This is not altogether surprising, since (as shown in Table 2) after the appropriate transformations the probabilities of both are identical functions of the respective transformed variates.

Of course the two systems are mutually exclusive: if the observed ranges can be reproduced by the first system we conclude that all moments in the initial distribution exist. If on the other hand, the observed geometric ranges can be represented by the second sytem we conclude that no moments of an order greater than k exist.

TABLE 2

RANGES AND GEOMETRIC RANGES

Type of Initial Distribution	Exponential	Gauchy
Variate	Range	Geometric Range
Definition	$w = x_n + (-x_1)$	$\rho = \sqrt{x_n(-x_1)}$
Transformation	$z = 2\exp\left[-\frac{\alpha}{2}(x_n - x_1 - 2u)\right]$	$\xi_k = 2u^k\rho^{-k}$
Logarithm	$\lg z = \lg 2 - \frac{\alpha}{2}(x_n - x_1 - 2u)$	$\lg \xi_k = \lg 2 - \frac{k}{2}(\lg x_n + \lg(-x_1) - 2\lg u)$
Probability	$G(w) = z\,K_1(z)$	$G_1(\rho) = \xi_k\,K_1(\xi_k)$
Distribution	$g(w) = \frac{\alpha z^2}{2}K_0(z)$	$g_1(\rho) = \frac{4k}{u}\left(\frac{\xi_k}{2}\right)^{2k+1}K_0(\xi_k)$
Median	$\tilde{w} = 2u + .9286/\alpha$	$2\lg\tilde{\rho} = 2\lg u + .9286/k$
Mean	$\bar{w} = 2u + 2\gamma/\alpha$	$\lg\overline{\rho^{-1}} = -\lg u + 2\lg\Gamma(1 + \frac{1}{2}k)$

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REMARK ON W. M. KINCAID'S "NOTE ON THE ERROR IN INTERPOLATION OF A FUNCTION OF TWO INDEPENDENT VARIABLES"

By T. N. E. Greville

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In a review of Dr. W. M. Kincaid's "Note on the Error in Interpolation of a Function of Two Independent Variables," (*Annals of Math. Stat.*, Vol. 19 (1948), pp. 85–88) which appeared in *Mathematical Reviews*, Vol. 9 (1948), p. 470, I stated that "a more simple and elegant, and equally general, expression is obtainable by a simple adaptation of formula (41), p. 215, of J. F. Steffensen's book, *Interpolation*."

This statement is not entirely correct and is also misleading in its implications since Dr. Kincaid's expressions are actually more general in certain respects, and simplicity and generality are not the only considerations nor, in this case, the most important ones. In setting up an expression for the remainder in an interpolation formula, the primary objective is to secure an efficient appraisal of the remainder. In this respect, Dr. Kincaid's expressions are superior as they involve only the higher derivatives of the function it is desired to represent, whereas Steffensen's method would always involve a first derivative term in such a way as to prevent any refinement of estimates of the error by introducing additional given values.

REMARK ON MY PAPER "ON A THEOREM OF HSU AND ROBBINS"

By P. Erdös

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Professor Robbins kindly pointed out that in my paper mentioned in the title (*Annals of Math. Stat.*, Vol. 20 (1949), p. 286–291) I have misquoted a statement in the paper of Hsu and Robbins ("Complete Convergence and the Law of Large Numbers" *Proc. Nat. Acad. of Sci.*, Vol. 33 (1947), p. 25–31). I attribute to Hsu and Robbins the conjecture (notations of my paper) that if $\sum_{n=1}^{\infty} M_n < \infty$ then (1) and (2) hold, and proceed to give a counter example. However, the conjecture of Hsu and Robbins is not the above false one but the following: If $\sum_{n=1}^{\infty} M_n < \infty$ and (1) holds then (2) also holds. This conjecture is true and is in fact proved in my paper.

Professor Robbins also points out that a slight modification of my theorem can be stated in a more concise form as follows: Let $X_1, X_2, \cdots$ be a sequence of independent random variables having the same distribution function $F(x)$, and let

$$Y_n = (1/n)\ (X_1 + \cdots + X_n)$$

Then the necessary and sufficient condition that

$$\sum_{n=1}^{\infty} P_r\{|Y_n| > \epsilon\} < \infty, \qquad \text{for every } \epsilon > 0,$$

is that

$$\int_{-\infty}^{\infty} x\, dF(x) = 0, \qquad \int_{-\infty}^{\infty} x^2\, dF(x) < \infty.$$