MINIMUM VARIANCE UNBIASED ESTIMATION FOR THE TRUNCATED POISSON DISTRIBUTION

By R. F. Tate[1] and R. L. Goen[2]

University of Washington

1. Summary. A minimum variance unbiased estimator is provided for the parameter of a truncated Poisson distribution in the case of truncation on the left. In this connection the distribution is obtained for the sum of n independent identically distributed truncated Poisson random variables, and then well-known properties of sufficient statistics are employed to obtain the estimator. For the case of truncation away from the zero value results are expressed in terms of Stirling numbers of the second kind. The estimator has a particularly simple form and tables are available for its computation. For the general case results are expressed in terms of what we call generalized Stirling numbers. As a by-product of the statistical considerations there arises an identity between generalized Stirling numbers which may be useful in the study of Difference Equations.

2. Introduction. Numerous articles have been written on the subject of the estimation of the parameter of a truncated or censored Poisson distribution. Our work concerns the former distribution. The two types of distributions can be distinguished as follows: Consider an ordinary Poisson random variable with range $\{0, 1, 2, \cdots\}$, and let A be a subset of this range. If values in the set A cannot be members of a sample, then a random observation of the restricted variable is said to have a truncated Poisson distribution or to be truncated away from A. On the other hand there is the possibility for values in the set A to be members of a sample, but for some reason not distinguishable from one another. In this case a random observation of the restricted variable is said to have a censored Poisson distribution.

A situation calling for the truncated Poisson distribution would occur when one wishes to fit a distribution to Poisson-like data consisting of numbers of individuals in certain groups which possess a given attribute, but in which a group cannot be sampled unless at least a specified number of its members have the attribute. For example, the group may be a household of people, and the attribute measles; the specified number would then be one. A censored Poisson distribution is used most often in connection with pooled data.

The estimation problem for both the truncated and the censored cases has been discussed extensively from the point of view of maximum likelihood by Cohen [1]. Earlier results based on maximum likelihood were obtained by

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Tippett [9], David and Johnson [2], and Rider [8]. Various other estimators were proposed by Moore [5], [6], Rider [8], and Plackett [7]. Plackett appears to be the only writer ever to propose an unbiased estimator for any of the cases of a truncated or censored Poisson distribution. His estimator for the parameter of a Poisson distribution truncated away from 0, which will arise several times during our discussion, is

$$\lambda^* = \frac{1}{n} \sum X_k,$$

where the summation is taken over all $X_k \geqq 2$.

The present paper is concerned with unbiased estimators for the case of tail truncation. It can readily be shown that truncation on the right, that is away from $A = \{c, c + 1, \cdots \}$, precludes the existence of an unbiased estimator. The argument is based on the identity of two power series; details will be omitted.

Assume that $A = \{0, 1, 2, \cdots , c\}$ for some $c \geqq 0$. Let the Poisson density be denoted by

$$f(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, \cdots .$$

The density of the restricted random variable which is truncated away from A is then

$$g(x; \lambda, c) = \frac{e^{-\lambda}\lambda^x}{x! \sum_{c+1}^{\infty} e^{-\lambda} \frac{\lambda^i}{i!}}, \qquad x = c + 1, c + 2, \cdots .$$

Consider a sample of n independent observations $X_1, X_2, \cdots, X_n$, each with density $g(x; \lambda, c)$, and let

$$T_c = \sum_1^n X_k .$$

It is well known that $\sum X_k$ is a sufficient statistic for the family $\{f(x; \lambda)\}$. A result of Tukey [10] states that sufficiency is preserved under truncation away from any Borel set in the range of X. Hence, in the case at hand T_c is sufficient for $\{g(x; \lambda, c)\}$. It can be verified that T_c is also complete.

For the case $c = 0$ the distribution of T_0 and the minimum variance unbiased estimator $\tilde{\lambda}_0$ are derived in Section 3. This is at the same time the most important case for applications and the easiest with which to deal. A recent extension of the table of Stirling numbers of the second kind makes $\tilde{\lambda}_0$ easy to compute for many values of n and T_0.

In order to express the results for the general case $c \geqq 1$ in a simple form it is necessary to introduce the notion of a generalized Stirling number. This will be done in Definition 3 below.

The following relations are quoted here for later reference.

Definition 1 (Jordan [3], p. 169): Stirling number of the second kind.

$$\mathfrak{S}_t^n = \frac{(-1)^n}{n!} \sum_0^\infty \binom{n}{k} (-1)^k (k)^t \qquad t = n, n+1, \cdots,$$

$\mathfrak{S}_t^n = 0$ for $t < n$.

Definition 2 (Jordan, p. 185):

$$\bar{C}_{p,i} = \sum_{j=p+1}^{2p-i} (-1)^{j+1} \binom{2p-1}{j} \mathfrak{S}_j^{j-p}.$$

Property 1 (Jordan, p. 169):

$$\mathfrak{S}_t^n = \mathfrak{S}_{t-1}^{n-1} + n\, \mathfrak{S}_{t-1}^n.$$

Property 2 (Jordan, p. 186):

$$\mathfrak{S}_{t+1}^{n+1} = \sum_{j=n}^{t} \binom{t}{j} \mathfrak{S}_j^n.$$

Property 3 (Jordan, p. 171):

$$\bar{C}_{t-n,t-2n} = n\, \bar{C}_{t-n-1,t-2n-1} + (t-1)\bar{C}_{t-n-1,t-2n}.$$

The generalized Stirling number will be introduced by

Definition 3:

$$\mathfrak{G}_{n,t}^c = \frac{(-1)^n t!}{n!} \sum \frac{n!}{k_1! \cdots k_{c+2}!} \frac{(-1)^{k_1} k_1^{(t-\Sigma_0^c jk_{j+2})}}{\left(t - \sum_0^c jk_{j+2}\right)! \prod_0^c (j!)^{k_{j+2}}},$$

where $k_i = 0, 1, \cdots, n$; $i = 1, 2, \cdots, c+2$; $t = n(c+1), n(c+1)+1, \cdots$; and the summation is taken over all $(k_1, \cdots, k_{c+2})$ such that $k_1 + \cdots + k_{c+2} = n$.

Property 4:

$$\mathfrak{G}_{n,t}^0 = \mathfrak{S}_t^n.$$

Property 5:

$$\mathfrak{G}_{n,t}^1 = \bar{C}_{t-n,t-2n}.$$

To verify Property 5 write $\mathfrak{G}_{n,t}^1$ as an iterated sum over k_1 and k_2, and use Definition 2.

In Section 4 the distribution of T_c and the minimum variance unbiased estimator $\tilde{\lambda}_c$ are derived for the general case. There, also, a simple unbiased estimator based on one observation is given for λ, and is used, via the Lehmann-Scheffé-Blackwell method, to reproduce $\tilde{\lambda}_c$. When equated, the two expressions for $\tilde{\lambda}_c(t)$ provide an identity for the numbers $\mathfrak{G}_{n,t}^c$. The estimator used is related to Plackett's estimator λ^*.

3. The case $c = 0$. Let $X_1, X_2, \cdots, X_n$ be independent random variables, each with density $g(x; \lambda, 0)$ and characteristic function $\phi_0(\alpha)$. Then T_0 has the characteristic function

$$\psi_0(\alpha) = [\phi_0(\alpha)]^n = \left(\sum_1^\infty \frac{\lambda^x e^{i\alpha x - \lambda}}{x!\,(1 - e^{-\lambda})}\right)^n.$$

Using the fact that $f(x; \lambda)$ has characteristic function $\exp\lambda(e^{i\alpha} - 1)$, and simplifying, we have

$$\psi_0(\alpha) = \left(\frac{e^{\lambda e^{i\alpha}} - 1}{e^\lambda - 1}\right)^n.$$

The inversion formula for characteristic functions shows that T_0 has the density

$$p_0(t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \psi_0(\alpha)e^{-i\alpha t}\,d\alpha.$$

A binomial expansion for the numerator of $\psi_0(\alpha)$ shows that $p_0(t)$ is

$$\frac{(-1)^n}{(e^\lambda - 1)^n}\sum_{k=0}^n \binom{n}{k}(-1)^k \int_{-\pi}^{+\pi} \frac{e^{k\lambda e^{i\alpha} - i\alpha t}}{2\pi}\,d\alpha.$$

Since inversion of $\exp\lambda(e^{i\alpha} - 1)$ results in $e^{-\lambda}\lambda^t/t!$, the integral in $p_0(t)$ is $(k\lambda)^t/t!$, and from Definition 1 we finally arrive at

$$p_0(t) = \frac{\lambda^t n!}{(e^\lambda - 1)^n t!}\mathfrak{S}_t^n, \qquad t = n, n + 1, \cdots.$$

It was noted in the introduction that T_0 is a complete sufficient statistic for the family $\{g(x; \lambda, 0)\}$. It then follows that if an unbiased estimator based on T_0 exists for λ, it will be unique and have the property of minimum variance (See Lehmann [4]). The condition for unbiasedness of $\tilde{\lambda}_0$ is

$$\sum_{t=n}^\infty \tilde{\lambda}_0(t)\frac{\lambda^t n!}{t!(e^\lambda - 1)^n}\mathfrak{S}_t^n \equiv \lambda.$$

In view of that fact that

$$(e^\lambda - 1)^n = \sum_{t=n}^\infty \frac{\lambda^t n!}{t!}\mathfrak{S}_t^n,$$

the condition becomes

$$\sum_{t=n}^\infty \tilde{\lambda}_0(t)\frac{\lambda^t}{t!}\mathfrak{S}_t^n \equiv \sum_{t=n}^\infty \frac{\lambda^{t+1}}{t!}\mathfrak{S}_t^n.$$

Comparing coefficients of powers of λ, we have the minimum variance unbiased estimator

$$\tilde{\lambda}_0(t) = t\frac{\mathfrak{S}_{t-1}^n}{\mathfrak{S}_t^n}.$$

Property 1 gives the alternative form[3]

$$\tilde{\lambda}_0(t) = \frac{t}{n}\left(1 - \frac{\mathfrak{S}_{t-1}^{n-1}}{\mathfrak{S}_t^n}\right).$$

Mr. Francis L. Miksa has computed the most complete table to date of Stirling numbers of the second kind.[4] Miksa's table gives $\mathfrak{S}_t^n$ for $n = 1(1)t$, $t = 1(1)50$. The quantity needed for the estimation of λ, the parameter of a Poisson distribution truncated away from zero, is

$$\tilde{\lambda}_0(t) = \frac{t}{n}\left(1 - \frac{\mathfrak{S}_{t-1}^{n-1}}{\mathfrak{S}_t^n}\right) = \frac{t}{n}\,C(n, t).$$

A table of $C(n, t)$ for $n = 2(1)t - 1$, $t = 3(1)50$ appears at the end of this paper. Note that for certain values of (n, t), $C(n, t)$ has not been tabulated, since

$$C(n, t) = 0 \quad \text{when} \quad n = t \geqq 1$$

$$C(1, t) = 1 \quad \text{when} \quad t \geqq 2.$$

All other missing entries are 1 (correct to 5 decimals); for example, $C(2, t) = 1$ for $t \geqq 19$.

For values of t which are large compared to n, the asymptotic expression $\mathfrak{S}_t^n \sim n^t/n!$ is available (Jordan [3], p. 173). Thus we have

$$\tilde{\lambda}_0(t) = \frac{t}{n}\left(1 - \left(\frac{n-1}{n}\right)^{t-1}\right).$$

The percentage error of approximation, $E(n, t)$, decreases with increasing t when n is fixed. For fixed t/n the percentage error increases with increasing n; however, the rate of increase falls off rapidly, as can be seen from the following short table, computed for $t/n = 4$.

(n, t)	(2, 8)	(4, 16)	(6, 24)	(8, 32)	(10, 40)	(12, 48)
$E(n, t)$	.006%	.046%	.073%	.090%	.101%	.107%

Since $E(15, 50) = .4\%$, we may consider the approximation quite satisfactory for $2 \leqq n \leqq 15$, $t \geqq 51$. For larger values of n we must have t/n larger than 50/15, but not necessarily much larger, in view of the above table. For larger values of n one may also resort to the use of the unbiased estimator of Plackett, which was defined in the introduction and can also be written[5]

$$\lambda^* = \frac{t}{n}\left(1 - \frac{n_1}{t}\right),$$

[3] This form may be thought of as a slight change of the usual estimator t/n due to the missing zero class.

[4] This table is as yet unpublished.

[5] In this connection see also the definition of V_c in Section 4.

where n_1 is the number of observations in the sample which have the value 1; or one can use the maximum likelihood estimator $\hat{\lambda}$, which is the solution of the equation

$$\frac{t}{n} = \frac{\hat{\lambda}}{1 - e^{-\hat{\lambda}}}.$$

In summary, the following is an improved procedure for estimating λ, which in many cases yields a minimum variance unbiased estimator.

Estimate λ by

$$\frac{t}{n} C(n, t), \qquad 1 \leqq n \leqq t, 1 \leqq t \leqq 50; n = t \geqq 51; n = 1, t \geqq 51,$$

$$\frac{t}{n}\left(1 - \left(\frac{n-1}{n}\right)^{t-1}\right), \qquad 2 \leqq n \leqq 15, t \geqq 51; n \geqq 16, t \gg n,$$

$$\lambda^* \quad \text{or} \quad \hat{\lambda}, \qquad \text{otherwise.}$$

An extended table of $C(n, t)$ would be quite useful. However, in order to obtain such a table it is necessary to devise a method for computing $C(n, t)$ which does not depend on entries in the table of $\mathfrak{S}_t^n$, since, for example, $\mathfrak{S}_{50}^{16}$ in Miksa's table is an integer of forty-seven digits. The authors have been unable to do this.

The following facts should be observed in comparing our estimator with the estimator λ^* of Plackett [7] and the maximum likelihood estimator $\hat{\lambda}$.

1. Plackett's estimator λ^* and $\tilde{\lambda}_0$ are different, and λ^* has exact variance

$$\frac{1}{n}\left(\lambda + \frac{\lambda^2}{e^{\lambda} - 1}\right).$$

2. $\hat{\lambda}$ was shown by David and Johnson [2] to be the solution of the equation

$$\bar{x} = \frac{\lambda}{1 - e^{-\lambda}},$$

so it is obviously a function of T_0. A simple numerical calculation shows that $\hat{\lambda}$ and $\tilde{\lambda}_0$ are different. Therefore, by uniqueness of unbiased estimators based on T_0, we see that $\hat{\lambda}$ is a biased estimator of λ.

3. David and Johnson also showed that the asymptotic variance of $\hat{\lambda}$ is

$$\frac{\lambda(1 - e^{-\lambda})^2}{n(1 - e^{-\lambda} - \lambda e^{-\lambda})}.$$

This is then also, for each fixed n, the Cramér-Rao lower bound for exact variances of unbiased estimators. The following calculations show that there is no unbiased estimator whose variance attains this lower bound: Let $J_\lambda(\mathbf{x})$ denote the joint density of n independent truncated Poisson random variables, each with density $g(x; \lambda, 0)$. Then, a necessary and sufficient condition for a Cramér-Rao estimator to exist is that there exist a function $h(\lambda)$ such that the expression

$$\lambda + h(\lambda) \frac{\partial \log J_\lambda(\mathbf{x})}{\partial \lambda}$$

is independent of λ for all values of $\mathbf{x}$. It can be verified that

$$\frac{\partial \log J_\lambda(\mathbf{x})}{\partial \lambda} = \left(\frac{\sum x_k}{n} - \frac{n}{1 - e^{-\lambda}} \right),$$

and that no such function $h(\lambda)$ exists. Moreover, since $\tilde{\lambda}_0$ and λ^* are different functions of t, the variance of λ^* must exceed that of $\tilde{\lambda}_0$. Consequently, we may write (for each fixed n)

$$\frac{\lambda(1 - e^{-\lambda})^2}{n(1 - e^{-\lambda} - \lambda e^{-\lambda})} < \sigma^2_{\tilde{\lambda}_0} < \frac{1}{n} \left(\lambda + \frac{\lambda^2}{e^\lambda - 1} \right).$$

4. The general case. The derivation of the distribution of T_c and the minimum variance unbiased estimator $\tilde{\lambda}_c$ proceeds in a manner analogous to the case $c = 0$, except that here we use a multinomial expansion and generalized Stirling numbers. More precisely, let

$$F(c) = \sum_{j=0}^{c} \frac{e^{-\lambda}\lambda^j}{j!}.$$

Then, T_c will have characteristic function

$$\psi_c(\alpha) = \left(\sum_{c+1}^{\infty} \frac{e^{i\alpha x - \lambda}\lambda^x}{x!\,[1 - F(c)]} \right)^n = \frac{e^{-n\lambda}}{[1 - F(c)]^n} \left(e^{\lambda e^{i\alpha}} - \sum_{0}^{c} \frac{e^{i\alpha x}\lambda^x}{x!} \right)^n.$$

After performing the multinomial expansion, employing the inversion formula, and evaluating the same types of integrals as before, we use Definition 3 and arrive at the following expression for the density of T_c:

$$p_c(t) = \frac{n!\,\lambda^t \mathfrak{S}^c_{n,t}}{t! \left(e^\lambda - \sum_0^c \frac{\lambda^j}{j!} \right)^n}, \qquad t = n(c + 1), n(c + 1) + 1, \cdots.$$

In the same way as before the condition of unbiasedness now yields

$$\tilde{\lambda}_c(t) = t \frac{\mathfrak{S}^c_{n,t-1}}{\mathfrak{S}^c_{n,t}}.$$

It is clear from Property 4 that for $c = 0$, $\tilde{\lambda}_c(t)$ reduces to the expression of Section 3. Also, from Property 5 we see that

$$\tilde{\lambda}_1(t) = t \frac{\bar{C}_{t-n-1,t-2n-1}}{\bar{C}_{t-n,t-2n}}.$$

At the present time there appears to be only one available table (Jordan, p. 172) for evaluating $\bar{C}_{p,i}$. This table handles the estimation problem for $n = 1, \cdots, 5$, $2n + 1 \leqq t \leqq n + 6$.

One simple unbiased estimator for λ is

$$U_c(x_1) = \begin{cases} 0 & x_1 = c + 1 \\ x_1 & x_1 \geqq c + 2. \end{cases}$$

Now we use the Lehmann-Scheffé-Blackwell Method (see Lehmann [4]):

$$\tilde{\lambda}_c(t) = E(U_c \mid T_c = t) = \sum_{c+2}^{t-(n-1)(c+1)} xP(X_1 = x \mid T_c = t).$$

$P(X_1 = x \mid T_c = t)$ can be written as $P(X_1 = x)\, P(\sum_2^n X_j = t - x)/P(T_c = t)$, and then simplified by the use of $p_c(t)$ to the form

$$P(X_1 = x \mid T_c = t) = \frac{\binom{t}{x} \mathfrak{S}_{n-1,t-x}^c}{n\mathfrak{S}_{n,t}^c}.$$

Substituting this expression in the above, and equating the result with the earlier form of $\tilde{\lambda}_c(t)$, we obtain the identity

$$\sum_{(c+1)(n-1)}^{t-c-2} \binom{t-1}{j} \mathfrak{S}_{n-1,j}^c = n\mathfrak{S}_{n,t-1}^c.$$

For $c = 0$ this reduces to a combination of Property 1 and Property 2.

The natural unbiased estimator based on the whole sample, which may be generated from U_c, is

$$V_c(x_1, x_2, \cdots, x_n) = \frac{1}{n} \sum_1^n U_c(x_j).$$

V_0 is precisely Plackett's λ^*.

TABLE OF $10^5\, C(n, t) = 10^5 \left(1 - \frac{\mathfrak{S}_{t-1}^{n-1}}{\mathfrak{S}_t^n}\right)$

(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(ν, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)
(2, 3)	66667	(3, 7)	89701	(3, 27)	99997	(4, 19)	99428	(4, 39)	99998	(5, 20)	98497	(5, 40)	99983
(2, 4)	85714	(3, 8)	93478	(3, 28)	99998	(4, 20)	99572	(4, 40)	99998	(5, 21)	98807	(5, 41)	99987
(2, 5)	93333	(3, 9)	95802	(3, 29)	99999	(4, 21)	99680	(4, 41)	99999	(5, 22)	99052	(5, 42)	99989
(2, 6)	96774	(3, 10)	97267	(3, 30)	99999	(4, 22)	99761	(4, 42)	99999	(5, 23)	99245	(5, 43)	99991
(2, 7)	98413	(3, 11)	98207	(3, 31)	99999	(4, 23)	99821	(4, 43)	99999	(5, 24)	99399	(5, 44)	99993
(2, 8)	99213	(3, 12)	98818	$n = 4$		(4, 24)	99866	$n = 5$		(5, 25)	99521	(5, 45)	99995
(2, 9)	99608	(3, 13)	99218	(4, 5)	40000	(41 25)	99898	(5, 6)	33333	(5, 26)	99618	(5, 46)	99996
(2, 10)	99804	(3, 14)	99481	(4, 6)	61538	(4, 26)	99925	(5, 7)	53571	(5, 27)	99695	(5, 47)	99997
(2, 11)	99902	(3, 15)	99655	(4, 7)	74286	(4, 27)	99943	(5, 8)	66667	(5, 28)	99756	(5, 48)	99997
(2, 12)	99951	(3, 16)	99771	(4, 8)	82305	(4, 28)	99958	(5, 9)	75529	(5, 29)	99805	(5, 49)	99998
(2, 13)	99976	(3, 17)	99847	(4, 9)	87568	(4, 29)	99968	(5, 10)	81728	(5, 30)	99844	(5, 50)	99998
(2, 14)	99988	(3, 18)	99898	(4, 10)	91130	(4, 30)	99976	(5, 11)	86177	(5, 31)	99876	$n = 6$	
(2, 15)	99994	(3, 19)	99932	(4, 11)	93586	(4, 31)	99982	(5, 12)	89434	(5, 32)	99901	(6, 7)	28571
(2, 16)	99997	(3, 20)	99955	(4, 12)	95339	(4, 32)	99987	(5, 13)	91851	(5, 33)	99921	(6, 8)	47368
(2, 17)	99998	(3, 21)	99970	(4, 13)	96583	(4, 33)	99990	(5, 14)	93686	(5, 34)	99936	(6, 9)	60317
(2, 18)	99999	(3, 22)	99980	(4, 14)	97482	(4, 34)	99992	(5, 15)	95070	(5, 35)	99949	(6, 10)	69549
$n = 3$		(3, 23)	99987	(4, 15)	98137	(4, 35)	99994	(5, 16)	96136	(5, 36)	99959	(6, 11)	76307
(3, 4)	50000	(3, 24)	99991	(4, 16)	98618	(4, 36)	99996	(5, 17)	96961	(5, 37)	99968	(6, 12)	81360
(3, 5)	72000	(3, 25)	99994	(4, 17)	98971	(4, 37)	99997	(5, 18)	97602	(5, 38)	99974	(6, 13)	85202
(3, 6)	83333	(3, 26)	99996	(4, 18)	99233	(4, 38)	99998	(5, 19)	98104	(5, 39)	99979	(6, 14)	88164

TABLE—*Continued*

(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)
(6, 15)	90474	(7, 34)	99368	(9, 12)	46666	(10, 34)	96576	(12, 18)	59241	(13, 43)	96100	(15, 33)	84943
(6, 16)	92294	(7, 35)	99460	(9, 13)	55765	(10, 35)	96946	(12, 19)	64243	(13, 44)	96427	(15, 34)	86196
(6, 17)	93738	(7, 36)	99558	(9, 14)	63008	(10, 36)	97273	(12, 20)	68514	(13, 45)	96726	(15, 35)	87330
(6, 18)	94893	(7, 37)	99605	(9, 15)	68847	(10, 37)	97564	(12, 21)	72182	(13, 46)	96998	(15, 36)	88360
(6, 19)	95822	(7, 38)	99662	(9, 16)	73607	(10, 38)	97822	(12, 22)	75348	(13, 47)	97246	(15, 37)	89297
(6, 20)	96573	(7, 39)	99711	(9, 17)	77523	(10, 39)	98052	(12, 23)	78095	(13, 48)	97473	(15, 38)	90149
(6, 21)	97182	(7, 40)	99753	(9, 18)	80771	(10, 40)	98257	(12, 24)	80489	(13, 49)	97680	(15, 39)	90926
(6, 22)	97678	(7, 41)	99788	(9, 19)	83485	(10, 41)	98439	(12, 25)	82582	(13, 50)	97869	(15, 40)	91636
(6, 23)	98084	(7, 42)	99820	(9, 20)	85765	(10, 42)	98602	(12, 26)	84419	$n = 14$		(15, 41)	92285
(6, 24)	98417	(7, 43)	99845	(9, 21)	87693	(10, 43)	98747	(12, 27)	86036	(14, 15)	13333	(15, 42)	92878
(6, 25)	98690	(7, 44)	99867	(9, 22)	89331	(10, 44)	98877	(12, 28)	87465	(14, 16)	24419	(15, 43)	93422
(6, 26)	98915	(7, 45)	99886	(9, 23)	90727	(10, 45)	98993	(12, 29)	88730	(14, 17)	33725	(15, 44)	93921
(6, 27)	99101	(7, 46)	99902	(9, 24)	91924	(10, 46)	99096	(12, 30)	89853	(14, 18)	41607	(15, 45)	94379
(6, 28)	99254	(7, 47)	99916	(9, 25)	92952	(10, 47)	99189	(12, 31)	90852	(14, 19)	48331	(15, 46)	94799
(6, 29)	99381	(7, 48)	99928	(9, 26)	93838	(10, 48)	99272	(12, 32)	91743	(14, 20)	54107	(15, 47)	95186
(6, 30)	99485	(7, 49)	99939	(9, 27)	94605	(10, 49)	99347	(12, 33)	92539	(14, 21)	59098	(15, 48)	95542
(6, 31)	99572	(7, 50)	99947	(9, 28)	95269	(10, 50)	99413	(12, 34)	93251	(14, 22)	63434	(15, 49)	95870
(6, 32)	99644	$n = 8$		(9, 29)	95846	$n = 11$		(12, 35)	93890	(14, 23)	67220	(15, 50)	96172
(6, 33)	99704	(8, 9)	22222	(9, 30)	96348	(11, 12)	16667	(12, 36)	94463	(14, 24)	70539	$n = 16$	
(6, 34)	99754	(8, 10)	38400	(9, 31)	96787	(11, 13)	29864	(12, 37)	94978	(14, 25)	73462	(16, 18)	21769
(6, 35)	99795	(8, 11)	50505	(9, 32)	97170	(11, 14)	40476	(12, 38)	95442	(14, 26)	76044	(16, 19)	30341
(6, 36)	99830	(8, 12)	59763	(9, 33)	97505	(11, 15)	49120	(12, 39)	95860	(14, 27)	78333	(16, 20)	37735
(6, 37)	99858	(8, 13)	66972	(9, 34)	97799	(11, 16)	56240	(12, 40)	96238	(14, 28)	80369	(16, 21)	44153
(6, 38)	99882	(8, 14)	72670	(9, 35)	98057	(11, 17)	62162	(12, 41)	96579	(14, 29)	82185	(16, 22)	49754
(6, 39)	99902	(8, 15)	77229	(9, 36)	98283	(11, 18)	67127	(12, 42)	96887	(14, 30)	83809	(16, 23)	54665
(6, 40)	99918	(8, 16)	80916	(9, 37)	98482	(11, 19)	71323	(12, 43)	97166	(14, 31)	85264	(16, 24)	58991
(6, 41)	99932	(8, 17)	83925	(9, 38)	98658	(11, 20)	74890	(12, 44)	97419	(14, 32)	86571	(16, 25)	62818
(6, 42)	99943	(8, 18)	86400	(9, 39)	98812	(11, 21)	77942	(12, 45)	97648	(14, 33)	87747	(16, 26)	66215
(6, 43)	99953	(8, 19)	88451	(9, 40)	98949	(11, 22)	80565	(12, 46)	97856	(14, 34)	88808	(16, 27)	69242
(6, 44)	99961	(8, 20)	90159	(9, 41)	99069	(11, 23)	82831	(12, 47)	98045	(14, 35)	89767	(16, 28)	71946
(6, 45)	99967	(8, 21)	91590	(9, 42)	99175	(11, 24)	84797	(12, 48)	98217	(14, 36)	90635	(16, 29)	74370
(6, 46)	99973	(8, 22)	92795	(9, 43)	99269	(11, 25)	86508	(12, 49)	98373	(14, 37)	91421	(16, 30)	76508
(6, 47)	99977	(8, 23)	93813	(9, 44)	99352	(11, 26)	88003	(12, 50)	98514	(14, 38)	92135	(16, 31)	78510
(6, 48)	99981	(8, 24)	94676	(9, 45)	99426	(11, 27)	89314	$n = 13$		(14, 39)	92783	(16, 32)	80281
(6, 49)	99984	(8, 25)	95411	(9, 46)	99491	(11, 28)	90466	(13, 14)	14286	(14, 40)	93374	(16, 33)	81884
(6, 50)	99987	(8, 26)	96037	(9, 47)	99548	(11, 29)	91481	(13, 15)	26000	(14, 41)	93912	(16, 34)	83337
$n = 7$		(8, 27)	96574	(9, 48)	99599	(11, 30)	92377	(13, 16)	35714	(14, 42)	94403	(16, 35)	84657
(7, 8)	25000	(8, 28)	97034	(9, 49)	99644	(11, 31)	93171	(13, 17)	43849	(14, 43)	95851	(16, 36)	85859
(7, 9)	42424	(8, 29)	97429	(9, 50)	99684	(11, 32)	93875	(13, 18)	50719	(14, 44)	95260	(16, 37)	86954
(7, 10)	55000	(8, 30)	97770	$n = 10$		(11, 33)	94501	(13, 19)	56565	(14, 45)	95635	(16, 38)	87953
(7, 11)	64326	(8, 31)	98063	(10, 11)	18182	(11, 34)	95058	(13, 20)	61573	(14, 46)	95978	(16, 39)	88867
(7, 12)	71392	(8, 32)	98317	(10, 12)	32258	(11, 35)	95555	(13, 21)	65888	(14, 47)	96293	(16, 40)	89703
(7, 13)	76841	(8, 33)	98536	(10, 13)	43357	(11, 36)	95999	(13, 22)	69627	(14, 48)	96581	(16, 41)	90469
(7, 14)	81104	(8, 34)	98726	(10, 14)	52242	(11, 37)	96395	(13, 23)	72881	(14, 49)	96846	(16, 42)	91173
(7, 15)	84480	(8, 35)	98890	(10, 15)	59447	(11, 38)	96750	(13, 24)	75726	(14, 50)	97089	(16, 43)	91819
(7, 16)	87181	(8, 36)	99033	(10, 16)	65354	(11, 39)	97068	(13, 25)	78224	$n = 15$		(16, 44)	92413
(7, 17)	89362	(8, 37)	99157	(10, 17)	70243	(11, 40)	97354	(13, 26)	80424	(15, 16)	12500	(16, 45)	92960
(7, 18)	91135	(8, 38)	99265	(10, 18)	74324	(11, 41)	97610	(13, 27)	82369	(15, 17)	23018	(16, 46)	93464
(7, 19)	92586	(8, 39)	99359	(10, 19)	77755	(11, 42)	97841	(13, 28)	84093	(15, 18)	31944	(16, 47)	93929
(7, 20)	93780	(8, 40)	99441	(10, 20)	80657	(11, 43)	98048	(13, 29)	85625	(15, 19)	39578	(16, 48)	94358
(7, 21)	94769	(8, 41)	99512	(10, 21)	83127	(11, 44)	98235	(13, 30)	86991	(15, 20)	46150	(16, 49)	94754
(7, 22)	95589	(8, 42)	99574	(10, 22)	85239	(11, 45)	98403	(13, 31)	88211	(15, 21)	51843	(16, 50)	95130
(7, 23)	96274	(8, 43)	99628	(10, 23)	87054	(11, 46)	98555	(13, 32)	89304	(15, 22)	56801	$n = 17$	
(7, 24)	96846	(8, 44)	99675	(10, 24)	88619	(11, 47)	98691	(13, 33)	90283	(15, 23)	61139	(17, 18)	11111
(7, 25)	97327	(8, 45)	99716	(10, 25)	89975	(11, 48)	98815	(13, 34)	91164	(15, 24)	64953	(17, 19)	20648
(7, 26)	97731	(8, 46)	99752	(10, 26)	91152	(11, 49)	98926	(13, 35)	91957	(15, 25)	68319	(17, 20)	28888
(7, 27)	98072	(8, 47)	99783	(10, 27)	92178	(11, 50)	99027	(13, 36)	92672	(15, 26)	71301	(17, 21)	36054
(7, 28)	98360	(8, 48)	99810	(10, 28)	93074	$n = 12$		(13, 37)	93318	(15, 27)	73951	(17, 22)	42317
(7, 29)	98603	(8, 49)	99834	(10, 29)	93859	(12, 13)	15385	(13, 38)	93902	(15, 28)	76314	(17, 23)	47820
(7, 30)	98810	(8, 50)	99855	(10, 30)	94548	(12, 14)	27799	(13, 39)	94430	(15, 29)	78428	(17, 24)	52676
(7, 31)	98985	$n = 9$		(10, 31)	95154	(12, 15)	37949	(13, 40)	94910	(15, 30)	80322	(17, 25)	56979
(7, 32)	99134	(9, 10)	20000	(10, 32)	95688	(12, 16)	46340	(13, 41)	95345	(15, 31)	82025	(17, 26)	60807
(7, 33)	99260	(9, 11)	35065	(10, 33)	96159	(12, 17)	53344	(13, 42)	95740	(15, 32)	83559	(17, 27)	64222

TABLE—*Continued*

(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)
(17, 28)	67281	(19, 25)	44360	(21, 27)	41355	(23, 33)	54244	(25, 43)	70434	(28, 34)	33396	(31, 34)	17279
(17, 29)	70027	(19, 26)	49087	(21, 28)	45940	(23, 34)	57325	(25, 44)	72104	(28, 35)	37471	(31, 35)	22157
(17, 30)	72499	(19, 27)	53319	(21, 29)	50080	(23, 35)	60153	(25, 45)	73662	(28, 36)	41233	(31, 36)	26667
(17, 31)	74731	(19, 28)	57120	(21, 30)	53829	(23, 36)	62753	(25, 46)	75117	(28, 37)	44711	(31, 37)	30842
(17, 32)	76750	(19, 29)	60543	(21, 31)	57232	(23, 37)	65147	(25, 47)	76478	(28, 38)	47931	(31, 38)	34715
(17, 33)	78581	(19, 30)	63636	(21, 32)	60330	(23, 38)	67356	(25, 48)	77751	(28, 39)	50919	(31, 39)	38313
(17, 34)	80245	(19, 31)	66436	(21, 33)	63155	(23, 39)	69397	(25, 49)	78943	(28, 40)	53694	(31, 40)	41662
(17, 35)	81759	(19, 32)	68978	(21, 34)	65736	(23, 40)	71285	(25, 50)	80061	(28, 41)	56275	(31, 41)	44782
(17, 36)	83140	(19, 33)	71291	(21, 35)	68101	(23, 41)	73035	n = 26		(28, 42)	58680	(31, 42)	47693
(17, 37)	84401	(19, 34)	73399	(21, 36)	70270	(23, 42)	74659	(26, 27)	07407	(28, 43)	60923	(31, 43)	50413
(17, 38)	85555	(19, 35)	75325	(21, 37)	72264	(23, 43)	76168	(26, 28)	14105	(28, 44)	63018	(31, 44)	52958
(17, 39)	86612	(19, 36)	77086	(21, 38)	74091	(23, 44)	77571	(26, 29)	20179	(28, 45)	64976	(31, 45)	55341
(17, 40)	87582	(19, 37)	78701	(21, 39)	75791	(23, 45)	78877	(26, 30)	25703	(28, 46)	66810	(31, 46)	57576
(17, 41)	88472	(19, 38)	80184	(21, 40)	77353	(23, 46)	80095	(26, 31)	30740	(28, 47)	68527	(31, 47)	59673
(17, 42)	89292	(19, 39)	81547	(21, 41)	78798	(23, 47)	81231	(26, 32)	35343	(28, 48)	70138	(31, 48)	61643
(17, 43)	90046	(19, 40)	82802	(21, 42)	80135	(23, 48)	82293	(26, 33)	39561	(28, 49)	71651	(31, 49)	63497
(17, 44)	90742	(19, 41)	83960	(21, 43)	81375	(23, 49)	83285	(26, 34)	43434	(28, 50)	73072	(31, 50)	65241
(17, 45)	91383	(19, 42)	85028	(21, 44)	82525	(23, 50)	84213	(26, 35)	46996	n = 29		n = 32	
(17, 46)	91976	(19, 43)	86015	(21, 45)	83593	n = 24		(26, 36)	50279	(29, 30)	06667	(32, 33)	06061
(17, 47)	92524	(19, 44)	86929	(21, 46)	84587	(24, 25)	08000	(26, 37)	53311	(29, 31)	12757	(32, 34)	11643
(17, 48)	93031	(19, 45)	87775	(21, 47)	85512	(24, 26)	15174	(26, 38)	56114	(29, 32)	18333	(32, 35)	16797
(17, 49)	93501	(19, 46)	88560	(21, 48)	86374	(24, 27)	21630	(26, 39)	58711	(29, 33)	23452	(32, 36)	21562
(17, 50)	93936	(19, 47)	89288	(21, 49)	87178	(24, 28)	27458	(26, 40)	61121	(29, 34)	28160	(32, 37)	25977
n = 18		(19, 48)	89965	(21, 50)	87929	(24, 29)	32736	(26, 41)	63359	(29, 35)	32491	(32, 38)	30075
(18, 17)	11765	(19, 49)	90594	n = 22		(24, 30)	37529	(26, 42)	65441	(29, 36)	36506	(32, 39)	33883
(18, 18)	21769	(19, 50)	91179	(22, 23)	08696	(24, 31)	41893	(26, 43)	67381	(29, 37)	40213	(32, 40)	37429
(18, 19)	30341	n = 20		(22, 24)	16418	(24, 32)	45875	(26, 44)	69189	(29, 38)	43647	(32, 41)	40734
(18, 20)	37735	(20, 21)	09524	(22, 25)	23304	(24, 33)	49519	(26, 45)	70878	(29, 39)	46835	(32, 42)	43820
(18, 21)	44153	(20, 22)	17884	(22, 26)	29469	(24, 34)	52858	(26, 46)	72457	(29, 40)	49798	(32, 43)	46704
(18, 22)	49754	(20, 23)	25259	(22, 27)	35007	(24, 35)	55926	(26, 47)	73933	(29, 41)	52556	(32, 44)	49404
(18, 23)	54665	(20, 24)	31795	(22, 28)	39998	(24, 36)	58748	(26, 48)	75316	(29, 42)	55127	(32, 45)	51938
(18, 24)	58991	(20, 25)	37611	(22, 29)	44509	(24, 37)	61350	(26, 49)	76613	(29, 43)	57526	(32, 46)	54306
(18, 25)	62818	(20, 26)	42806	(22, 30)	48598	(24, 38)	63753	(26, 50)	78298	(29, 44)	59768	(32, 47)	56534
(18, 27)	58910	(20, 27)	47463	(22, 31)	52315	(24, 39)	65975	n = 27		(29, 45)	61866	(32, 48)	58629
(18, 28)	62333	(20, 28)	51651	(22, 32)	55701	(24, 40)	68034	(27, 28)	07143	(29, 46)	63830	(32, 49)	60600
(18, 29)	65413	(20, 29)	55428	(22, 33)	58792	(24, 41)	69943	(27, 29)	13625	(29, 47)	65672	(32, 50)	62457
(18, 30)	68191	(20, 30)	58845	(22, 34)	61621	(24, 42)	71716	(27, 30)	19524	(29, 48)	67401	n = 33	
(18, 31)	70703	(20, 31)	61943	(22, 35)	64214	(24, 43)	73364	(27, 31)	24906	(29, 49)	69025	(33, 34)	05882
(18, 32)	72979	(20, 32)	64759	(22, 36)	66595	(24, 44)	74899	(27, 32)	29829	(29, 50)	70551	(33, 35)	11314
(18, 33)	75047	(20, 33)	67324	(22, 37)	68787	(24, 45)	76330	(27, 33)	34342	n = 30		(33, 36)	16340
(18, 34)	76929	(20, 34)	69665	(22, 38)	70807	(24, 46)	77665	(27, 34)	38488	(30, 31)	06452	(33, 37)	20998
(18, 35)	78645	(20, 35)	71807	(22, 39)	72671	(24, 47)	78912	(27, 35)	42305	(30, 32)	12363	(33, 38)	25323
(18, 36)	80213	(20, 36)	73770	(22, 40)	74394	(24, 48)	80077	(27, 36)	45826	(30, 33)	17791	(33, 39)	29344
(18, 37)	81647	(20, 37)	75571	(22, 41)	75989	(24, 49)	81168	(27, 37)	49078	(30, 34)	22786	(33, 40)	33090
(18, 38)	82962	(20, 38)	77227	(22, 42)	77467	(24, 50)	82190	(27, 38)	52088	(30, 35)	27393	(33, 41)	36583
(18, 39)	84170	(20, 39)	78752	(22, 43)	78839	n = 25		(27, 39)	54879	(30, 36)	31649	(33, 42)	39846
(18, 40)	85279	(20, 40)	80158	(22, 44)	80114	(25, 26)	07692	(27, 40)	57469	(30, 37)	35588	(33, 43)	42897
(18, 41)	86300	(20, 41)	81457	(22, 45)	81300	(25, 27)	14620	(27, 41)	59878	(30, 38)	39240	(33, 44)	45754
(18, 42)	87241	(20, 42)	82657	(22, 46)	82404	(25, 28)	20879	(27, 42)	62120	(30, 39)	42632	(33, 45)	48432
(18, 43)	88110	(20, 43)	83768	(22, 47)	83433	(25, 29)	26552	(27, 43)	64209	(30, 40)	45786	(33, 46)	50946
(18, 44)	88912	(20, 44)	84798	(22, 48)	84393	(25, 30)	31707	(27, 44)	66160	(30, 41)	48724	(33, 47)	53307
(18, 45)	89653	(20, 45)	85754	(22, 49)	85289	(25, 31)	36404	(27, 45)	67982	(30, 42)	51463	(33, 48)	55528
(18, 46)	90340	(20, 46)	86641	(22, 50)	86127	(25, 32)	40694	(27, 46)	69686	(30, 43)	54022	(33, 49)	57619
(18, 47)	90976	(20, 47)	87466	n = 23		(25, 33)	44622	(27, 47)	71282	(30, 44)	56414	(33, 50)	59589
(18, 48)	91565	(20, 48)	88233	(23, 24)	08333	(25, 34)	48225	(27, 48)	72778	(30, 45)	58653	n = 34	
(18, 49)	92113	(20, 49)	88948	(23, 25)	15771	(25, 35)	51538	(27, 49)	74181	(30, 46)	60751	(34, 35)	05714
(18, 50)	92621	(20, 50)	89614	(23, 26)	22436	(25, 36)	54589	(27, 50)	75499	(30, 47)	62720	(34, 36)	11003
n = 19		n = 21		(23, 27)	28428	(25, 37)	57403	n = 28		(30, 48)	64568	(34, 37)	15907
(19, 20)	10000	(21, 22)	09091	(23, 28)	33834	(25, 38)	60004	(28, 29)	06897	(30, 49)	66306	(34, 38)	20462
(19, 21)	18719	(21, 23)	17120	(23, 29)	38725	(25, 39)	62412	(28, 30)	13176	(30, 50)	67940	(34, 39)	24700
(19, 22)	26364	(21, 24)	24242	(23, 30)	43162	(25, 40)	64644	(28, 31)	18910	n = 31		(34, 40)	28648
(19, 23)	33099	(21, 25)	30588	(23, 31)	47199	(25, 41)	66716	(28, 32)	24157	(31, 32)	06250	(34, 41)	32332
(19, 24)	39061	(21, 26)	36263	(23, 32)	50880	(25, 42)	68641	(28, 33)	28971	(31, 33)	11992	(34, 42)	35774

TABLE—*Concluded*

(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)	(n, t)	C(n, t)
(34, 43)	38995	(35, 48)	49075	(37, 40)	14737	(38, 38)	38800	(40, 46)	25076	(42, 48)	24074	(45, 48)	12319
(34, 44)	42011	(35, 49)	51411	(37, 41)	19008	(38, 49)	41517	(40, 47)	28422	(42, 49)	27320	(45, 49)	15977
(34, 45)	44840	(35, 50)	53613	(37, 42)	23002	(38, 50)	44081	(40, 48)	31578	(42, 50)	30387	(45, 50)	19437
(34, 46)	47497	$n = 36$		(37, 43)	26744	$n = 39$		(40, 49)	34557	$n = 43$		$n = 46$	
(34, 47)	49993	(36, 37)	05405	(37, 44)	30253	(39, 40)	05000	(40, 50)	37369	(43, 44)	04545	(46, 47)	04255
(34, 48)	52343	(36, 38)	10430	(37, 45)	33547	(39, 41)	09673	$n = 41$		(43, 45)	08821	(46, 48)	08273
(34, 49)	54555	(36, 39)	15107	(37, 46)	36643	(39, 42)	14048	(41, 42)	04762	(43, 46)	12846	(46, 49)	12071
(34, 50)	56640	(36, 40)	19469	(37, 47)	39557	(39, 43)	18147	(41, 43)	09227	(43, 47)	16640	(46, 50)	15664
$n = 35$		(36, 41)	23542	(37, 48)	42302	(39, 44)	21994	(41, 44)	13420	(43, 48)	20221	$n = 47$	
(35, 36)	05556	(36, 42)	27350	(37, 49)	44889	(39, 45)	25608	(41, 45)	17361	(43, 49)	23602	(47, 48)	04167
(35, 37)	10709	(36, 43)	30916	(37, 50)	47332	(39, 46)	29007	(41, 46)	21070	(43, 50)	26800	(47, 49)	08106
(35, 38)	15497	(36, 44)	34258	$n = 38$		(39, 47)	32208	(41, 47)	24565	$n = 44$		(47, 50)	11833
(35, 39)	19953	(36, 45)	37395	(38, 39)	05128	(39, 48)	35225	(41, 48)	27860	(44, 45)	04444	$n = 48$	
(35, 40)	24107	(36, 46)	40343	(38, 40)	09913	(39, 49)	38071	(41, 49)	30971	(44, 46)	08630	(48, 49)	04082
(35, 41)	27984	(36, 47)	43116	(38, 41)	14384	(39, 50)	40760	(41, 50)	33910	(44, 47)	12577	(48, 50)	07945
(35, 42)	31608	(36, 48)	45727	(38, 42)	18567	$n = 40$		$n = 42$		(44, 48)	16302	$n = 49$	
(35, 43)	35000	(36, 49)	48188	(38, 43)	22487	(40, 41)	04878	(42, 43)	04651	(44, 49)	19821	(49, 50)	04000
(35, 44)	38179	(36, 50)	50510	(38, 44)	26164	(40, 42)	09445	(42, 44)	09019	(44, 50)	23149		
(35, 45)	41161	$n = 37$		(38, 45)	29617	(40, 43)	13727	(42, 45)	13127	$n = 45$			
(35, 46)	43962	(37, 38)	05263	(38, 46)	32864	(40, 44)	17745	(42, 46)	16993	(45, 46)	04348		
(35, 47)	46596	(37, 39)	10165	(38, 47)	35920	(40, 45)	21522	(42, 47)	20637	(45, 47)	08448		

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