

TABLE FOR ESTIMATING THE GOODNESS OF FIT OF EMPIRICAL DISTRIBUTIONS

BY N. SMIRNOV

1. **Editorial Note.** The table presented on pp. 280-281 was originally published in [1]. It gives values of

$$L(z) = 1 - 2 \sum_{\nu=1}^{\infty} (-1)^{\nu-1} e^{-\nu^2 z^2} = (2\pi)^{\frac{1}{2}} z^{-1} \sum_{\nu=1}^{\infty} e^{-(2\nu-1)^2 \pi^2 / 8z^2},$$

which is also derived in [2].

Let (X_1, \dots, X_n) be a sample of independent variables with the same continuous cumulative distribution function $F(x)$, and let $N(z)$ be the number of X_k which are $\leq z$. By empirical distribution is meant the step-function $F_n^*(z) = N(z)/n$. The maximum D_n of the difference $|F_n^*(z) - F(z)|$ is a random variable and $L(z)$ is the limiting cumulative distribution function of $n^{1/2}D_n$. If $D_{m,n}$ is the maximum of the difference $|F_m^*(z) - F_n^{**}(z)|$ between the empirical distributions of two independent samples of sizes m and n , respectively, then $L(z)$ is also the limiting cumulative distribution function of $(mn/(m+n))^{1/2}D_{m,n}$.

REFERENCES

- [1] N. SMIRNOV, "On the estimation of the discrepancy between empirical curves of distribution for two independent samples," *Bulletin Mathématique de l'Université de Moscou*, Vol. 2 (1939), fasc. 2.
 [2] W. FELLER, "On the Kolmogorov-Smirnov limit theorems for empirical distributions," *Annals of Math. Stat.*, Vol. 19 (1948), pp. 177-189.

TABLE of $L(z)$			TABLE of $L(z)$ — <i>Continued</i>			TABLE of $L(z)$ — <i>Continued</i>		
z	$L(z)$		z	$L(z)$		z	$L(z)$	
.28	.000	001	.69	.272	189	1.09	.814	342
.29	.000	004	.70	.288	765	1.10	.822	282
.30	.000	009	.71	.305	471	1.11	.829	950
.31	.000	021	.72	.322	265	1.12	.837	356
.32	.000	046	.73	.339	113	1.13	.844	502
.33	.000	091	.74	.355	981	1.14	.851	394
.34	.000	171	.75	.372	833	1.15	.858	038
.35	.000	303	.76	.389	640	1.16	.864	442
.36	.000	511	.77	.406	372	1.17	.870	612
.37	.000	826	.78	.423	002	1.18	.876	548
.38	.001	285	.79	.439	505	1.19	.882	258
.39	.001	929	.80	.455	857	1.20	.887	750
.40	.002	808	.81	.472	041	1.21	.893	030
.41	.003	972	.82	.488	030	1.22	.898	104
.42	.005	476	.83	.503	808	1.23	.902	972
.43	.007	377	.84	.519	366	1.24	.907	648
.44	.009	730	.85	.534	682	1.25	.912	132
.45	.012	590	.86	.549	744	1.26	.916	432
.46	.016	005	.87	.564	546	1.27	.920	556
.47	.020	022	.88	.579	070	1.28	.924	505
.48	.024	682	.89	.593	316	1.29	.928	288
.49	.030	017	.90	.607	270	1.30	.931	908
.50	.036	055	.91	.620	928	1.31	.935	370
.51	.042	814	.92	.634	286	1.32	.938	682
.52	.050	306	.93	.647	338	1.33	.941	848
.53	.058	534	.94	.660	082	1.34	.944	872
.54	.067	497	.95	.672	516	1.35	.947	756
.55	.077	183	.96	.684	636	1.36	.950	512
.56	.087	577	.97	.696	444	1.37	.953	142
.57	.098	656	.98	.707	940	1.38	.955	650
.58	.110	395	.99	.719	126	1.39	.958	040
.59	.122	760	1.00	.730	000	1.40	.960	318
.60	.135	718	1.01	.740	566	1.41	.962	486
.61	.149	229	1.02	.750	826	1.42	.964	552
.62	.163	225	1.03	.760	780	1.43	.966	516
.63	.177	753	1.04	.770	434	1.44	.968	382
.64	.192	677	1.05	.779	794	1.45	.970	158
.65	.207	987	1.06	.788	860	1.46	.971	846
.66	.223	637	1.07	.797	636	1.47	.973	448
.67	.239	582	1.08	.806	128	1.48	.974	970
.68	.255	780						

TABLE of $L(z)$ —
Continued

z	$L(z)$
1.49	.976 412
1.50	.977 782
1.51	.979 080
1.52	.980 310
1.53	.981 476
1.54	.982 578
1.55	.983 622
1.56	.984 610
1.57	.985 544
1.58	.986 426
1.59	.987 260
1.60	.988 048
1.61	.988 791
1.62	.989 492
1.63	.990 154
1.64	.990 777
1.65	.991 364
1.66	.991 917
1.67	.992 438
1.68	.992 928
1.69	.993 389
1.70	.993 823
1.71	.994 230
1.72	.994 612
1.73	.994 972
1.74	.995 309
1.75	.995 625
1.76	.995 922
1.77	.996 200
1.78	.996 460
1.79	.996 704
1.80	.996 932
1.81	.997 146
1.82	.997 346
1.83	.997 533
1.84	.997 707
1.85	.997 870
1.86	.998 023
1.87	.998 145
1.88	.998 297

TABLE of $L(z)$ —
Continued

z	$L(z)$
1.89	.998 421
1.90	.998 536
1.91	.998 644
1.92	.998 744
1.93	.998 837
1.94	.998 924
1.95	.999 004
1.96	.999 079
1.97	.999 149
1.98	.999 213
1.99	.999 273
2.00	.999 329
2.01	.999 380
2.02	.999 428
2.03	.999 474
2.04	.999 516
2.05	.999 552
2.06	.999 588
2.07	.999 620
2.08	.999 650
2.09	.999 680
2.10	.999 705
2.11	.999 728
2.12	.999 750
2.13	.999 770
2.14	.999 790
2.15	.999 806
2.16	.999 822
2.17	.999 838
2.18	.999 852
2.19	.999 864
2.20	.999 874
2.21	.999 886
2.22	.999 896
2.23	.999 904
2.24	.999 912
2.25	.999 920
2.26	.999 926
2.27	.999 934
2.28	.999 940

TABLE of $L(z)$ —
Concluded

z	$L(z)$
2.29	.999 944
2.30	.999 949
2.31	.999 954
2.32	.999 958
2.33	.999 962
2.34	.999 965
2.35	.999 968
2.36	.999 970
2.37	.999 973
2.38	.999 976
2.39	.999 978
2.40	.999 980
2.41	.999 982
2.42	.999 984
2.43	.999 986
2.44	.999 987
2.45	.999 988
2.46	.999 989
2.47	.999 990
2.48	.999 991
2.49	.999 992
2.50	.999 9925
2.55	.999 9956
2.60	.999 9974
2.65	.999 9984
2.70	.999 9990
2.75	.999 9994
2.80	.999 9997
2.85	.999 99982
2.90	.999 99990
2.95	.999 99994
3.00	.999 99997