for a sample of size n. By reversing the process, it is clear that if $\nu_{1|n}$, $\nu_{1|n-1}$, $\cdots$, $\nu_{1|1}$ are known, the normalized moments for all samples of size no greater than n can be determined by successive differencing, although in this case there is a progressive loss of significant figures.

CORRECTION TO "THE PROBLEM OF THE GREATER MEAN"

By Raghu Raj Bahadur and Herbert Robbins

University of Chicago and University of North Carolina

In the paper mentioned in the title (*Annals of Mathematical Statistics*, Vol. 21 (1950), pp. 469–487), the paragraph on page 484 beginning "We have given no criterion . . ." is erroneous, and should be omitted. The following paragraph would then read: "Let us suppose that Ω is given by (33). Then $f^0(v)$ is admissible and minimax, by the preceding paragraph. There is, however, another reason for preferring $f^0(v)$ . . .."

We remark that in case a point on the plane $\{\omega : m_1 = m_2\}$ is an interior point of Ω and the risk function is $\bar{r}$, then (contrary to statements in the erroneous paragraph) $f^0(v)$ possesses the following property. *If $f(v)$ is a decision function such that $f(v) \not\equiv f^0(v)$ and*

$$\sup_{\omega \epsilon \Omega} \bar{r}(f \mid \omega) \leq \sup_{\omega \epsilon \Omega} \bar{r}(f^0 \mid \omega)(= \tfrac{1}{2}),$$

then $\bar{r}(f^0 \mid \omega) \leq \bar{r}(f \mid \omega)$ for all ω in Ω, the inequality being strict whenever $m_1 \neq m_2$. It follows that $f^0(v)$ is the unique decision function which is admissible and minimax. A proof of this remark is contained in an unpublished paper by R. R. Bahadur entitled "A Property of the t Statistic."

ERRATA

By P. V. Krishna Iyer

University of Oxford

In the author's paper "The theory of probability distributions of points on a lattice" (*Annals of Math. Stat.*, Vol. 21 (1950), pp. 198–217), read "2 × 2 × 3" for "2 × 3 × 3" on page 211, line 22, and on page 213, Table 8, heading.

ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Oak Ridge meeting of the Institute, March 15–17, 1951)

1. **Confidence Intervals for the Mean Rate at Which Radioactive Particles Impinge on a Type I Counter.** **(Preliminary Report.)** G. E. Albert, University of Tennessee and Oak Ridge National Laboratory.

The number of particles impinging on a Geiger-Mueller counter in a time interval of length t is assumed to be a random variable with a Poisson distribution of mean at. Starting with Feller's results for a Type I counter given in his paper "On probability problems in the theory of counters" in the *Courant Anniversary Volume*, 1948, it is shown that the count N registered by the counter in time t has the distribution: $Pr(N \geqq m) = 0$ if $m \geqq (t/u) + 1$, and $Pr(N \geqq m) = \exp(-\lambda)\Sigma_{k=m}^{\infty}\lambda^k/k!$, $\lambda = a[t - (m - 1)u]$, if $m < (t/u) + 1$, where u is the dead time of the counter. Confidence interval charts for the parameter $b = au$ for various values of $t' = t/u$ are prepared by the usual inversion procedure. If N and $t-Nu$ are both large, approximate confidence intervals for the parameter a take the simple form

$$(N \pm x_p N^{\frac{1}{2}})/[t - (N - 1)u],$$

where x_p is the two-tailed percentage point of the normal distribution for the confidence level $1 - p$.

2. A Problem of Elapsed Times in a Sequence of Events. OSMER CARPENTER, Carbide and Carbon Chemicals Division, Oak Ridge.

The problem considered refers to a series of random events forming a sequence in time or space, for example, the emission of particles by radioactive matter. From a sequence f of such events, a derived sequence g is formed by selecting from f all those events which follow the preceding event by an elapsed time greater than a given constant, $U \geqq 0$. The times between successive events in the sequence f are given to the independently distributed by a known distribution function, $F(t)$. It is required to find the distribution functions of elapsed time and of the number of counts per fixed time interval for the derived sequence, g. A general method is applied to the solution of the exponential case, $F(t) = ke^{-kt}$.

3. On the Existence of Unbiased Tests for Testing Composite Hypotheses. ESTHER SEIDEN, University of Buffalo.

The following problem was suggested by J. Neyman. Let X be an observable random variable, multivariate or not, and H a composite hypothesis concerning X. Let $\bar{H}$ denote a hypothesis, concerning X, alternative to H. Finally, let α be a chosen level of significance. What restriction should one impose on the hypotheses H and $\bar{H}$ in order that there exists a critical region w such that (i) $P(X \varepsilon w \mid H) \equiv \alpha$, and, whatever be the simple hypothesis $h \varepsilon \bar{H}$, (ii) $P(X \varepsilon w \mid h) > \alpha$? It is shown now that if H as well as $\bar{H}$ consists in assuming that the random variable X follows a continuous distribution law, then there exists always the most powerful region w satisfying conditions (i) and (ii), provided that the distributions belonging to H and $\bar{H}$ are linearly independent. If H and $\bar{H}$ are infinite families of absolutely continuous distributions and condition (i) is replaced by (i') $P(X \varepsilon w \mid H) \leqq \alpha$, then for some α less than $\frac{1}{2}$ there exists a region w satisfying conditions (i') and (ii), provided that the convex closures of H and $\bar{H}$ are disjoint.

4. Group Divisible Incomplete Block Designs. R. C. BOSE, University of North Carolina.

An incomplete block design with v treatments each replicated r times in b blocks of size k is said to be group divisible if the treatments can be divided into m groups each with n treatments, so that the treatments of the same group occur together in λ_1 blocks and treatments of different groups occur together in λ_2 blocks, $\lambda_1 \neq \lambda_2$. The parameters are connected by the relations $v = mn$, $bk = vr$, $\lambda_1(n - 1) + \lambda_2 n(m - 1) = r(k - 1)$. It is shown that these designs fall into three classes: (i) singular for which $r = \lambda_1$, (ii) semiregular for which $r > \lambda_1$, $rk = v\lambda_2$, (iii) regular for which $r > \lambda_1$, $rk > v\lambda_2$. It is proved that for regular designs $b \geqq v$, and for semiregular designs $b \geqq v - m + 1$, every block containing the same number of treatments from each group. A singular design is always derivable from a balanced incomplete block design by replacing each treatment by a group of n new treatments. When $b = v$ the quantity $(r - \lambda_1)^{m(n-1)}(rk - v\lambda_2)^{m-1}$ must be a perfect square, and the Hasse invariant of NN', where N is the incidence matrix of the design, must be $+1$. The value of this invariant has been calculated in terms of the parameters. The parameters for all group divisible designs with $r \leqq 10$, $k \leqq 10$, whose existence is not ruled out by theorems stated above, have been listed. Combinatorial solutions for most of these have been derived, though there remain a number of unsolved cases. The analysis of variance and the equations for intra- and inter-block estimates have been given. These designs are likely to prove useful both in varietal trials and in factorial experiments.

5. **Orthogonal Arrays of Strength Two and Three.** R. C. Bose and Kenneth A. Bush, University of North Carolina.

Consider a matrix $A = ((a_{ij}))$ with m rows and N columns where each element a_{ij} represents one of the s integers $0, 1, 2, \cdots, s - 1$. The columns of any t-rowed submatrix of A provide N ordered t-plets. The matrix A is called an orthogonal array (N, m, s, t) of size N, m constraints, s levels, and strength t if each of the C_t^m partial t-rowed matrices formed from A contains all the s^t possible ordered t-plets each repeated λ times $(N = \lambda s^t)$. The known upper bounds for the number of constraints when $t = 2$ and 3 have been improved: If $\lambda - 1 = a(s - 1) + b$, $0 \leqq b < s - 1$, and n is the largest positive integer (including 0) consistent with $s(b - 2n) \geqq (b - n)(b - n + 1)$, then for the case $t = 2$, $m \leqq I[(\lambda s^2 - 1)/(s - 1)] - n - 1$, and for the case $t = 3$, $m \leqq I[(\lambda s^2 + s - 2)/(s - 1)] - n - 1$. Methods of constructing orthogonal arrays of strength 2 and 3 have been investigated. A difference theorem enabling the construction of the arrays (18, 7, 3, 2) and (32, 9, 4, 2) has been proved, and it is shown that if $s = p^n$, $\lambda = p^u$, where p is a prime, then we can construct the array $(\lambda s^2, m, s, 2)$, with

$$m = 1 + p^{n+d} + \cdots + p^{rn+d} + p^{rn+n+d},$$

where $u = rn + d$, $0 \leqq d < n$, $r \geqq 0$. Another theorem connects finite projective geometries with orthogonal arrays, and is used to construct the arrays (i) $(s^3, s + 2, s, 3)$ when $s = 2^n$; (ii) $(s^3, s + 1, s, 3)$ when $s = p^n$, p being an odd prime; (iii) $s^4, s^2 + 1, s, 3)$ when $s = p^n$, where p is a prime; (iv) $(s^r, s^{r-1}, s, 3)$ when $s = 2$. Orthogonal arrays are useful in connection with many problems of experimental design.

6. **The Structure of Balanced Incomplete Block Designs, and the Impossibility of Certain Unsymmetrical Cases.** William S. Connor, University of North Carolina.

If α_{ij} is the number of treatments common to the ith and jth blocks of a balanced incomplete block design with v treatments, b blocks, r replications, and k treatments per block, with any two treatments occurring together in the same block λ times, then the characteristic matrix $C = ((c_{ij}))$ of the design may be defined by $c_{ii} = (r - k)(r - \lambda)$, $i = 1, 2, \cdots, v$, $c_{ij} = k\lambda - r\alpha_{ij}$, $i, j = 1, 2, \cdots, v$, $i \neq j$. If $| C_t |$ is any symmetrically chosen partial determinant of order r belonging to C, we prove that (i) $| C_t |$ is nonnegative; (ii) if $t = b - v$, then $| C_t | k(r - \lambda)^{v-t-1}/r^{t-1}$ is a perfect square; (iii) if $t > b - v$, $| C_t | = 0$. From (i) Fisher's inequality $b \geqq v$ is deduced and it is shown that

$$k + \lambda - r \leqq \alpha_{ij} \leqq r - \lambda - k + 2k\lambda/r.$$

The structure of the designs (a) $v = 15$, $b = 21$, $r = 7$, $k = 5$, $\lambda = 2$; (b) $v = 36$, $b = 45$, $r = 10$, $k = 8$, $\lambda = 2$; (c) $v = 21$, $b = 28$, $r = 8$, $k = 6$, $\lambda = 2$ is studied. For (a) and (b) it is proved that there must exist $b - v$ blocks, the $| C_{b-v} |$ for which contradicts (ii). For

(c) it is shown that if the incidence matrix N is augmented to N_0 by adding 7 suitably chosen row vectors then the Hasse invariant $C_p(N_0N_0')$ for N_0N_0' is -1, when $p = 3$. This demonstrates the impossibility of (a), (b), and (c). The last two results are new. (Research carried on under the sponsorship of the Office of Naval Research.)

7. Some Bounded Significance Level Tests for the Median. JOHN E. WALSH, The Rand Corporation.

In practice it is often permissible to assume that the observations of a set are statistically independent and from continuous populations with a common median. This is the case, for example, if the observations are a sample from a continuous population. Then the population median can be investigated by using the sign test. For small numbers of observations, however, the sign test does not furnish very many suitable significance levels. Also, some of the sign tests with suitable significance levels are not very efficient. This note presents some tests whose significance levels are only approximate but cover a wide range of suitable values. The significance levels of these tests are exactly determined if the populations are also symmetrical; they are bounded otherwise. Some of these bounded significance level tests have high efficiencies.

8. Joint Sampling Distribution of the Mean and Standard Deviation for Distribution Functions of the First Kind. MELVIN D. SPRINGER, U. S. Naval Ordnance, Indianapolis.

Consider a universe characterized by the distribution function $f(x)$, $-\infty < x < \infty$. If n variates x_i, $i = 1, 2, \cdots, n$, are selected at random from this universe, the probability that they will fall simultaneously within the intervals dx_i, $i = 1, 2, \cdots, n$, is given, to within infinitesimals of higher order, by $f(x_1)f(x_2) \cdots f(x_n)\, dx_1\, dx_2 \cdots dx_n$. As an immediate consequence of the definitions of $\bar{x}$ and s one may employ the transformation T: $x_1 = x_1$, $x_2 = x_2$, $\cdots$, $x_{n-2} = x_{n-2}$, $x_{n-1} = (n\bar{x} - \Sigma_1^{n-2} x_i \pm \Omega_1)/n$, $x_n = n\bar{x} - \Sigma_1^{n-1} x_i$, where $\Omega_1 = [-3\Sigma_1^{n-2} x_i^2 - 2\Sigma_{i=1}^{n-3}\Sigma_{j=i}^{n-3} x_i x_{j+1} + 2n\bar{x}\,\Sigma_1^{n-2} x_i - n(n-2)\bar{x}^2 + 2ns^2]^{\frac{1}{2}}$. Application of this transformation gives

$$f(x_1)f(x_2) \cdots f(x_n)\, dx_1\, dx_2 \cdots dx_n$$
$$= f(x_1)f(x_2) \cdots f(x_{n-2}) f([n\bar{x} - \Sigma_1^{n-2} x_i - \Omega_1]/2)$$
$$\cdot f([n\bar{x} - \Sigma_1^{n-2} x_i + \Omega_1]/2) \mid J \mid dx_1\, dx_2 \cdots dx_{n-2}\, d\bar{x}\, ds,$$

where $\mid J \mid = \mid \text{Jacobian of } T \mid = n^2 s/\Omega_1$. Evaluation of the multiple integral

$$F(\bar{x}, s) = \int\int \cdots \int f(x_1)f(x_2) \cdots f(x_{n-2}) f([n\bar{x} - \Sigma_1^{n-2} x_i - \Omega_1]/2)$$
$$\cdot f([n\bar{x} - \Sigma_1^{n-2} x_i + \Omega_1]/2)\; 2n^2 s/\Omega_1\, dx_{n-2} \cdots dx_2\, dx_1$$

yields the joint distribution function $F(\bar{x}, s)$. The limits of integration are established by employing the relationships $\Sigma_1^n x_i = n\bar{x}$ and $\Sigma_1^n x_i^2 = ns^2 + n\bar{x}^2$, together with mathematical induction, to prove that x_{n-r}, $r = 2, 3, \cdots, n-1$, is restricted to the closed interval

$$([n\bar{x} - \Sigma_1^{n-r-1} x_i - \Omega_r]/(r+1),\ [n\bar{x} - \Sigma_1^{n-r-1} x_i + \Omega_r]/[r+1]),$$

where

$$\Omega_r = [-r(r+2)\Sigma_1^{n-r-1} x_i^2 - 2r\Sigma_{i=1}^{n-r-2}\Sigma_{j=i}^{n-r-2} x_i x_{j+1}$$
$$+ 2rn\bar{x}\Sigma_1^{n-r-1} x_i - rn(n-r-1)\bar{x}^2 + (r+1)rns^2]^{\frac{1}{2}}.$$

9. On Certain Distribution Problems in Multivariate Analysis. (Preliminary Report.) Ingram Olkin, University of North Carolina.

This paper is concerned with the derivation of the joint distributions of (i) rectangular coordinates, (ii) correlation coefficients, (iii) characteristic roots of a matrix, and (iv) roots of a determinantal equation, starting in each case from the multivariate normal distribution. Consider a set of pn random variables following the distribution law $f(X, n) = K \exp(-\frac{1}{2} tr XX')$, X a $p \times n$ matrix, and the real transformations: (1) $X = (T\,O)e^{A}$, where T $(p \times p)$ is a triangular matrix with $t_{ij} = 0$ $(i < j)$, A $(n \times n)$ is a skew-symmetric matrix; (2) $X = D_\alpha(U\,O)e^{B}$, $UU' = R$, where D_α is a diagonal matrix with elements $\alpha_1, \cdots, \alpha_p$, U $(p \times p)$ is a triangular matrix with $u_{ij} = 0$ $(i < j)$, $\Sigma_j^p u_{ij}^2 = 1$, $i = 1, \cdots, p$, B $(n \times n)$ is a skew-symmetric matrix, R $(p \times p)$ is a symmetric matrix; (3) $X = e^{C}(D_\mu\, O)e^{D}$, where C $(p \times p)$ and D $(n \times n)$ are skew-symmetric, D_μ is a diagonal matrix with elements $\mu_1, \cdots, \mu_p$, where μ^2 are the characteristic roots of XX'. Using these transformations on $f(X, n)$, (i), (ii), and (iii) are obtained. From the distribution law

$$f(X_1, n_1)f(X_2, n_2)$$

and the transformation (4) $X_1 = Y(D_s O)e^{E}$, $X_2 = Y(D_c O)e^{F}$, where Y $(p \times p)$, E $(n_1 \times n_1)$ and F $(n_2 \times n_2)$ are skew-symmetric matrices, D_s and D_c are diagonal matrices with elements $s_1, \cdots, s_p$ and $c_1, \ldots, c_p$ respectively, such that $s^2 + c^2 = 1$, the joint distribution of $\theta = s^2$ is found, where θ are the roots of $| X_1X_1' - \theta(X_1X_1' + X_2X_2') | = 0$.

10. A Unified Approach to a Wide Class of Distribution Problems in Multivariate Analysis. S. N. Roy, University of North Carolina.

(1) X being a $p \times n$ matrix of random observations (reduced to means) from a p-variate normal population, and it being known that there exist a $p \times p$ triangular matrix T, and a $p \times n$ matrix L (both ordinarily uniquely determined) such that $X = TL$ and $LL' = I$, it is of interest to obtain the sampling distribution of T from which various distributions, including those of partial and multiple correlations, would easily follow. (2) X_1 and X_2 being $p \times n_1$ and $p \times n_2$ matrices of random observations (reduced to means) from two p-variate normal populations and it being known that there exist (ordinarily uniquely) a $p \times p$ matrix Z, and a $p \times n_1$ matrix L_1, a $p \times n_2$ matrix L_2 (with the constraints $L_1L_1' = L_2L_2' = I$), and $p \times p$ diagonal matrices D_s and D_c (where $S_i = \text{Sin}\,\theta_i$; $C_i = \text{Cos}\,\theta_i$; $i = 1, 2, \cdots, p$) such that $X_1 = ZD_sL_1$, $X_2 = ZD_cL_2$, it is of interest in multivariate analysis to obtain the sampling distribution of θ $(\equiv \theta_1, \cdots, \theta_p)$. (3) With the same X matrix as in (1), and it being known that there exist (ordinarily uniquely) a $p \times p$ orthogonal matrix Γ, a $p \times n$ matrix M (such that $MM' = I$), and a diagonal matrix $D_t(p \times p)$ such that $X = \Gamma D_t M$, it is of interest to obtain the sampling distribution of $t(\equiv t_1, t_2, \cdots, t_p)$. With the help of the constraints indicated one could knock out any $p(p + 1)/2$ out of L in (1), out of each of L_1 and L_2 in (2), and of each of Γ and M in (3); denote the remaining elements respectively by L_R, $(L_{1R}, L_{2R}($, and (Γ_R, M_R). Then in (1), (2), and (3), respectively, we change over from X to (T, L_R), from (X_1, X_2) to $(Z, \theta, L_{1R}, L_{2R})$, and from X to (Γ_R, t, M_R). This is made easy by an artifice discussed in the paper, and the way L in (1) (L_1, L_2) in (2), and M in (3) occur, makes it easy to integrate out over them leaving us with the distributions of T in (1), (Z and θ) in (2), and (Γ and t) in (3). From this the null distributions of T in (1), θ in (2), and t in (3) follow with great ease. Certain nonnull distributions would also come out without much difficulty.

11. An Extension of the Buffon Needle Problem. Nathan Mantel, National Cancer Institute.

Historically, the Buffon needle problem is concerned with the estimation of the value of π from the probability of intersection of a straight line of fixed length (< 1) with a series of equally unit-spaced parallel lines, on which the straight line is allowed to fall at random. The present paper extends the problem to the estimation of π from the average number of intersections of a straight line of any fixed length with a series of equally spaced parallel and perpendicular lines on which the straight line is allowed to fall at random. It is also shown that, comparatively, very precise estimates of π can be made, for long straight lines, from the variation in number of intersections rather than from the average number of intersections. From purely statistical considerations it is demonstrated that π must lie between 3.1231 and 3.1752, with no necessity for any measurements being made.

12. A Generalization of Sampling without Replacement from a Finite Universe. D. G. Horvitz and D. J. Thompson, Iowa State College.

Let the finite universe consist of N elements U_i $(i = 1, 2, \cdots, N)$. A sample of n elements is to be drawn without replacement and the total T of some character X of the elements estimated from the sample. Denote by $P(U_i)$ the probability that the ith element will be included in a sample of size n. An unbiased estimator $\hat{T} = \Sigma_{i=1}^{n} x_i/P(U_i)$ is proposed, and expressions for the variance of this estimator as well as an unbiased estimator of this variance are given. An extension to a two-stage sampling scheme is presented. Consideration is given to methods of determining selection probabilities which will result in optimum probabilities $P(U_i)$ on the basis of the prior information available on the elements of the universe, and two approximate methods are illustrated.

13. A Problem in Two-Stage Sampling. B. M. Seelbinder, University of North Carolina.

Charles Stein has suggested a two-stage sampling plan, the size of the second part of the sample depending on the information supplied about the variance of the population by the first part of the sample. In his work, the size n_1 of the first part of the sample is left to the discretion of the experimenter. This study is designed to throw further light on the choice of the value of n_1. For this purpose the expected value of the total sample size n for given n_1 has been computed for four different significance levels $\alpha = .1, .05, .02, .01$ and varying $c = d/\sigma$, where d is the allowable discrepancy. These values are presented in four tables where c ranges from .01 to 1.0 and n_1 ranges from 5 to 72,000. It is shown that the computation can be made to depend on the knowledge of Pearson's incomplete Gamma function. An approximation whereby the computation can be made to depend only on the knowledge of the normal distribution function has also been developed. Numerical evidence for the adequacy of the approximation for moderately large values of n_1 $(n_1 \geqq 61)$ has been adduced. Limiting values for the expected value of the total sample size are given for fixed n_1 and α with varying c. The discussion of the use of the tables covers the different sampling situations which may arise: (i) an approximate estimate of σ is available, (ii) only a rough estimate of σ is available. Reasons are given which point to 250 as the upper limit for n_1 in a two-stage sampling plan.

14. Bounds on a Distribution Which Are a Function of Moments to Order Four. (Preliminary Report.) Marvin Zelen, University of North Carolina.

Let $F(y)$ be a cumulative distribution function defined for the random variable $a < y < b$, and x be a known quantity. Markov and Stieltjes considered the problem of finding the $\inf_{F(y)} F(x)$ and the $\sup_{F(y)} F(x)$ as a function of a finite number of moments of the distribution. This present paper investigates the explicit expressions for these bounds if the moments to order four are known (1) in the case when the random variable has finite range, (2) in the case when the random variable has infinite range. In the applications of these bounds, it is necessary to order roots of certain orthogonal polynomials. It is suggested that for ready application, a nomograph be used. These bounds would be useful when one is confronted with a cumulative distribution function which is unknown or difficult to handle.

15. An Inconsistency among Type A Regions. HERMAN CHERNOFF, University of Illinois.

In a test of a hypothesis one may regard a sample in the critical region as evidence that the hypothesis is false. Let us assume that for some reason it is desired to increase the critical size of the test, i.e., to make rejection of the hypothesis more probable. Then one may expect that an observation which led to rejection in the first test should still lead to rejection in the new test. In other words, one should expect $W_\alpha \supset W_{\alpha'}$ if $\alpha > \alpha'$ where W_α is the critical region of size α. An example is given where regions of Type A fail to have this property.

16. Stochastic Approximation. (Preliminary Report.) HERBERT ROBBINS AND S. MONRO, University of North Carolina.

We consider the general problem of estimating a constant associated with a function $M(x)$, e.g., the root of an equation $M(x) = \alpha$ or the abscissa of a maximum of $M(x)$. When $M(x)$ is observable there are methods of determining the constant by "successive approximation." We suppose, on the contrary, that $M(x)$ is unknown but that to each value of x corresponds an observable random variable $Y = Y(x)$ with distribution function $P[Y(x) \leqq y] = H(y \mid x)$ such that $M(x) = \int y\, dH(y \mid x)$ is the expected value of Y for the given x. In the case where $M(x)$ is increasing and we wish to estimate the unique root, $x = \theta$, of $M(x) = \alpha$, we propose to let $x_{n+1} = x_n + a_n(\alpha - y_n)$, where x_1 is an arbitrary constant, $\{a_n\}$ is a sequence of positive constants, and y_n is a random variable such that $P[y_n \leqq y \mid x_n] = H(y \mid x_n)$. One of us has shown under certain conditions on $H(y \mid x)$ that, if $\{a_n\}$ is of the type $\{1/n\}$, then no matter what the initial value x_1,

$$\lim_{n\to\infty} E(x_n - \theta)^2 = 0,$$

so that x_n is a consistent estimator of θ. Work is in progress to establish in special cases bounds on $E(x_n - \theta)^2$. By replacing $(\alpha - y_n)$ by $\operatorname{sgn}(\alpha - y_n)$, less severe restrictions need be imposed on $H(y \mid x)$ in order to obtain the same result. This sequential type of design can be applied to estimation in regression problems, to "all or nothing" response experiments (where y_n is limited to the values 0 and 1), and to the experimental determination of the maximum of a function (cf. Harold Hotelling's paper in *Annals of Mathematical Statistics*, vol. 12 (1941), pp. 20–45). (This research was done in part under an Office of Naval Research contract.)

17. On the Properties and Statistical Purposes of Some Well-known and Some New Tests in Multivariate Analysis. S. N. ROY, University of North Carolina.

Consider two problems of multivariate analysis, each of which could be made to cover a wide number of situations. (1) With two random samples of sizes n_1 and n_2 and dispersion matrices (a_{1ij}) and (a_{2ij}) from two p-variate normal populations with dispersion matrices (α_{1ij}) and (α_{2ij}), $i, j = 1, 2, \cdots, p$, an infinite number of similar region tests could be constructed for the composite hypothesis $(\alpha_{1ij}) = (\alpha_{2ij})$, among which there is none having the strong optimum properties of the usual F-test in the anlogous univariate problem. Among these similar region tests, however, there is a subset based on F_i, $i = 1, 2, \cdots, p$, where F_i's are the roots of the equation in F: $|a_{1ij} - Fa_{2ij}| = 0$, such that the largest root has moderate optimum properties with respect to one class of alternatives, the smallest for another class and the product of the roots (which is the likelihood ratio test) for another class of alternatives—all discussed in this paper. (2) With k random samples of sizes n_r from k p-variate normal populations with means m_{ri} and a common dispersion matrix (α_{ij}), $r = 1, 2, \cdots, k$; $i, j = 1, 2, \cdots, p$, an infinite number of similar region tests could be constructed for the composite hypothesis $(m_{1i}) = (m_{2i}) = \cdots = (m_{ki})$, $i = 1, 2, \cdots, p$, among which there is none having the strong optimum properties of the F-test in the analogous univariate problem. Among these similar region tests, however, there is a subset based on F_i, $i = 1, 2, \cdots, q \leqq p$, where F_i's are the nontrivial roots of the equations in F: $|b_{1ij} - Fb_{2ij}| = 0$ (where (b_{1ij}) is the matrix of the sample means reduced to the grand means and (b_{2ij}) is the pooled dispersion matrix from the different samples), such that the largest and smallest roots have moderate optimum properties with respect to two different classes of alternatives and the sum of the roots for a third class of alternatives—all discussed in this paper. The likelihood ratio test, however, leads to the product. The wide variety of situations each problem could be made to cover is also discussed in this paper.

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest.

Personal Items

Dr. K. S. Banerjee, Statistician at the Central Sugarcane Research Station, Bihar, India, received his doctorate degree from the Calcutta University in January of this year. His thesis covered his contributions to "weighing designs."

Mr. Lyle D. Calvin, formerly at the Institute of Statistics, North Carolina State College, has accepted the position of Biometrician with the Division of Biological Research, G. D. Searle & Co., Chicago, Illinois.

Dr. Robert J. Hader has accepted a position on the staff of the Institute of Statistics, North Carolina State College. He leaves Los Alamos, New Mexico, where he has been employed as statistician for the Los Alamos Scientific Laboratory for the past two years.

Mr. Bernard Hecht has joined the Victor Division of RCA, Camden, New Jersey, as Manager, Assembly Quality Control, after five years as Quality Control Manager of the International Resistance Company of Philadelphia, Pennsylvania.

Dr. Edward L. Kaplan has received his doctorate degree in mathematics from Princeton University and is now a member of the Technical Staff, Bell Telephone Laboratories, Murray Hill, New Jersey.

Dr. Eugene Lukacs has joined the staff of the Statistical Engineering Laboratory of the National Bureau of Standards. At the Bureau he will be engaged in research in mathematical statistics, particularly autoregressive series and stochastic processes.

Mr. A. W. Marshall, formerly at the Washington, D. C., office of the Rand Corporation, has now moved to its Santa Monica, California, office.