THE ELIMINATION OF PERPETUAL CALENDARS

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If we wish to find the day of the week for any date, one way to solve the problem is to use a perpetual calendar. Another way to solve the problem is to calculate the day of the week by mathematical methods. In the past these mathematical methods have been so complicated that it has been much more convenient to use a perpetual calendar. This explains why some people have put themselves to the expense of buying perpetual calendars. The purpose of this article is to provide a mathematical method which is so simple that the entire calculation can be done mentally and which is as convenient as a perpetual calendar. In this article this mathematical method is applied to the Gregorian, Julian, and World calendars. Since a great many records have been made using the Julian and Gregorian calendars, the adoption of the World calendar would not completely eliminate the usefulness of applying the mathematical method to the historical calendars. The mathematical method also shows to what extent the World calendar is a simplification; this is important because proposals to reform the present calendar are attracting world-wide attention.

In the theory of numbers occurs the expression,

$$a \equiv b \bmod p, \tag{1}$$

which is read a is congruent to b modulo p, and which means that the difference of a and b is divisible by p. Since p in this article is always equal to 7, it is convenient to represent (1) by

$$a \equiv b. \tag{2}$$

Assume m stands for any number which represents any monthday of any month. Assume w stands for any number which represents any day of the week. It is assumed that 7 stands for Sunday, 1 for Monday, 2 for Tuesday, etc. It is assumed that the constant c for any month is the value of m at the first Sunday in that month. Then (2) becomes

$$w \equiv m - c, \tag{3}$$

which enables us to find w if m is known provided the constant c is known for the month in question. Consequently, all we need to complete our theory is to discover a method of finding c for any possible month.

First, there will be discussed rules for finding c for any month of the Gregorian calendar in 1935. An inspection of the calendar shows that c for December is

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equal to 1. Since November has 30 days, we can find c for it by adding 2, which is congruent to 30, to the c for December. Since the number of days in September, October, and November is 91, which is congruent to zero, the c's for September and December have the same value. In like manner, since c for September is 1, the c for June is 2, and the c for March is 3. We now have all the theory which is necessary to find w at any date in 1935. For example, suppose we wish to find w for April 17, and know that the c for December is 1. Then, by adding 2 we find that the c for March is 3. We are now in position easily to calculate that the c for April is 7. Applying (3) we find that w at April 17 is 3, which stands for Wednesday.

All that is necessary to complete our theory of the Gregorian calendar is to find rules for finding c for December of any possible year, because, if this is known, we can find c for any month in that year by the method used for 1935. It is convenient to represent the expression, "c for December 1935" by "C for 1935." In like manner C for any calendar year means c for December of that year. Since C for 1935 is 1 and since the number of days in 1936 is 366, which is congruent to 2, subtracting this 2, we find that C for 1936 is 6, because -1 is congruent to 6. Knowing C for 1936, we deduce that C for 1940, which is four years later, is 1, because $6 + 2$ is congruent to 1; and that C for 1928 is 2, found by subtracting 4. The C's for 1900, 1928, 1956, and 1984 are equal. Full centuries in order to be leap years must be divisible by 400. Since C for 1900 is 2, we find by adding 1 that C for 2000 is 3. Knowing C for 2000, we deduce by adding 2 that C for 2100 is 5. 1600, 2000, and 2400 have the same value of C. If it is assumed that the length of the tropical year is exactly 365.2425 days, we have all the theory which is necessary to find C for any possible year. Although this assumption contains a small error, any further discussion of it would hardly be of any practical interest. The foregoing theory provides complete methods for finding w by means of a series of steps, which are so simple that the entire calculation can be done mentally. For example, suppose we wish to find w for November 29, 1888. Each of the C's for 1800 and 1884 is 7. Therefore, C for 1888 is 2, which is congruent to $7 + 2$. Adding 2, c for November of this year is 4. Applying (3), we find that w at November 29, 1888 is 4, which stands for Thursday. In order to calculate mentally w for any date of the Gregorian calendar, it is only necessary for me to remember the foregoing mathematical method and to remember I was born on November 29, 1888, a Thanksgiving Day.

Deplorable changes were made in the Julian calendar between 45 B.C. and 1 A.D. Also it was not until 325 A.D. that the use of the 7-day week became general throughout the Roman Empire, gradually supplanting the old division of the month into Calends, Nones, and Ides. Therefore, in order to save space, the application of our theory prior to 1 A.D. is left to the reader. Starting with this year it is only necessary to discover a rule for finding the C's of the Julian calendar for the full centuries, because the rules of the Gregorian calendar apply to all other years. October 5, 1582, Old Style was the same day as October 15, 1582, New Style; the Gregorian calendar was born at this date. December 17, 1600, New Style was a Sunday, and was the same day as December 7, 1600, Old Style. Therefore, C for 1600, Old Style is 7. It is now a very simple matter to complete our theory of the Julian calendar. Since C for 1600 is 7, subtracting 1, C for 1500 is 6. 200, 900, and 1600 have the same value of C.

In the case of the World calendar the c's for the three months of each of the equal quarters can be found as follows. For the first month c is 1. Therefore, c for the second month is 5, which is congruent to $1 - 3$. Subtracting 2 from this 5, we find that c for the third month is 3.