FUNDAMENTAL FORMULAS FOR THE DOOLITTLE METHOD, USING ZERO-ORDER CORRELATION COEFFICIENTS

By

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So far as the writer has been able to determine, fundamental formulas for the Doolittle method as applied to the solution of normal linear equations expressed in correlation coefficients have never before been developed. Because of their peculiar telescoping qualities, the writer has termed them "endothetic formulas." Perhaps the best way to judge the respective merits of three methods of solving simultaneous linear equations to obtain the coefficient of partial regression (the β's)—determinants, Kelley's partial regression method,[1] and Doolittle's direct substitution method[2]—is to compare the formulas by which each might be expressed.

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[1]Kelley, T. L. Chart to Facilitate the Calculation of Partial Coefficients of Correlation and Regression Equations. 1st ed. School of Education, Special Monograph No. 1. Palo Alto: Stanford University Publications, 1921.

[2]Wallace, H. A., and Snedecor, G. W. Correlation and Machine Calculation. 1st ed. Official Publ. Vol. 23, No. 35. Ames: Iowa State College of Agriculture, 1925.

THREE-VARIABLE FORMULAS

Determinants

$$\beta_{02.1} = \frac{r_{02} - r_{01}r_{12}}{1 - r_{12}^2} \qquad \beta_{01.2} = \frac{r_{01} - r_{02}r_{12}}{1 - r_{12}^2}$$

Kelley's

$$\beta_{02.1} = \frac{r_{02} - r_{01}r_{12}}{1 - r_{12}^2} \qquad \beta_{01.2} = \frac{r_{01} - r_{02}r_{12}}{1 - r_{12}^2}$$

Doolittle's

$$\beta_{02.1} = \frac{r_{02} - r_{01}r_{12}}{1 - r_{12}^2} \qquad \beta_{01.2} = r_{01} - r_{12}\beta_{02.1}$$

OPERATIONS REQUIRED IN SOLVING A THREE-VARIABLE PROBLEM

	Determinants	Kelley's	Doolittle's
Consulting tables	1	1	1
Adding	0	0	0
Subtracting	2	2	2
Multiplying	2	2	2
Dividing	2	2	1

In a three-variable problem the Doolittle method has but a very slight advantage over the Determinant method and the method used by Kelley in his *Chart.*

FOUR-VARIABLE FORMULAS

Determinants

$$\beta_{03.12} = \frac{r_{03}(1-r_{12}^2)-r_{01}\,r_{13}-r_{02}\,r_{23}+r_{12}(r_{01}\,r_{23}+r_{02}\,r_{13})}{1-r_{12}^2-r_{13}^2-r_{23}^2+2\,r_{12}\,r_{13}\,r_{23}}$$

$$\beta_{02.13} = \frac{r_{02}(1-r_{13}^2)-r_{01}\,r_{12}-r_{03}\,r_{23}+r_{13}(r_{01}\,r_{23}+r_{03}\,r_{12})}{1-r_{12}^2-r_{13}^2-r_{23}^2+2\,r_{12}\,r_{13}\,r_{23}}$$

$$\beta_{01.23} = \frac{r_{01}(1-r_{23}^2)-r_{02}\,r_{12}-r_{03}\,r_{13}+r_{23}(r_{02}\,r_{13}+r_{03}\,r_{12})}{1-r_{12}^2-r_{13}^2-r_{23}^2+2\,r_{12}\,r_{13}\,r_{23}}$$

Kelley's

$$\beta_{03.12} = \frac{\dfrac{r_{03}-r_{01}\,r_{13}}{1-r_{13}^2}-\dfrac{r_{02}-r_{01}\,r_{12}}{1-r_{12}^2}\times\dfrac{r_{23}-r_{12}\,r_{13}}{1-r_{13}^2}}{1-\dfrac{r_{23}-r_{12}\,r_{13}}{1-r_{12}^2}\times\dfrac{r_{23}-r_{12}\,r_{13}}{1-r_{13}^2}}$$

$$\beta_{02.13} = \frac{\dfrac{r_{02}-r_{01}\,r_{12}}{1-r_{12}^2}-\dfrac{r_{03}-r_{01}\,r_{13}}{1-r_{13}^2}\times\dfrac{r_{23}-r_{12}\,r_{13}}{1-r_{12}^2}}{1-\dfrac{r_{23}-r_{12}\,r_{13}}{1-r_{13}^2}\times\dfrac{r_{23}-r_{12}\,r_{13}}{1-r_{12}^2}}$$

$$\beta_{01.23} = \frac{\dfrac{r_{01}-r_{02}\,r_{12}}{1-r_{12}^2}-\dfrac{r_{03}-r_{02}\,r_{23}}{1-r_{23}^2}\times\dfrac{r_{13}-r_{12}\,r_{23}}{1-r_{12}^2}}{1-\dfrac{r_{13}-r_{12}\,r_{23}}{1-r_{23}^2}\times\dfrac{r_{13}-r_{12}\,r_{23}}{1-r_{12}^2}}$$

Doolittle's

$$\beta_{03.12} = \frac{r_{03} - r_{01} r_{13} - \dfrac{r_{02} - r_{01} r_{12}}{1 - r_{12}^2} \times (r_{23} - r_{12} r_{13})}{1 - r_{13}^2 - \dfrac{r_{23} - r_{12} r_{13}}{1 - r_{12}^2} \times (r_{23} - r_{12} r_{13})}$$

$$\beta_{02.13} = \frac{r_{02} - r_{01} r_{12}}{1 - r_{12}^2} - \frac{r_{23} - r_{12} r_{13}}{1 - r_{12}^2} \times \beta_{03.12}$$

$$\beta_{01.23} = r_{01} - r_{12} \beta_{02.13} - r_{13} \beta_{03.12}$$

OPERATIONS REQUIRED IN SOLVING A FOUR-VARIABLE PROBLEM

	Determinants	Kelley's	Doolittle's
Consulting tables	6	3	2
Adding	12	0	1
Subtracting	4	12	4
Multiplying	18	12	8
Dividing	3	11	3

In a four-variable problem the Doolittle method is seen to have a decided advantage over the other two. An examination and comparison of these fundamental formulas for three and four variables would seem to justify the conclusion that an increasing number of variables would but enhance the manifest superiority of the Doolittle method.

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