

comes to the same thing in view of unbiasedness—some multiple of the mean square of the estimator. We find that

$$\begin{aligned} p^2 E(\{\varphi w^*\}^2) &= \lambda^2 \int_0^\infty \varphi(y)^2 f(y) dy + (1 - \lambda)^2 \int_0^\infty \frac{f(y)}{f(-y)} \varphi(y)^2 f(y) dy \\ &= \lambda^2 J_1 + (1 - \lambda)^2 J_2, \end{aligned}$$

say, which is minimized by setting

$$\lambda = J_2 / (J_1 + J_2).$$

In case  $f$  is symmetric  $J_1 = J_2$  and optimum  $\lambda = 1/2$ ; here the naive procedure of rejecting negative  $x$ 's corresponds to  $\lambda = 1$  and maximizes the mean square! However, if  $f(-y) = 0$  over a stretch in which  $f(y) > 0$  then  $J_2 = \infty$ , and we must take  $\lambda = 1$ , adopt the naive solution, in order to obtain finite variance of estimate. Finally, in case  $\varphi(y)$  and  $f(-y)/f(y)$  have large similar peaks near some  $y_0 > 0$  then  $J_1$  may be very much larger than  $J_2$  and optimum  $\lambda$  very close to 0.

#### REFERENCES

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### TABLES FOR TYPE A CRITICAL REGIONS

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1. This note provides tables connected with work by Neyman [1] and Johnson [2] on testing hypotheses, expanding the table given in [2]. This table, as expanded provides solutions for the values of  $A$  satisfying,

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{A-Bu^2}^{\infty} e^{-\frac{1}{2}u^2 - \frac{1}{2}v^2} dv du = \alpha$$

for  $\alpha = .01, .05$ , and  $B = 0(.1)5, 5(1)10, 10(10)100$ .  
 When  $\alpha = .05$  set  $A = 3.8414588B + \rho_{.05}$ , and when  
 $\alpha = .01$  set  $A = 6.6348966B + \rho_{.01}$

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TABLE 1

B	$\rho_{.05}$	$\rho_{.01}$	B	$\rho_{.05}$	$\rho_{.01}$
0.0	1.64484	2.32634	3.3	0.09559	0.08721
0.1	1.37901	1.79269	3.4	0.09277	0.08464
0.2	1.15224	1.34283	3.5	0.09012	0.08222
0.3	0.96047	0.99547	3.6	0.08761	0.07993
0.4	0.79869	0.75054	3.7	0.08523	0.07777
0.5	0.66460	0.59093	3.8	0.08299	0.07572
0.6	0.55679	0.48717	3.9	0.08085	0.07378
0.7	0.47283	0.41535	4.0	0.07883	0.07193
0.8	0.40873	0.36237	4.1	0.07690	0.07018
0.9	0.35972	0.32153	4.2	0.07507	0.06851
1.0	0.32153	0.28902	4.3	0.07332	0.06691
1.1	0.29097	0.26252	4.4	0.07165	0.06539
1.2	0.26590	0.24049	4.5	0.07006	0.06393
1.3	0.24491	0.22188	4.6	0.06853	0.06254
1.4	0.22704	0.20595	4.7	0.06707	0.06121
1.5	0.21164	0.19216	4.8	0.06568	0.05994
1.6	0.19821	0.18011	4.9	0.06434	0.05871
1.7	0.18640	0.16948	5.0	0.06305	0.05754
1.8	0.17593	0.16004	6.0	0.05254	0.04794
1.9	0.16657	0.15160	7.0	0.04504	0.04108
2.0	0.15817	0.14400	8.0	0.03943	0.03594
2.1	0.15058	0.13713	9.0	0.03506	0.03194
2.2	0.14368	0.13088	10.0	0.03157	0.02874
2.3	0.13739	0.12518	20.0	0.01591	0.01432
2.4	0.13163	0.11996	30.0	0.01075	0.00949
2.5	0.12633	0.11515	40.0	0.00821	0.00706
2.6	0.12145	0.11072	50.0	0.00673	0.00559
2.7	0.11693	0.10661	60.0	0.00576	0.00461
2.8	0.11273	0.10280	70.0	0.00510	0.00389
2.9	0.10883	0.09925	80.0	0.00463	0.00335
3.0	0.10519	0.09594	90.0	0.00429	0.00292
3.1	0.10178	0.09284	100.0	0.00401	0.00257
3.2	0.09859	0.08994			

This work was done on the Univac located in the Applied Mathematics Laboratory, David Taylor Model Basin, Navy Department.

## REFERENCES

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