NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

ON THE ANALYSIS OF A CERTAIN SIX-BY-SIX FOUR-GROUP LATTICE DESIGN USING THE RECOVERY OF INTER-BLOCK INFORMATION

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1. Introduction. A detailed description for a six-by-six four-group lattice design is given in a recent article [1] by the author, and the analysis is developed which uses only the intra-block information to correct the varieties for the block effects. Here is developed the analysis that makes use of both the intra- and the inter-block information.

Referring to Group X on page 307, [1], since block (1) contains varieties 1 to 6, and block (2) contains varieties 7 to 12, the difference between the means of these two blocks is also an estimate of the difference between the first six varieties and the second six varieties. The information obtained from such inter-block comparisons was ignored in the previous analysis. In attempting to use this information, the chief difficulty is to decide how estimates derived from the comparison of block totals shall be combined with the previous estimates. Since each block consists of six plots, comparisons between block totals may be expected to have a higher error variance than the within-block comparisons, just as in split-plot designs the main block comparisons usually have a higher error than the sub-plot comparisons. The problem is, therefore, to estimate the relative error variances of the inter- and intra-block comparisons, and then to combine the two types of estimates to the best advantage.

2. Calculations of the adjusted varietal totals. In addition to the equations (7), [1], which contain all the intra-block information, we now have the additional set of equations,

$B_i = 6\mu +$ (sum varietal constants in this block) $+ \epsilon_i$, which are estimated by

$$B_i = 6m + \Sigma v_{bi} + E_i .$$

In these equations and all the following equations, the double prime symbol ($''$) used in [1] is omitted, but the statistics have the same meaning as in equations (7), [1] except in this paper they are adjusted by both inter- and intra-block information.

[1] The author wishes to express his appreciation to W. G. Cochran of Iowa State College, who advised in the preparation of this analysis.

The general problem is to minimize the function,

$$F = WS(y_{ij} - m - v_j - b_i)^2 + \frac{W'}{6}S(B_i - 6m - \Sigma v_{bi})^2$$

subject to the restriction $\sum_{j=1}^{36} v_j = 0$ and $\sum_{e=x}^{u} \sum_{i=1}^{k} b_{ei} = 0$, and where $W = \frac{1}{\sigma^2}$ and $W' = \frac{1}{\sigma_b^2}$.

Following the method given in [1] the typical block equations for $b_{x1} \cdots b_{x6}$ is

$$b_{x1} = \frac{1}{6}\frac{W}{3W + W'}(4B_{x1} - T_{x1}) = \frac{1}{6}\frac{W}{3W + W'}C_{x1}$$

and for $b_{z1} \cdots b_{u6}$ is

$$B_{z1} = \frac{1}{144}\left\{\frac{1}{(W + W')(3W + W')}[(25W^2 + 22\,WW' + W'^2)C_{z1} + (W - W')^2(C_{z3} + C_{z5})] + \frac{W - W'}{W + W'}(C_{u2} + C_{u4} + C_{u6})\right\}.$$

It can be seen that for $W' = 0$, b_{x1} and b_{z1} are the intra-block values given in [1] and for $W' = W$ they are the randomized block values.

A typical adjustment varietal total then becomes

$$4v_1 + 4m = V_1 - \frac{W - W'}{W}(b_{x1} + b_{y1} + b_{z1} + b_{u2}).$$

3. Estimation of W and W'. Following the method presented by Cochran [6] and Yates [3], the error of a block total may be written as

$$E_i = e_{i1} + e_{i2} + \cdots + e_{i6} + 6b_i^e$$

where

$$V(e) = \sigma^2 \text{ and } V(b^e) = \sigma_b^2.$$

Hence $V(E_i) = 6\sigma^2 + 36\sigma_b^2$ and component (a) is thus an estimate of $\sigma^2 + 6\sigma_b^2$. One finds from evaluating the expected value of (15), [1] corrected for replicates, $E\left(\Sigma bC - \frac{\Sigma b \Sigma C}{6}\right)$, that the expected value of component (b) is $\sigma^2 + \frac{3}{4}\cdot 6\sigma_b^2$. In the analysis of variance if components (a) and (b) are pooled, one obtains the block variance B as an estimate of $\sigma^2 + \frac{7}{8}\cdot 6\sigma_b^2$. Since the intra-block variance is an estimate of σ^2 the estimates of the true variance between blocks, $\sigma^2 + 6\sigma_b^2$, is $\frac{8B - E}{7} = \frac{1}{W'}$.

4. Standard error of adjusted varietal means. The standard error of the difference between the adjusted means of two varieties which appear together in the same blocks in groups Z or U, is

$$\frac{1}{4kW}\left[(k - 2) + \frac{8W}{3W + W'}\right],$$

obtained by the method outlined by Cochran. Similarly, for the case in which the varieties are together in the same block in groups Z or U.

When an attempt is made to express the difference between these two adjusted varieties which appear together in the same block in groups X or Y in terms of the levels of the main effects and interactions, the interactions are no longer unconfounded and the method employed above breaks down.

If one is willing to assume that the formula for the variance of the difference between two adjusted varietal means for varieties which appear together in the same block in the groups X or Y is of the form $\frac{1}{24\,W}\left(A + \frac{BW}{3W + W'}\right)$ the constants may be determined by the values already known, [1]. This form can be shown to be that for a quadruple lattice.

The formula $\frac{1}{24W}\left(A + \frac{BW}{3W + W'}\right)$ must reduce to the value for intra-block analysis [1] when $W' = 0$, and when $W = W'$ to the value for complete randomized blocks. When these conditions are imposed, the formula becomes

$$\frac{1}{144W}\left(16 + \frac{80W}{3W + W'}\right).$$

This value is slightly larger than the value obtained when the adjusted varieties appear together in the same block in groups Z or U, as should be the case. This gives us a lower limit. One can arrive at the upper limit in the following manner: suppose the variance $(\text{intra})_1$ obtained in the intra-block analysis for the difference between two varietal means such as v_1 and v_2 is greater than that for varietal means v_8 and v_1 $(\text{intra})_2$, then it follows that:

$$(\text{inter} + \text{intra})_1 \leqq (\text{inter} + \text{intra})_2 \times \frac{(\text{intra})_1}{(\text{intra})_2}.$$

Using this relation, the upper limit for two varieties together in the same block in groups X or Y is

$$\frac{1}{24W}\left(3 + \frac{12W}{3W + W'}\right)\frac{64}{63},$$

which gives a value slightly greater than the formula derived, as it should if it is to be the upper limit. In a similar manner one gets the variance for the difference between varietal means not appearing together in the same block.

5. Efficiency of the design to the randomized complete blocks. By the method outlined by Cochran [6] the efficiency can be shown to be measured by the ratio of

$$\frac{\frac{k}{W} + \frac{1}{W'}}{k + 1}$$ to 4 (average error variance of the difference between two plots).

It will be noted, by using the above formula, that the gain in efficiency for the numerical problem given in [1] is 1.003, which for our purpose here is zero.

This, in general, will not be the case, for on most soils there is a block difference. In this particular test the ground used had been previously filled in with well mixed soil. The efficiency for the analysis given in [1] relative to the randomized complete blocks was less than 1.00.

This paper and the previous one show what a long tedious procedure is necessary to analyze the data, when the design does not follow the rules for the construction of the lattice, triple lattice, etc. The complexity of these methods stresses the importance, to those designing experiments, of not deviating from the established design if the most information is to be secured from the data with simple calculations.

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FURTHER REMARKS ON LINKAGE THEORY IN MENDELIAN HEREDITY

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In the following an explicit formula for the distribution of genotypes in case of three Mendelian characters will be given [formula (5)]. The complete discussion of the case $m = 3$ suggests a supplement (as stated in the last paragraph of this paper) to the general limit theorem dealing with m characters.

In an earlier paper[1] recurrence formulae have been derived which furnish the distribution of genotypes in the nth generation if the distribution in the $(n - 1)$th generation and the "linkage distribution" (l.d.) are known. It was also shown how to "integrate" this system of difference equations so as to determine the distribution in the nth generation directly from that in the 0th generation. This last method, though straightforward, requires however in each particular case quite a few operations.

In case m, the number of Mendelian characters, equals two, an explicit formula for the problem in question had been known. Denote by $p(x_1, x_2)$,

[1] HILDA GEIRINGER, *Annals of Math. Stat.* Vol. 15 (1944), pp.25–57. The notation in the present Note will be the same as in this paper.