out is formal, and yet inspiring, rigorous and yet never pedantic. It will serve as an example worthy of imitation and is an achievement on which the author deserves our sincere congratulations.

The Advanced Theory of Statistics. Vols. I and II. *Maurice G. Kendall.* London: C. Griffin and Co., Ltd. Vol. I. Second ed. revised, 1945; pp. xii, 457, 50 shillings. Vol. II. 1946; pp. viii, 521; 42 shillings.

REVIEWED BY M. S. BARTLETT

Cambridge University and *The University of North Carolina*

With the recent appearance of the second volume, it is now possible to review as one work this comprehensive treatise. To quote the author's opening remarks to the Preface to Volume I: "The need for a thorough exposition of the theory of statistics has been repeatedly emphasized in recent years. The object of this book is to develop a systematic treatment of that theory as it exists at the present time." An outline of the contents, which in the two volumes make up just on a thousand pages, will indicate that this formidable task has been squarely faced by the author, who, when a tentative co-operative venture of writing such a treatise was upset by the outbreak of the war, continued alone with the project.

Volume I contains sixteen chapters. The first six introduce the concept of frequency distributions via observational data on groups and aggregates, and their mathematical representation (Ch. 1), measures of location and dispersion (Ch. 2) and moments and cumulants in general (Ch. 3), characteristic functions (Ch. 4), and ending with a description of the standard distribution functions, such as the binomial, Poisson, hypergeometric and normal distributions, and the Pearson and Gram-Charlier systems. The next section opens with probability (Ch. 7) and proceeds to sampling theory (Chs. 8–11), including a chapter (Ch. 10) on exact sampling distributions, many of the standard sampling distributions being used in this chapter to illustrate the mathematical methods available for obtaining sampling distributions. Chapter 11 deals with the general sampling theory of cumulants, including a useful reference list of formulae and a demonstration, due to the author, of the validity of Fisher's combinatorial rules for obtaining these formulas. The section concludes with a chapter on the Chi-square distribution and some of its applications. The last four chapters of Volume I deal with association and contingency, correlation, including partial and multiple correlation, and rank correlation; this last chapter being a comprehensive treatment including comparatively recent results of the author.

It will be convenient to list also the contents of Volume II before any critical comment on either volume. The first section of the second volume comprises four chapters on the theory of estimation, including a derivation of the properties of the maximum likelihood estimate (Ch. 17) and separate chapters on Fisher's theory of fiducial probability and Neyman's theory of confidence intervals. The second main section, according to the author's remarks in the preface to Volume II, deals with the theory of statistical tests and comprises chapters 21, 23, 24, 26, 27 and 28; of these after an introductory chapter (Ch. 21) on tests of significance, chapters 23 and 24 cover analysis of variance, chapters 26 and 27 give a fairly detailed account of the general theory of significance-tests originated by Neyman and Pearson and Chapter 28 deals with the recently developed techniques of multivariate analysis. The remaining chapters are 22 on regression, 25 on the design of sampling enquiries, and Chapters 29 and 30 on time-series, another subject in which the author has himself taken an active interest. Finally, there are two appendices, A consisting of a few addenda to Volume I, and B an extensive bibliography of theoretical statistical papers.

The volumes are attractively printed; and each chapter concludes with a useful collection of examples for the reader.

In any comprehensive treatment of a wide subject there can be no clearly defined order of presentation; nevertheless, the author's order of chapters in Volume II and in particular his inclusion of analysis of variance among the chapters on the theory of statistical tests is a little puzzling, and the reviewer's preference would have been to see this important subject treated earlier, together with regression analysis, and their link with the classical method of least squares more firmly outlined. Incidentally, there appears to be no mention of the Fourier analysis of observational data except in its relation to periodogram analysis (Ch. 30). This change of order would perhaps also have allowed a shift forward of Chapter 25 on the design of sampling enquiries, and a more compact section on multiple correlation, culminating with the chapter on multivariate analysis before the chapters on the general theory of statistical inference were begun.

Another arrangement of rather doubtful value in Volume II is the allocation of separate chapters to fiducial probability and to the theory of confidence intervals. The problem of how to deal with a field which is still a battleground is admittedly not an easy one, and this particular one is an embarrassment at present to many teachers, but it may be questioned whether strict impartiality is the best answer. To take a hypothetical example, there would seem to be no particular virtue in a textbook which expounded, in parallel, statistical methods of inference using direct probabilities and the method of "inverse probability", leaving the reader to decide at the end which he should adopt.

The most criticizable arrangement, however, occurs in Volume I with the late and rather scanty treatment of probability in Chapter 7. To begin with examples of statistical data is sound, but since the whole conceptual model erected to deal with such data is based on probability theory, it does not seem sufficient for a reader who "feels keenly on the subject" to do as the author suggests in the Preface and read Chapters 7 and 8 after Chapter 1. Even if he does so, he will find no very clear exposition of the statistical theory of probability,—no mention, for example, of the laws of large numbers, whether for simple dichotomies or for entire continuous distribution functions, that show how the conceptual model adequately corresponds with the empirical notions of "in the long run" or "for a large enough sample". The actual arrangement, moreover, leads to an apparently rather arbitrary treatment of theorems on limiting distributions; the First Limit Theorem, which deals with the equivalence of the limits of distribution function and corresponding characteristic function sequences, is given in the chapter on characteristic functions (Ch. 4), and the Central Limit Theorem, dealing with the convergence to normality of a sum of n independent random variables, is given in the chapter on probability.

In the proof of the second part of the First Limit Theorem, dealing with the conditions under which a sequence $\phi_n(t)$ of characteristic functions determine the limiting distribution function $F(x)$, the author has not yet corrected an error that occurred in Cramér's original version, which Kendall follows (section 4.12). Correct conditions for convergence of the distribution function sequence $F_n(x)$ to $F(x)$ (at all continuity points of F) are convergence of the characteristic function sequence to $\phi(t)$ for all real t, uniformly in at least some finite t interval (cf. H. Scheffé, *Math. Reviews*, Vol. 6 (1945), p. 89).

Another proof in Volume I which appears to need clarification is the geometrical derivation of the distribution of the multiple correlation coefficient in the case of a non-zero true correlation (section 15.21). The blunt statement is made, following equation (15.51), that the sample correlation coefficient R and an angle ψ (defined in the text) are independent, a statement which is incorrect. However, if the logic of Fisher's original derivation is examined, it turns out that the relation of R and ψ is only required when the true correlation is zero; under such conditions R and ψ are independent.

In Volume II there is a sentence requiring correction and amplification in the derivation (in the case of zero true canonical correlations) of the sampling canonical correlation distribution (section 28.30). The sentence "Consider the distribution for a given value of t_{ij} and $z_{ij} \cdots$" should be corrected to read "Consider the distribution for a given value of $t_{ij} + z_{ij} \cdots$". Some justification that the distribution is independent of $t_{ij} + z_{ij}$ is then still needed.

There is inevitably, owing to the time the book was written, no mention of sequential analysis, the sampling technique developed during the war by Wald and others and only recently "derestricted". Again, in chapter 18, where the work of Aitken and Silverstone on unbiased estimates with minimum variance is referred to, the simple inequality connecting the variance of any unbiased estimate with Fisher's information function throws an interesting new light on this aspect of the estimation problem (see, for example, H. Cramér, *Mathematical Methods of Statistics*, section 32.3, or C. R. Rao, *Bulletin Calcutta Math. Soc.*, Vol. 37 (1945), p. 81), but was not known to the author when this chapter was written. Such omissions are merely an indication of the developing nature of the subject, and it is hoped they can be remedied in later editions. There is, however, especially in Volume II, an occasional impression of patchiness in the treatment not altogether excusable on such grounds. This can perhaps be illustrated from the last chapter, a valuable contribution to the still-growing subject of time-series, but where the importance of some known results does not always seem sufficiently stressed; in particular, the Wiener-Khintchine relation between the periodogram and correlogram is noted (section 30.68) as "an interesting relation", whereas it is a fundamental relation in the modern method of approach to time-series, giving much deeper insight into the correct interpretation of classical periodogram analysis.

These criticisms, which could be extended to cover minor errors and misprints, are not intended to detract seriously from what is a remarkable achievement. An excellent sense of proportion has been maintained throughout between mathematical theory and illustrative discussion and examples. This makes this treatise, if both the breadth and level of the subject matter are taken into account, at present unique. It will be an indispensable reference book to every teacher and advanced student of the theory of statistics.

Sequential Analysis of Statistical Data: Applications. Prepared by the Statistical Research Group, Columbia University for the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development. SRG Report 255, Revised; AMP Report 30.2R, Revised. New York: Columbia University Press, September 1945. pp. vii, 17; iv, 80; v, 57; iii, 25; iii, 18; iii, 39; ii, 41. $6.25. (London: Oxford University Press, 1946.)

REVIEWED BY JOHN W. TUKEY

Princeton University

Many of the features of this compendium are familiar to most of the readers of this review, but for the benefit of the others I shall enumerate them briefly. It consists of a heavy looseleaf binder containing 7 booklets of distinctive colors—each saddle stitched and usable separately. It is the last word (to date) in presenting sequential analysis to the statistician who may wish to use it in practice. It covers five elementary cases (each in a booklet, the two others being used for introduction and appendices):

Acceptance or rejection by percent defective (Sec. 2)
Comparative percent satisfactory (Double dichotomy) (Sec. 3)
Acceptance or rejection by the adequacy of the mean (with known variability) (Sec. 4)
Acceptance or rejection by the exact value of the mean (with known variability) (Sec. 5)
Acceptance or rejection by the smallness of the variability (Sec. 6)

These cases are covered in complete detail, with illustrative examples, tables and charts. A copy should be accessible to every teacher of statistics and to every statistician in industry or experimental work who can propose new techniques of testing.