0}$ is the annulus with inner and outer radii $2R_0$ and $4R_0$, respectively. As ε tends to zero the second and the third terms have finite limits P-a.s. by (Y2) of the main lemma, page 757, and Proposition 4, page 765. The first term can be shown to stay bounded as follows.

By (34), page 765, we have

$$\text{(E4)} \qquad \int_{B_R} y^0_{k,\varepsilon}(x)\, dx = \int_0^1 d\sigma \int_{B_R} x \cdot \nabla y^0_{k,\varepsilon/\sigma}(x)\, dx \qquad \forall R > 0.$$

Moreover, we know that

$$\sup_{\rho>0} \int_{B_R} |\nabla y^0_{k,\rho}(x)|^2\, dx = M(w) < +\infty, \qquad P\text{-a.s.}$$

from the individual ergodic theorem, Proposition 4, equation (33) (for small ρ), and the almost sure, local boundedness of $\nabla y^0_k(\cdot)$ (for intermediate and large ρ). Thus σ-integral of the right-hand side of (E4) can be bounded by

$$\sup_{\rho>0} \|\nabla y^0_{k,\rho}\|_{L^2(B_{2R_0})} \|x\|_{L^2(B_{2R_0})} < +\infty \qquad \forall R \in [R_0, 2R_0].$$

Therefore the first term on the right-hand side of (E3) remains bounded P-a.s., as $\varepsilon \downarrow 0$.

Summarizing, we have from (E1) that

$$\lim_{\varepsilon\to 0} \sup_{[0,T]} \int_{B_R} u_{k,\varepsilon}\, dx < +\infty, \qquad P\text{-a.s.}$$

The proof of (41), page 767, is thus complete in view of the point-wise estimate $|y_{k,\varepsilon}| \le |u_{k,\varepsilon}|$.

REFERENCES

[1] KESAVAN, S. (1989). *Topics in Functional Analysis and Applications*. Wiley, New York.

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