A NOTE ON BIBDS

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It is well known that the parameters (v, b, r, k, λ) of a balanced incomplete block design (BIBD) satisfy the relations

$$bk = rv, \tag{1}$$

$$\lambda(v - 1) = r(k - 1), \tag{2}$$

$$r - \lambda = rk - \lambda v. \tag{2'}$$

Fisher [2] proved that

$$b \geqq v, \tag{3}$$

and Bose [1] showed that for a resolvable BIBD one has

$$b \geqq v + r - 1. \tag{4}$$

Nair [3] proved the inequality

$$b \geqq 1 + \frac{k(r-1)^2}{(r-k) + \lambda(k-1)} \tag{5}$$

for any BIBD, and

$$b \geqq \frac{rk(r-1)}{(r-k) + \lambda(k-1)} \tag{6}$$

for a resolvable BIBD. While it was originally claimed that (5) and (6) are sharper results than (3) and (4), it is the purpose of this note to show that this is not so; (5) and (6) are completely equivalent to (3) and (4).

We first put (4) in a neater form by writing it as $(b - r)k \geqq k(v - 1)$; using (1) and (2),

$$rv - \lambda(v-1) - r \geqq k(v-1),$$

$$(v-1)(r-\lambda) \geqq k(v-1).$$

Since $v - 1 > 0$, (4) is equivalent to

$$r - \lambda \geqq k. \tag{7}$$

We now take Nair's inequality (6); using (1), it is equivalent to $v(r - k + \lambda k - \lambda) \geqq k^2(r - 1)$. Applying (2′), and the fact that $v - k > 0$, we then obtain

$$k(r-1)(v-k) \geqq \lambda v(v-k),$$

$$k(r-1) \geqq \lambda v,$$

$$r - \lambda \geqq k.$$

This demonstrates the equivalence of (4), (6), and (7).

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Finally, employing (1) and (2′), (5) is equivalent to $rv(rk - \lambda v - k + \lambda k) \geqq k(\lambda k - \lambda v + kr^2 - kr)$. Grouping the terms in λ, we have

$$kr(r-1)(v-k) \geqq \lambda(v-k)(vr-k). \tag{8}$$

If we apply (1) to (8), we get relation

$$r(r-1) \geqq \lambda(b-1); \tag{9}$$

however, applying (2′) to (8) gives

$$kr^2 - \lambda rv \geqq k(r-\lambda) = k^2r - \lambda kv,$$

$$(kr - \lambda v)(r-k) \geqq 0,$$

$$(r-\lambda)(r-k) \geqq 0.$$

It is trivial that $r - \lambda > 0$; hence

$$r - k \geqq 0, \tag{10}$$

which is equivalent, by (1), to Fisher's inequality (3). Thus we conclude that (5) and (3) are equivalent.

This completes the proof that inequalities (5) and (6) are in reality no more general than (3) and (4).

REFERENCES

[1] R. C. Bose, "A note on the resolvability of balanced incomplete designs," *Sankhyā*, Vol. 6 (1942), pp. 105–110.

[2] R. A. Fisher, "An examination of the different possible solutions of a problem in incomplete blocks," *Ann. Eugenics*, Vol. 10 (1940), pp. 52–75.

[3] K. R. Nair, "Certain inequality relationships among the combinatorial parameters of incomplete block designs," *Sankhyā*, Vol. 6 (1943), pp. 255–259.

CORRECTIONS TO "THE SURPRISE INDEX FOR THE MULTIVARIATE NORMAL DISTRIBUTION"

By I. J. Good

In the paper cited in the title (*Ann. Math. Stat.* Vol. 27 (1956), pp. 1130–1135):

Sec. 1, line 4. For **E** read $\mathbf{E}_i$. (This was correct on some prints.)

P. 1132, line 7. For λ_0 read λ_u.

Two lines above Sec. 4. For λ read λ_1.

End of paper. The remark concerning Hotelling's generalised "Student" test is misleading and should be deleted.