THE STANDARD ERROR OF A "SOCIAL FORCE"

By Stuart C. Dodd

I. Definitions

In the theory of measurement of social forces certain special cases of frequent occurrence where the population shifts from one date of measurement to the next require the derivation of appropriate standard error formulae.

The theory may be briefly restated[1] in equations as follows: any measurable social change, C, in a population, P, may be defined as the difference in mean scores, S, from surveys or measurements on the dates denoted by subscripts

$$C_{2-1} = S_2 - S_1 = \frac{\Sigma s_2}{P} - \frac{\Sigma s_1}{P} \tag{1}$$

The momentum of a social change may be defined as the product of its time rate in years and the population that is being changed

$$M_{2-1} = PV_{2-1} \tag{2}$$

$$= \frac{PC_{2-1}}{Y_{2-1}} = \frac{P}{Y_{2-1}}(S_2 - S_1) \tag{2a}$$

where Y_{2-1} is the period from date 1 to date 2 and V is the velocity, or speed of change, in that period. The acceleration of a social change is definable as the rate of change of the velocity of change

$$A = \frac{V_{4-3} - V_{2-1}}{.5Y_{(4-3-2+1)}} \tag{3}$$

where each velocity, being an average for its period, is taken as representing the mid-date of that period.

The resultant social force which produces a measured change is now definable as that which accelerates the change in a population. It is measurable as the product of the acceleration and the population.[2]

$$F = AP \tag{4}$$

$$= \frac{P}{.5Y_{(4-3-2+1)}}\left(\frac{S_1}{Y_{2-1}} - \frac{S_2}{Y_{2-1}} - \frac{S_3}{Y_{4-3}} + \frac{S_4}{Y_{4-3}}\right) \tag{5}$$

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[1] *A Controlled Experiment on Rural Hygiene in Syria*, Dodd, S. C., Publications of the American University of Beirut, Syria, Social Science Series No. 7, 1934, pp. 336.

Also, *A Theory for the Measurement of Some Social Forces*, Dodd, S. C., Scientific Monthly, Vol. XLIII, No. 1, July 1936, pp. 58–62

[2] Force thus defined in terms of its effect is a resultant force, i.e., the residual force after deducting all resisting forces from the total force in the direction of the change observed. This formula defines quantitatively and exactly the "net" force not the "gross" force

II. The Sampling error of one case (momentum)

The formulae for the standard errors of sampling for the above concepts, social change, velocity, momentum, acceleration and force, (C, V, M, A, and F) have been published for the case where the population, P, is the same on all dates of measurement. But it is not always possible to observe the ideal experimental technic of holding the population unchanged in number nor to select out individuals common to all the surveys and to neglect the rest. Ordinarily there will be different P's, P_1, P_2, P_3, and P_4, at the different dates.

To derive the standard errors of (2) and (4) when P shifts, each P is considered to be a sub-sample[3] of the main sample which is $(P_1 + P_2 + P_3 + P_4)$. The orthodox view of sampling is taken where the sub-samples may differ in size but maintain fixed proportions in each main sample which is drawn from the "parent" population.

Let primes denote an M, or other function of (1) to (5), which is an approximation due to the shifting of the population and the use of an average P.

To simplify and generalize the notation, let k denote the constant term compounded of P's and Y's which is associated with each S. The first subscript of k denotes the function, f, which is any particular one of the left hand members of equations (1) to (5) and the second subscript denotes the date of its S. Thus, from (2a)

$$k_{M1} = \frac{-P_1 + P_2}{2Y_{2-1}} = -k_{M2} \tag{6}$$

Then (2) may be rewritten:

$$M'_{2-1} = S_1 k_{M1} + S_2 k_{M2} \tag{7}$$

$$= \sum_1^2 Sk_M. \tag{7a}$$

To derive the standard error of (7) the total differential is:

$$dM'_{2-1} = k_{M1}\, d\left(\frac{\Sigma s_1}{P_1}\right) + k_{M2}\, d\left(\frac{\Sigma s_2}{P_2}\right) \tag{8}$$

If Q_{12} denotes the population common to both dates of measurement so that:

$$\begin{aligned} P_1 &= Q_{12} + Q_1 \\ P_2 &= Q_{12} + Q_2 \end{aligned} \tag{9}$$

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producing the change. It thus measures only the *observable part* of the total forces in the situation. The fundamental problem remains, as always in science, to observe more adequately, to devise experimental and statistical technics for measuring the different forces (in isolation and in combinations) which facilitate or resist the measured change.

[3] The author is indebted to Mr. S. S. Wilks (Princeton) for this method of deriving these standard errors in a fluctuating population.

and, since the differential of a sum is the sum of the differentials of the several terms, (8) becomes

$$dM'_{2-1} = \frac{k_{M1}}{P_1}\left(\sum_1^{Q_{12}} ds_1 + \sum_1^{Q_1} ds_1\right) + \frac{k_{M2}}{P_2}\left(\sum_1^{Q_{12}} ds_2 + \sum_1^{Q_2} ds_2\right) \tag{10}$$

Squaring gives

$$\begin{aligned}(dM'_{2-1})^2 &= \frac{k^2_{M1}}{P_1^2}\left(\sum ds_1\right)^2 + \frac{k^2_{M2}}{P_2^2}\left(\sum ds_2\right)^2 \\ &+ \frac{2\,k_{M1}\,k_{M2}}{P_1\,P_2}\left[\sum_1^{Q_{12}} ds_1 \sum_1^{Q_{12}} ds_2 + \sum_1^{Q_{12}} ds_1 \sum_1^{Q_2} ds_2 + \sum_1^{Q_1} ds_1 \sum_1^{Q_{12}} ds_2 + \sum_1^{Q_1} ds_1 \sum_1^{Q_2} ds_2\right]\end{aligned} \tag{11}$$

On summing and dividing by the number of cases to get the expected values, the last three terms in the square brackets vanish. Using the relation where, in random sampling, the correlation between two variables is the same as the correlation between their means

$$r_{12} = r_{s_1s_2} = \frac{\Sigma\, S_1S_2}{Q_{12}\,\sigma_1\sigma_2} = \frac{\Sigma\left(\frac{\Sigma\, s_1}{Q_{12}} \cdot \frac{\Sigma\, s_2}{Q_{12}}\right)}{Q_{12}\,\frac{\sigma_1\,\sigma_2}{\sqrt{Q_{12}\cdot Q_{12}}}} \tag{12}$$

gives

$$\sigma^2_{M'_{2-1}} = \frac{k^2_{M1}\,\sigma_1^2}{P_1} + \frac{k^2_{M2}\,\sigma_2^2}{P_2} + \frac{2\,k_{M1}\,k_{M2}\,Q_{12}\,\sigma_1\,\sigma_2\,r_{12}}{P_1\,P_2} \tag{13}$$

Standard error of momentum when the population shifts

The best estimates of σ_1 and σ_2 are the standard deviations of the scores, s_1 and s_2, and the best estimate of r_{12} is, strictly, the covariance of the common cases divided by the two sigmas. Unless the selection of Q_{12} out of P_1 and P_2 curtails the range in some way (i.e., Q_{12} is not a random selection), then, except for sampling variation, σ_1 and σ_2 are the same in the Q_{12} population as in the P_1 and P_2 populations so that there is only a sampling discrepancy between the ratio above and the r_{12}, the observed correlation between the s_1 and s_2 scores in the Q_{12} population.

III. The generalized standard error

The above standard error may be readily generalized. Any of the equations (1) to (5) may be expressed as a simple linear sum of the products of a variable, S, and its appropriate constant, k.

$$f = \sum_{i=1}^{i=n} S_i\, k_{fi} \tag{14}$$

where f is any one of the concepts S, C, V, A, M or F defined by (1) to (5) and n is the number of surveys, or different S's involved, and i denotes each survey in turn from 1 to n. Thus where f means F, (5) becomes:

$$f'_F = F' = k_{F1} S_1 + k_{F2} S_2 + k_{F3} S_3 + k_{F4} S_4 = \sum_{i=1}^{i=4} k_{Fi} S_i \tag{15}$$

where

$$k_{F1} = -\, k_{F2} = \frac{P_1 + P_2 + P_3 + P_4}{2\, Y_{(4-3-2+1)}\, Y_{(2-1)}} \tag{16a}$$

$$k_{F4} = -\, k_{F3} = \frac{P_1 + P_2 + P_3 + P_4}{2\, Y_{(4-3-2+1)}\, Y_{(4-3)}}. \tag{16b}$$

In the special case when a force, F, has been determined from only three surveys using two consecutive periods, $n = 3$ and

$$k_{1F} = \frac{P_1 + P_2 + P_3}{1.5\, Y_{(3-1)}\, (Y_{2-1})} \tag{16c}$$

$$k_{F2} = -\, \frac{(P_1 + P_2 + P_3)\, (Y_{(2-1)} + Y_{(3-2)})}{1.5\, Y_{(3-1)}\, Y_{(3-2)}\, Y_{(2-1)}} \tag{16d}$$

$$k_{F3} = \frac{P_1 + P_2 + P_3}{1.5\, Y_{(3-1)}\, Y_{(3-2)}} \tag{16e}$$

If the difference between two forces (or other functions, f) has been measured in either the same or in different populations and the significance of the difference in terms of its standard error is desired, f of (14) can also denote that difference.

$$f_{dF} = F_a - F_b; \qquad f_{dM} = M_a - M_b; \text{etc.} \tag{17}$$

It is only necessary to write the difference as a linear sum of products of S and k on the model of (2a) or (5) to get the k-values for that particular f.

It is now possible to write the standard error formula for f in a single generalized form that covers all the concepts and their differences as defined in equations (1) to (5), (14) and (17). Observing that (14) is the general case for n surveys of the particular case (7a) where $n = 2$, it becomes evident, that on taking differentials, squaring, summing, and dividing the linear sum of the n terms of (14) there results n^2 terms of which there are n that are variances (times constants) of the sort $\frac{k^2\sigma^2}{P}$ and $\frac{n^2 - n}{2}$ are different terms each occurring twice that are covariances (times constants) of the sort $\frac{kkQ\sigma\sigma r}{PP}$. From these rough considerations as well as from rigorous derivation, the generalized standard error of (14) is found to be:

$$\sigma_f^2 = \sum_{1}^{n^2} \frac{k_{fi}\, \sigma_i\, k_{fj}\, \sigma_j\, Q_{ij}\, r_{ij}}{P_i\, P_j}, \tag{18}$$

The generalized standard error.

Where i and j denote each of the n surveys in turn. There will thus be n^2 terms to be summed—the number of combinations of i with j including the cases where $i = j$.

The derivation of (18) as well as its computation from data and its interpretation in special cases can all be made clearer by arranging the terms in a square array as follows:

	$i \rightarrow$	1	2	......	n
j ↓	Coefficients ↓ →	$\frac{k_{f1}\sigma_1}{P_1}$	$\frac{k_{f2}\sigma_2}{P_2}$	......	$\frac{k_{fn}\sigma_n}{P_n}$
1	$\frac{k_{f1}\sigma_1}{P_1}$	P_1 ( )	$Q_{12}r_{12}$ ( )	......	$Q_{1n}r_{1n}$ ( )
2	$\frac{k_{f2}\sigma_2}{P_2}$	$Q_{12}r_{12}$ ( )	P_2 ( )	......	$Q_{2n}r_{2n}$ ( )
⋮	⋮	⋮	⋮	......	⋮
n	$\frac{k_{fn}\sigma_n}{P_n}$	$Q_{1n}r_{1n}$ ( )	$Q_{2n}r_{2n}$ ( )	......	P_n ( )

To get σ_f write the computed values of the coefficients $\frac{k\sigma}{P}$ as captions of rows and of columns and write each computed Qr value in its appropriate cell, noting that in the main diagonal cells the self-correlations are unities and the population common to both column and row surveys, Q_{ij} is the entire population of that survey as $Q_{ij} = P_i$ when $i = j$. Thus $Q_{11} = P_1$. Next in each cell's parenthesis enter the product of three factors, namely: a) the cell Qr term, b) the column coefficient, and c) the row coefficient. The sum of these products in the parentheses, n^2 in number, is σ_f^2 of (18).

From the above square array it becomes clear that whenever in (17) the difference of two observed forces, or other functions, is derived from *different* populations the Q between these populations is zero so that the entire product terms in those cells vanish. Thus in the very simplest and familiar case of comparing two means from different populations, $n = 2$, $Q_{12} = 0$, $k = 1$, and (18) reduces to the usual sum of the two variances of the two means

$$\sigma^2 \text{ difference in means} = \frac{\sigma_1^2}{P_1} + \frac{\sigma_2^2}{P_2} \tag{19}$$

IV. Some special cases

It should be observed that the above formulae for the standard errors when P shifts all become identical with the simpler formulae previously derived for the case of a constant P. In this case, every $Q_{pq} = P_p = P_q$ and in the square array (in addition to k's which no longer involve an average P), the Q or P of the cells and the P's in the row coefficients, may be omitted as they cancel each other out.

Another special but very frequent case is where the social change is not given in terms of a difference in means, S_1 and S_2, but in terms of a difference in percentages, as when a literacy rate rises from 30% to 40%. A percentage can be viewed as a mean of a two-category, all-or-none, present-or-absent variable such as: A, non-A (foreign or native born, literate or illiterate, etc), where A is assigned a value of 1 and non-A a value of 0. Then the sum of the values of A, each times its frequency, divided by the population is both a proportion and a mean. Its standard error in the percentage, p, form of expression is then equal to it in the mean form:

$$\sigma_p = \frac{p\sqrt{1.00 - p}}{\sqrt{P}} = \sigma_S = \frac{\sigma_s}{\sqrt{P}} \qquad \left(\text{where } s = 1 \text{ or } 0 \text{ and } p = \frac{\Sigma s}{P} = S\right) \tag{20}$$

so that where S_i in (14) is a percent $p(1.00 - p)$ should be substituted for σ_i (and σ_j) in (18). In this case the appropriate formula to use for getting r_{ij} in (18) depends on the nature of the distribution of the variable that is expressed in percentage form. If the distribution is normal, tetrachoric r may be appropriate, while if the S in percentage form is from a two point distribution, r from a four fold point surface may be appropriate.

In all the above cases the usual interpretation of the significance of f in respect to sampling errors may be used in entering a normal probability table with a given σ_f from (18) and reading the probability of such a f occurring by chance.[4]

For a numerical illustration of this formula (18), consider the case of two villages, the statistical significance of whose momentums of a social change are to be determined. The data are from a study[1] of Syrian villages where an

[4] Mr. Wilks comments here that, "there is a more exact and rigorous test for comparing the two sets of S's which enter into a pair of M's or F's which involves some recent statistical theory but it is doubtful if the extra refinement is worth while at this stage of sociometric development."

itinerant Health Clinic in two years changed the average hygienic status of the families in each village by amounts of score (on a scale of 1 to 1000 points, devised for this study) as indicated in the table below.

	Village A	Village B
Mean score in 1931 $= S_1 =$	253	321
" " " 1933 $= S_2 =$	304	528
Population (families) in 1931 $= P_1 =$	46	46
" " " 1933 $= P_2 =$	40	32
Standard deviation of scores in 1931 $= \sigma_1 =$	54	39
" " " " " 1933 $= \sigma_2 =$	58	70
Families common to both censuses $= Q_{12} =$	40	32
Correlation of scores from the 2 dates $= r_{12} =$	.00	.19
$k_{M1} = -(P_1 + P_2)/2Y_{(2-1)} =$	−21.5	−19.5
$k_{M2} = -k_{M1} =$	21.5	19.5
$k_{M1}\sigma_1/P_1 =$	−25.24	−16.53
$k_{M2}\sigma_2/P_2 =$	31.17	42.65
$Q_{12}r_{12} =$	0	6.08
$\sigma_{M'_{2-1}} =$	261	249*
Momentum $= M'_{2-1}$	1,097	4,037
Significance ratio $M'_{2-1}/\sigma_{M'_{2-1}}$	4.2	16.2

* The calculation of this σ by (18) may be illustrated in detail:

Village B

Coefficients, $\frac{k\sigma}{P}$ → ↓		1	2
		−16.53	42.65
1	−16.53	46 ($= P_1$) (12,571)	6.08 ($= Qr$) (−4,286)
2	42.65	6.08 ($= Qr$) (−4,286)	32 ($= P_2$) (58,208)

$\Sigma(\ ) = 62{,}207 = \sigma^2_{M'_{(2-1)}}$

$\sigma_{M'_{(2-1)}} = 249$

The momentum of the movement towards improved hygiene achieved in village A is 4.2 times its standard error, while that of village B is 16.2 times its standard error. The excess momentum of village A over village B is $8.1 \left(= \frac{2940}{361}\right)$ times the standard error of their difference in momenta. Since all three of these significance ratios are well over 3 the conclusion is that the observed momenta and difference of momenta are statistically significant and cannot reasonably be due to sampling fluctuations. It may be noted that the significance ratios for the amounts of this social *change*, the difference in mean scores, are in close agreement with the above figures, being 4.1 and 15.9 for villages A and B respectively, instead of 4.2 and 16.2 as above. These discrepancies of a .1 and .3 in the statistical significance of these social changes compared with the corresponding social momenta are accounted for by the fact that the shift in the size of the population is allowed for in our formula for the case of momenta and is not considered in the usual formula for the case of social change.

A minimum of three measurements of one population is necessary to determine a social force. To determine its standard error all the correlations must be secured between every pair of measurements, each correlation derived from the part of the total population that is common to that pair of measurements. Obviously the data as currently reported from surveys and censuses and statistical bureaus do not meet these specifications. More rigorous analysis of social data and reporting of correlations in it is a prerequisite to the measurement of social forces and their significance.

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