ABSTRACTS OF PAPERS

(Abstracts of papers presented at the New York meeting of the Institute on April 8–9, 1949)

1. Adjustment of an Inverse Matrix Corresponding to a Change in One Element of a Given Matrix. JACK SHERMAN and WINIFRED J. MORRISON, The Texas Company Research Laboratories, Beacon, New York.

If one element, a_{RS}, in a square matrix A is changed by an amount Δa_{RS}, all the elements b_{ij} in the inverse matrix B are generally changed. A simple equation has been derived by means of which the elements b_{ij} in the resulting inverse matrix B' can be computed directly in terms of Δa_{RS} and the elements of B. The equation is

$$b'_{ij} = b_{ij} - \frac{b_{Sj}\, b_{iR}\, \Delta a_{RS}}{1 + b_{SR}\Delta a_{RS}}$$

It follows that any given square matrix can be transformed into a singular matrix by increasing any one element in the transposed inverse matrix.

2. The Distribution of the Number of Exceedances. E. J. GUMBEL, New York and H. VON SCHELLING, Naval Research Laboratory, New London, Conn.

The probability for the mth observation in a sample of size n taken from a population with an unknown distribution of a continuous variate to be exceeded x times in N future trials is studied. The averages, moments, and the cumulative probability of the number of exceedances are calculated with the help of the hypergeometric series. The tolerance limits constructed by Wilks are special cases of the cumulative probability. The mean number of exceedances is the same as in Bernoulli's distribution. In some cases there are two modes, namely $m - 1$ and $m - 2$. If $n = N$, the most probable number of exceedances over the mth largest value is either m, or $m - 1$, and the median number of exceedances is equal to $m - 1$. In 50% of all cases, the largest (smallest) of n past observations will not (always) be exceeded in n future observations. If n and N are both large and equal, the distribution of the number of exceedances over the median is normal whereas the distribution of the extremes, similar to Poisson's distribution, has a mean m, and a variance $2m$. The variance of the number of exceedances is largest for the median, and smallest for the extremes of the previous sample. These distribution-free methods may be applied to meteorological phenomena, such as floods, droughts, extreme temperatures (the killing frost), largest precipitations, etc., and permit the forecasting of the number of cases surpassing a given severity.

3. Note on the Power Function of a Quality Control Chart. LEO A. AROIAN, Hunter College, New York.

The power function of a quality control chart is given for a sequence of N sample points in terms of α and γ, the probability of a Type I error and the power function respectively for a single sample point. Two different models are considered and the generalization to two quality control charts is indicated.

4. Tests Between Two Means or Regression Coefficients When Observations are of Unequal Precision. UTTAM CHAND, University of North Carolina, Chapel Hill.

Relative merits of different tests available for testing two means or two regression coefficients in relation to asymmetric and symmetric aspects of Student's hypothesis in case of unequal population variances have been reconsidered. In this connection the distribution of a certain quantity t_k where k is some inexact value of the unknown ratio of variances has been obtained. The hypothesis of the equality of two linear regression functions in case of unequal residual variances has also been considered.

5. Functional Expansions. EUGENE W. PIKE, Boston, Massachusetts.

This paper calls attention to a new type of estimation problem, arising both in the interpretation of experimental data from complex experiments, and in the design of analogue computers for functions of several independent variables.

It has long been known, though not widely recognized, that the partial sums of rows and columns arising in the bivariate analysis of variance represent the least squares fit of a functional form $[f(x) + g(y)]$ to a tabular function $F(x, y)$ of two independent variables, for example. More recently, several people have realized gradually that independent causes may combine in much more complicated ways to produce a common effect, and that correspondingly more complicated functional combinations, such as $[f(x) + g(y) + h(x) \cdot k(y)]$, can be fitted by least squares to tabular functions of x and y.

Examples of such expansions, as applied both to the design of computers and to the analysis of experimental data, will be given.

This presentation is based on work supported by the Air Materiel Command, USAF.

6. The Geometric Range for Distributions of Cauchy's Type. E. J. GUMBEL, New York City, and R. D. KEENEY, Metropolitan Life Insurance Comany, New York City.

From each of N samples of large size n the largest and the smallest values $X_{n,\nu}$ and $X_{1,\nu}$ $(\nu = 1, 2, \cdots N)$ are taken, where each X is measured from the central value of Nn observations. The sample size must be so large that the probability of any extreme $X_{n,\nu}$ and $-X_{1,\nu}$ being negative may be neglected. The distribution of the geometric means ρ of the N pairs of extremes henceforth called geometric ranges, is derived under the assumption that the initial distribution is symmetric, unlimited and of the Cauchy type which implies that the moments of an order equal to, or larger than $k(k > 0)$ diverge. Let u be the expected largest value. Then the probability density of $\xi_k = 2u^k\rho^{-k}$ obtained from a theorem of Elfving (*Biometrika*, Vol. 35) is $\xi_k K_0(\xi_k)$ where K_0 is a Bessel function. This permits calculation of all moments of ξ_k. Methods are given for estimating the parameters u and k. The distribution of the geometric ranges ρ is again a Bessel function. A probability paper is constructed for testing the hypothesis that the initial distribution is of Cauchy's type. A strict parallelism is established between the asymptotic distributions of the range for the exponential type, and of the geometric range for Cauchy's type. This provides a criterion to which of the two types the initial distribution belongs.

7. On Sums of Random Integers Reduced Modulo m. A. DVORETZKY, Institute for Advanced Study, Princeton and J. WOLFOWITZ, Columbia University, New York City.

Let X_n, $(n = 1, 2, \cdots)$ be an infinite sequence of independent, integral-valued, chance variables, and let m be any fixed integer greater than 1. Put $S_n = \sum_{\nu=1}^{n} X_\nu$ and denote S_n reduced mod. m by Y_n; i.e., Y_n is a random variable which assumes only the values $j = 1, 2, \cdots, m$ with respective probabilities $P_n(j) = \text{Prob}\,\{S_n \equiv j \pmod{m}\}$. Necessary and sufficient conditions are obtained for Y_n to be equidistributed in the limit, i.e., for $\lim_{n=\infty} P_n(j) = \frac{1}{m}$. $(j = 1, 2, \cdots, m.)$ Some easily applicable sufficient conditions are deduced and the cases $m = 2, 3, 4$ are studied in detail. The rapidity with which $P_n(j) \to \frac{1}{m}$ is also studied

8. The Corpuscle Problem: Estimating the Surface-Volume Ratio of a Corpuscle of Arbitrary Shape. JEROME CORNFIELD, National Institutes of Health and HAROLD W. CHALKLEY, National Cancer Institute, Bethesda, Md.

Consider a space containing F, a closed figure of arbitrary shape, volume V and surface area S. Let a line segment of length r be thrown in the space in such a fashion that we have uniform distribution of the probabilities that the end point P occupies any position in the space and that the other end point P' occupies any position on the surface of a sphere of radius r with center at P. Count the number of end points falling in F (0, 1 or 2 for a single throw), call it the number of hits, and denote it by h. Count the number of times the line intersects the surface (0, 1 or 2 times for a single throw for a non-reentrant figure, possibly more for a re-entrant one), call it the number of cuts and denote it by c. Then, it is proved that $rE(h)/E(c) = 4V/S$. This result is intended to provide a theoretical basis for estimating the surface-volume ratio of physical objects of any shape.

9. Generalized Hit Probabilities with a Gaussian Target. D. A. S. FRASER, Princeton University.

In the *Supplement to the Journal of the Royal Statistical Society*, Vol. 8 (1946), L. B. C. Cunningham and W. R. B. Hynd proposed a problem and gave an approximate solution covering a partial range of parameter values: to find the probability that a moving target will survive a burst of "n" rounds from a rapid-firing gun, account being taken of correlation between the different points of aim.

Generalizing from the case of a two dimensional target to "k" dimensions, this paper gives the probability for $0, 1, 2, \cdots n$ hits, under the following assumptions: the "n" points of aim have a Multivariate Gaussian Distribution, the dispersion error has a Gaussian Distribution, and the target is a Gaussian Diffuse Target, that is, the probability of a hit on a particular round as a function of the coordinates of the shell has the form of "a constant times a Gaussian probability density function."

Limiting distributions are obtained as $n \to \infty$, subject to a variety of limiting conditions.

Numerical values for the probability of at least one hit are plotted when $n = 5$, for a range of values, relative to the target size, of dispersion and aiming errors.

10. A New Continuous Sampling Inspection Plan Based on an Analysis of Costs. F. E. SATTERTHWAITE, General Electric Company, Bridgeport, Connecticut.

Inspection, like all other industrial operations, must be run to produce the most return for the lowest cost. The costs include overhead and running inspection costs; complaint costs; rework and scrap costs; and the costs of unnecessary process rejections. Also one must consider the frequencies of occurrence of these costs. These include the process average percent defective; the probability of occurrence of a complaint; and the frequency of occurrence of quality deteriorations.

For continuous inspection, the percentage of the product to be inspected has a very simple formula: $P = \sqrt{SC/HM}$, where S is the sensitivity of the sampling plan used, C is the complaint cost, H is the effective inspection cost, and $1/M$ is the quality deterioration rate.

It was also necessary to develop a new continuous sampling inspection plan which would be efficient over the *entire* range of continuous sampling applications. The plan presented is a sequential plan which, with suitable attention to details, is easily applied on the shop floor. The Dodge Plan is a special case and is efficient only in a small percentage of applications.

11. On the Levels of Significance of the F and Beta Distributions. Leo A. Aroian, Hunter College, New York.

Two formulas are given for the determinations of the levels of significance of the F and Beta distributions. In the case of the F distribution a previous set of formulas (*Biometrika*, Vol. 34, pp. 359–360) is modified to give 3 significant figure accuracy, $n_1, n_2 \geqq 24$. The set for the Beta distribution is of Cornish-Fisher type, $p, q \geqq 6$. The advantage of these over Paulson's F formula and Carter's z formula are the avoidance of the solution of a quadratic in the case of Paulson's formula, and the avoidance of the exponential tables in the case of Carter's z formula. A short numerical table compares the three methods for selected values of n_1 and n_2.

12. Certain Statistics for Samples of 3 From a Rectangular Population, Julius Lieblein, National Bureau of Standards.

A continuation of a study presented at the Madison meeting of the Institute of Mathematical Statistics last September. (For abstract see *Annals of Math. Stat.*, December 1948, p. 595.) The previous paper derived properties of the statistics

$$y_1 = \frac{x' - x''}{x_3 - x_1}, \qquad y_2 = \frac{x' - x''}{2}, \qquad y_3 = \frac{x' + x''}{2},$$

where x_1, x_2, x_3 are the observations, ordered by increasing size, in an independent random sample of three observations from a *normal* population, and x' and x'', $x' \geq x''$, are the two *closest* of the three. In the present paper distributions (joint as well as simple) are obtained for the above three statistics and also for x''', the remaining observation not included in the closest pair, for samples of 3 from a *rectangular* population, and a theorem is proved concerning the distribution of y_1 for a wide class of continuous populations.

13. The Choice of Lot Inspection Plans of the Basis of Cost. F. E. Satterthwaite, and Burton Grad, General Electric Company, Bridgeport, Connecticut.

An extension of the first paper to single sampling inspection plans. The important concepts involved are the break-even quality level, the operating ratio, and the weighted prior odds that a lot is a good lot. Charts are being prepared which can be entered with simple functions of the costs and which give directly the sample size and acceptance number for the most efficient single sampling inspection plan.

It appears promising that the method can be extended to double and sequential sampling plans. This is imperative because of the large portion of the time that "no-inspection" is the most efficient single sampling plan.