ABSTRACTS OF PAPERS

(Abstracts of papers not presented at any meeting of the Institute)

6. On a χ^2-Test with Cells Determined by Order Statistics. HERMANN WITTING, University of Freiburg. (By title)

Let $X_1, \cdots, X_n$ be a sample of a continuous one-dimensional probability distribution $Q(A)$; let $X_{n,1}, \cdots, X_{n,k-1}$ be order statistics for given ranks $r_{n,j}$ with $p_{n,j} = (r_{n,j} - r_{n,j-1})/(n+1) = p_j + o(1/\sqrt{n})$. Let $S_{n,j} = \{x: X_{n,j-1} < x \leqq X_{n,j}\}$. For testing the hypothesis that $Q(A)$ belongs to an s-parametric class of probability distributions $P(A, \theta)$ the test statistic $T_n = \sum_{j=1}^{k} n(P(S_{n,j}, \hat{\theta}_n) - p_{n,j})^2/p_{n,j}$ is used, where $\hat{\theta}_n$ is the minimum-χ^2-estimate. Then if $Q(A) = P(A, \theta_0)$ or $Q(A) = P(A, \theta_0) - q(A)/\sqrt{n}$, respectively, under certain regularity conditions T_n is asymptotically distributed as χ^2 with $(k - s - 1)$ degrees of freedom (and noncentrality parameter $\sum_{j=1}^{k} q_j^2/p_j$, $q_j = p\text{-lim } q(S_{n,j})$. Using $(k-1)$ continuous functions $\varphi_1(x), \cdots, \varphi_{k-1}(x)$, defining $\varphi_j(X_{n,j})$ successively by ordering the values $\varphi_j(X_i)$ and defining $S_{n,j} = \{x: \varphi_l(x) > \varphi_l(X_{n,l}), l = 1, \cdots, j-1; \varphi_j(x) \leqq \varphi_j(X_{n,j})\}$, the same limiting behaviour of T_n holds for probability distributions in a metric space. The proof is based on the fact that the $Q(S_{n,j})$ are jointly B-distributed (cf. J. W. Tukey, *Ann. Math. Stat.* 18(1947)529). Therefore $\sqrt{n}(Q(S_{n,j}) - p_{n,j})$ are asymptotically $N(0, C)$ where C is of rank $(k-1)$ and coincides with the covariance matrix of the multinomial distribution, underlying the corresponding classical χ^2-test with the cells $S_j = p\text{-lim } S_{n,j}$. While having the same power, this modified χ^2-procedure has certain advantages over the classical χ^2-test.

7. A Generalized Pitman Efficiency for Nonparametric Tests. HERMANN WITTING, University of Freiburg. (By title)

Asymptotic expressions up to terms of order n^{-2} are given for the efficiency of the Wilcoxon two-sample test relative to the $\bar{x}$- and t-tests for nearby alternatives. The first term is the well-known Pitman efficiency; the remaining terms are corrections for finite sample sizes. Efficiency values are given for finite sample sizes in the case of normal and rectangular distributions and comparisons with the exact values are made. In general the Wilcoxon test is shown to be nearly as good locally for moderate sample sizes as it is known to be asymptotically. A similar analysis is performed for the single-sample sign test.

(Abstracts of papers to be presented at the Washington, D. C., Annual Meeting of the Institute, December 27–30, 1959. Additional abstracts will appear in the March, 1960 issue.)

1. Some Nonparametric Problems: I. V. P. BHAPKAR, University of North Carolina and University of Poona. (By title)

Mood and Brown have considered a nonparametric test for the equality of row effects in the two-way classification with one observation per cell or the same number of observations per cell. In this paper, first their test has been extended to cover incomplete block situations. For the BIBD in the usual terminology, if m_i denotes the number of observations, for the ith 'treatment', that exceed the respective 'block'-medians, then to test the equality of 'treatment'-effects we have $(k^2(k-1))/(a(k-a)\lambda v) \sum_{i=1}^{v} (m_i - (ra/k))^2$ asymptotically distributed as a χ^2 with $v - 1$ d.f. for large r, where a is $k/2$ if k is even and $(k-1)/2$ otherwise. The χ^2 statistic appropriate for PBIBD is also given.

Next, Hoeffding's theorem on U-statistics extended by Lehmann to the case of two samples, has been extended to the case of c samples. This is then applied to derive a new test for the problem of c samples. The test criterion is in terms of the number of c-plets that can be formed by choosing one observation from each sample such that the observation from the kth sample is the least ($k = 1, 2, \cdots, c$).

2. **Some Nonparametric Problems: II.** V. P. Bhapkar, University of North Carolina and University of Poona. (By title)

Mood and Brown have considered some simple nonparametric regression problems. In this paper, their methods are extended to discuss some additional regression problems. Next some bivariate analysis of variance problems are considered. The step-down procedure is used to reduce the problem to one involving conditional univariate distributions, the other variate being regarded as a concomitant random variate. The regression methods developed earlier are used here in these bivariate problems. The method seems to be perfectly general and could be extended to the general multivariate situation.

3. **On the Foundations of the Theory of Testing Hypotheses** (Preliminary report). Allan Birnbaum, New York University.

For testing between simple hypotheses H_i, $i = 1, 2$, an experiment is called *simple* if it is equivalent, in the sense of the theory of comparison of experiments, to one observation on X, where $\text{Prob}[X = 1 \mid H_i] = p_i$, $\text{Prob}[X = 0 \mid H_i] = q_i = 1 - p_i$, $i = 1, 2$, with p_i's known, $0 \leqq p_1 \leqq p_2 \leqq 1$. If various experiments are possible for a given testing problem, and if one of these is selected by use of a definite random device unrelated to the hypotheses, the over-all procedure is called a mixture of experiments. It is proved that under minor restrictions every experiment is equivalent to a mixture of simple experiments called its *components*. The possible decompositions into components are characterized and shown to be not essentially unique, except for simple experiments, whose components are equivalent to the given experiment. It follows that customary interpretations of error-probabilities of a test, as indicators of strength of evidence provided by a test outcome, require critical and constructive revision which leads to a modified Neyman-Pearson theory in which the likelihood function holds a central position as a consistently interpretable primitive indicator of evidence relevant to hypotheses. Wald's sequential test is given an elementary justification on these terms as a technique for informative inference.

4. **Unbiased Sequential Estimation for Certain Two Parameter Problems** (Preliminary report). B. Brainerd, University of Western Ontario, I. Chorneyko, University of Alberta, and T. V. Narayana, University of Alberta.

The probability of a coin falling head is $p_1 (0 < p_1 < 1)$, if in the previous trial the outcome was tail and $p_2 (0 < p_2 < 1)$, if in the previous trial the outcome was head. At the first trial the probability of a head is p_1. Using a technique devised by one of the authors, sufficient partitions are obtained for a wide class of simple closed regions. The results of M. H. DeGroot (*Ann. Math. Stat.*, Vol. 30 pp. 80–102) are shown to generalize, with the proper modifications, to the two parameter case. Estimable functions are explicitly given, and completeness of sampling plans proved for various regions. An analogue of the necessary and sufficient conditions of Lehmann and Stein for simple closed regions is being studied.

5. **Mathematical Models for Ranking from Paired Comparisons.** H. D. Brunk, University of Missouri. (By title)

Several models are discussed in each of two categories: (I) Each possible ranking of items is assumed to have a "utility" (for some segment of the community) which depends on the expected scores of the items in paired comparisons. Special instances are models in which "worth" of an item is *defined* in terms of its expected scores in comparisons with other items. (II) Each item is *assumed to have an intrinsic worth*; these intrinsic worths determine the expected scores.

The concept of "regularity" is introduced. Let the expected scores of Item A be at least as large as those of Item B. A utility is *regular* if under these conditions every ranking in which Item A precedes Item B has at least as great utility as one in which they are interchanged. This concept specializes to rankings based on worths. A necessary and sufficient condition is given in order that a linear utility may be regular.

In the second category a "minimum assumption" model is introduced and discussed. Let $e(u, v)$ denote the expected score of an item of worth u when compared with one of worth v. The assumption is: $e(u, v)$ is non-decreasing in u, non-increasing in v.

6. Asymptotically Optimal Stopping Rules in Sequential Analysis (Preliminary Report). HERMAN CHERNOFF, Stanford University.

It is desired to decide sequentially whether the mean μ of a normal distribution with known variance is positive or negative. Suppose that an a priori distribution is given for μ which has positive density at $\mu = 0$. Suppose also that the loss due to coming to the wrong conclusion is given by $r(\mu) = k \mid \mu \mid + 0(1)$, as $\mu \rightarrow 0$. Finally suppose that the cost of sampling $c \rightarrow 0$. For the optimal sequential procedure the main contribution to the Bayes risk is given by those values of μ which are of the order of magnitude of $c^{1/3}$.

The optimal stopping rule is approximated by the solution of the analogous continuous problem involving a Wiener process. This problem in turn is reduced to the solution of a free boundary problem involving the heat equation. A method of constructing this boundary is proposed.

7. Cross-Compounded Distributions. RICHARD A. EPSTEIN AND LLOYD R. WELCH, Jet Propulsion Laboratory, California Institute of Technology. (Introduced by L. A. Zadeh)

It is known that the generating function of the compound Poisson distribution has the property that the Poisson variable can be expressed as the sum of two or more independent variables. A particular method of "cross-compounding" two distributions is suggested; the same property obtains. In theory, any two distributions can be cross-compounded to produce a third, unique distribution. However, frequently the mathematics become overly involved so that it is necessary to select the distributions with discretion. Examples are given wherein the negative binomial distribution is cross-compounded with the Exponential distribution and with the Poisson distribution. Other combinations are also suggested which might lend themselves to cross-compounding.

8. Examples of Two Independent Separable Processes Whose Sum Is Not Separable. T. FERGUSON, Princeton University.

Two examples are given of two independent stochastic processes, X_t and Y_t, both of which are separable in the sense defined in Doob's book, and yet whose sum $Z_t = X_t + Y_t$ is not a separable process. All processes considered are measurable. In the first example, X_t is a constant (i.e. non-random) function, while in the second example, X_t and Y_t are identically distributed.

9. On the Exactness of the Missing Plot Procedure in a Randomized Block Design. J. L. FOLKS AND D. L. WEST, Texas Instruments Incorporated.

The randomized block design is said to be an unbiased design in that it allows unbiased estimates of treatment differences, an unbiased estimate of the error variance and an unbiased test of treatment differences. This claim can be justified by assuming that the observations are generated by the model $y_{ij} = \mu + b_i + t_j + e_{ij}$ where $e_{ij} \sim N(0, \sigma^2)$. It can also be justified by considering the population of conceptual yields arising from all possible randomizations. In the case of a missing plot, the exact procedure described by Yates gives unbiased estimates of treatment differences, an unbiased estimate of the error variance, and an unbiased test of treatment differences. However, it is based only on the normal model, not upon randomization theory. The authors examine the missing plot procedure from the standpoint of randomization theory. The finite population of conceptual yields is examined where (1) the same block-treatment combination is always missing, and where (2) the same block-plot combination is always missing. In both cases, unbiased estimates of treatment differences are given by the usual estimates. The estimate of error variance and the test for treatment differences are unbiased in (1) under a restriction slightly weaker than homogeneity but appear not to be in (2) for any reasonable restriction.

10. First Emptiness of Two Dams in Parallel. Joseph M. Gani, Columbia University.

The paper considers the probabilities of first emptiness of two dams in parallel, both subject to steady releases at a constant unit rate, and fed by discrete Poisson inputs of unit size which are directed to the dam with lesser content. The problem is shown to be equivalent to that of the single dam fed alternately by the two ordered inputs $0 \leqq \alpha, \beta \leqq 1 (\alpha + \beta = 1)$; starting with an initial content z, the probabilities of first emptiness of the process beginning with an input α, at the times

$$T = z + [(n + 1)/2]\alpha + [n/2]\beta \ (n = 0, 1, 2, \cdots)$$

are given by $g_\alpha(z, T) = e^{-\lambda z}$ if $n = 0$, and $g_\alpha(z, T) = e^{-\lambda z}\{ \sum_{j=1}^{[(n+1)/2]} g_\beta(j\alpha + j\beta - \beta, [(n + 1)/2]\alpha + [n/2]\beta) (\lambda z)^{2j-1}/(2j - 1)! + \sum_{j=1}^{[n/2]} g_\alpha(j\alpha + j\beta, [(n + 1)/2]\alpha + [n/2]\beta) (\lambda z)^{2j}/(2j)!\}$ if $n = 1, 2, \cdots$, where $g_\beta(z, z + [(k + 1)/2]\beta + [k/2]\alpha) (k = 0, 1, 2, \cdots)$, the analogous probability beginning with an input β, is given by an interchange of β for α in the previous equation. These probabilities may be evaluated recursively. A more convenient method is found by reducing the process to an associated occupancy problem, when the probabilities can be obtained by a rapid computational procedure. Generating functions of the probabilities are derived, and the paper concludes with a general formulation of the dam problem when the times of arrival for two ordered non-negative inputs of random size form a Poisson process.

11. Stochastic Approximation and "Minimax" Problems. L. A. Gardner, Jr., MIT Lincoln Laboratory. (By title)

With the exception of the Robbins-Monro and Kiefer-Wolfowitz processes, the technique of stochastic approximation does not appear to have found a range of application consistent with the generality of its formulation (for exposition see C. Derman, "Stochastic approximation", *Ann. Math. Stat.*, Vol. 27 (1956), pp. 879–886). In this paper we consider such an iterative scheme designed to estimate the minimum of a curve which is not a regression function but the a.s. supremum of an observable random variable depending upon a parameter. The range of the parameter is a known finite interval, and the possibility of the solution being a boundary point is admitted. "Deterministic" conditions of the usual kind are imposed. The procedure is formally a truncated Kiefer-Wolfowitz process with the estimate of slope calculated from observed largest values in samples whose size tend to infinity as the iteration proceeds. Convergence with probability one is insured if this number increases sufficiently fast, or equivalently the differencing interval decreases sufficiently slow, relative to a measure of the amount of probability in left neighborhoods of the function to be minimized. Although it is easy to argue the existence of such a measure, it cannot be assumed that anything is known concerning its (finite) value. This difficulty is resolved by having the differencing interval to be used for obtaining the next iterant depend in an appropriate way on a sample of largest values at the present. Estimates of convergence rates are made and optimum values obtained for certain constants of the process. Examples show the applicability of the theory to diverse problems.

12. Some Asymptotic Results for a Coverage Problem. MAX HALPERIN, Knolls Atomic Power Laboratory. (By title)

Let $\Delta_1, \Delta_2, \cdots, \Delta_n$ be a random sample from a population with probability density $p(\Delta)$, $0 \leqq \Delta \leqq \Delta_M$, Δ_M finite. The set of line segments corresponding to the Δ_i are cast on the interval $(0, L)$, $L \geqq n\Delta_M$, in such a way that every admissible configuration of the segments is equally likely. A configuration is admissible if (a) there is no overlapping of segments with each other. (b) there is no overlapping of segments with 0 or L. Now suppose a line of length λ is cast at random in the interval $(0, L)$, $\lambda < L$; i.e. y, the coordinate of the midpoint of the line of length λ is distributed uniformly on $(\lambda/2; L - (\lambda/2)$. We define fractional converage, F, as the fraction of the line of length λ which is covered by the segments of length $\Delta_1, \Delta_2, \cdots, \Delta_n$ and consider the probability distribution of F as $n, L \to \infty$ and $n\mu/L \to V$ where $\mu = E\Delta$ and $0 < V < 1$. It is shown that $\Pr\{F = 0\} = (1 - V) \exp - (V\lambda/(1 - V)\mu)$, $\Pr\{F = 1\} = V/\mu \int_\lambda^{\Delta_M} (y - \lambda)p(y)\, dy$, if $\lambda > \Delta_M$ and in zero if $\lambda \geqq \Delta_M$; for $0 < F < 1$, there are further (continuous) contributions to the cumulative probability which unfortunately are critically dependent upon the nature of $p(\Delta)$. One can show that $EF = V$ independently of the specific nature of $p(\Delta)$ for $\lambda > \Delta_M$ but the variance is a complex function of $p(\Delta)$ which is not simply expressible even for specific $p(\Delta)$. It can be shown that for large λ, F is normally distributed with mean V and variance $\mu V(1 - V)^2[1 + \sigma^2/\mu^2]/\lambda$ where $\sigma^2 = E\Delta^2 - \mu^2$.

The above work was motivated by the need for a plausible graduation function to fit the distribution of Boron Carbide intercepted by neutron paths (Boron Carbide is used to control reactor power output). Although the above assumptions are quite naive relative to the actual complexity of the problem, preliminary experimental data suggests that use of the results to match a graduation function to two moments may adequately describe observed frequency distributions.

13. Polya Type Distributions of Convolutions. SAMUEL KARLIN, Stanford University, AND FRANK PROSCHAN, Sylvania Electric Products, Inc., Mt. View, California.

This paper obtains several useful new theorems concerning successive convolutions of Polya frequency densities, such as: If $f_1, f_2, \cdots$ are density of non-negative random variables with each f_i a Polya frequency density of order k, then $g(n, x) = f_1^* f_2^* \cdots {}^*f_n(x)$ (the n-fold convolution) is Polya type of order k in the variables n and x, where n ranges over the positive integers and x traverses the positive real line. More generally, the following theorem is derived: Let $f_1, f_2, \cdots$ be a sequence of Polya frequency densities of order k for corresponding general real valued (not necessarily positive) random variables X_1, $X_2, \cdots$. Then $h(n, x) = P[\sum_{i=1}^n X_i \geqq x;\ \sum_{i=1}^j X_i < x, j = 1, 2, \cdots, n - 1]$ is totally positive of order k in n and x, n ranging over the positive integers and x over the positive axis. Applications of these theorems are given in inventory theory, probability, and mathematics.

14. A New Proof of the Continuity Theorem of Probability Theory. Emanuel Parzen, Stanford University. (By title)

The continuity theorem states that if a sequence of characteristic functions $\varphi_n(t)$ converge to a characteristic function $\varphi(t)$ at each real t, then the corresponding distribution functions converge, $F_n(x) \to F(x)$ at all continuity points x of F. The presently known proofs of this theorem are not constructive, but rather involve compactness arguments. This paper gives a new constructive proof of the continuity theorem, based on the observation that $\int_{-\infty}^{\infty} g(x)\, dF_n(x) \to \int_{-\infty}^{\infty} g(x)\, dF(x)$ for any bounded continuous function g with integrable Fourier transform. Details of the proof are given in Chapter 10 of my book *Modern Probability Theory and its Applications*, John Wiley, New York, 1960.

15. A New Inversion Formula. Emanuel Parzen, Stanford University. (By title)

Let g be a bounded integrable Borel function of a real variable which possesses right and left hand limits at every real x. Let $g^*(x) = \{g(x + 0) + g(x - 0)\}/2$. Let

$$\gamma(u) = (1/2\pi) \int_{-\infty}^{\infty} e^{-iux} g(x)\, dx.$$

Then for any distribution function F (with corresponding characteristic function φ) $\int_{-\infty}^{\infty} g^*(x)\, dF(x) = \lim_{U\to\infty} \int_{-U}^{U} (1 - (|u|/U))\gamma(u)\varphi(u)\, du$. The proof is given in Chapter 9 of my book *Modern Probability Theory and its Applications*, John Wiley, New York, 1960.

16. A Law of Large Numbers for Dependent Random Variables. Emanuel Parzen, Stanford University. (By title)

Let $X_1, X_2, \cdots$ be random variables with zero means and uniformly bounded variances. Let $Z_n = (X_1 + \cdots + X_n)/n$. Let $C_n = E[X_n Z_n]$. *Quadratic Mean Law of Large Numbers.* $Z_n \to 0$ in mean square as $n \to \infty$ if and only if $C_n \to 0$ as $n \to \infty$. *Strong Law of Large Numbers.* $Z_n \to 0$ with probability one if $C_n = 0(n^q)$ for some positive q. These results generalize some of the known laws of large numbers for orthogonal and stationary sequences of random variables. The proof is based on the identity $n^2 E[Z_n^2] + \Sigma_{k=1}^{n} E[X_k^2] = 2\, \Sigma_{k=1}^{n} kC_k$. Details are given in Chapter 10 of my book *Modern Probability Theory and its Applications*, John Wiley, New York, 1960.

17. Inference in Stochastic Processes I: Testing Composite Hypotheses (Preliminary report). M. M. Rao, Carnegie Institute of Technology.

Let $\{x(t), t \varepsilon T\}$ be a (real) stochastic process where T is a linear Borel set. For any n, let $t_1 < t_2 < \cdots < t_n$ be in D, a dense subset of T, and $f_{t_1,\cdots,t_n}(x_{t_1}, \cdots, {}_{t_n}; \theta)$, or $f_n(x, \theta)$ say, be the finite dimensional density function (w.r.t. Lebesgue meas.), of the process, which depends on $\theta = (\theta_1, \cdots, \theta_k)$, k being independent of n. Suppose the testing problem consists of the hypotheses $H_0 : \theta \varepsilon \omega_a$ vs. $H_1 : \theta \varepsilon \omega_r$ (based on one realization), where ω_a and ω_r are closed disjoint subsets of the (real) Euclichian k-space. Assume the following conditions on the densities: (a) for all n, the carriers of $f_n(x, \theta)$ remain invariant for all θ in $\Omega = \omega_a + \omega_r$, and f_n are Baire densities, (b) if θ_1, and θ_2 in Ω are distinct, then $f_n(x, \theta_1) \neq f_n(x, \theta_2)$ a.e., and (c) if $\xi(\theta)$ is any distribution function (d.f.) on Ω which assigns positive probability to both ω_a and ω_r, then $(f_{n+1}(x, \theta) \int_\omega f_n(x, \theta)\, d\xi(\theta) - f_n(x, \theta) \int_\omega f_{n+1}(x, \theta)\, d\xi(\theta))$, for any θ in $\omega (= \omega_a$ or $\omega_r)$, is either non-negative or non-positive for all n.

Theorem: If $\{x(t),\ t \varepsilon T\}$ *is a real separable stochastic process without fixed points of discontinuity and with the finite dimensional density functions* $f_n(x, \theta)$ *satisfying the conditions* (a)–(c), *then, for a sufficiently large number of observations on the process at* t_i *of* D, *there exists an essentially unique Bayes solution, relative to an a priori distribution* $\xi(\theta)$ *on* Ω, *satisfying* (c), *for testing the composite hypotheses* H_0 *against* H_1. Instead, θ, being a vector of k components, may depend on t (or k or n). Then, if the condition (c) is suitably modified, an analogous result obtains. Some applications are considered.

18. **Testing of Hypotheses on Categorical Data.** S. N. Roy, University of North Carolina, and V. P. Bhapkar, University of Poona.

In an earlier paper, we have posed hypotheses, which might be considered to be generalizations, appropriate to the categorical data (structured or unstructured), of the usual hypotheses in classical 'normal' univariate and multivariate analysis of variance and in analysis of various kinds of 'normal' association. The large sample tests for some such hypotheses have been offered earlier and for most of the rest are offered here. The theorem on minimum χ_1^2 is proved along Cramér's lines and an independent justification for Neyman's 'linearization' technique is given. It is also shown that for linear hypotheses the minimum χ_1^2 is exactly the same expression as the minimum sum of squares obtained by the "general least squares" approach to a model involving some asymptotically normal variables.

19. **On Tests of Certain Types of Hypotheses Involving the Dispersion Matrices of Two or More Multivariate Normal Distributions and the Associated Confidence Bounds.** S. N. Roy, University of North Carolina, and R. Gnanadesikan, Bell Telephone Laboratories.

For $N\left(\underset{p\times 1}{\xi_i}, \underset{p\times p}{\Sigma_i}\right)$ $(i = 1, 2)$, one of the authors derived several years ago, on a certain principle, a test for $H_0 : \Sigma_1 = \Sigma_2$ against $H: \Sigma_i \neq \Sigma_2$, with an acceptance region $\mu_1 \leqq$ all ch $(S_1 S_2^{-1}) \leqq \mu_2$, where S_1 and S_2 are the sample dispersion matrices, and also the associated confidence bounds. In this paper the same principle is used to derive tests for $H_0 : \Sigma_1 = \Sigma_2$ against the respective alternatives (i) H: all ch $(\Sigma_1\Sigma_2^{-1}) > 1$, (ii) H: all ch $(\Sigma_1\Sigma_2^{-1}) < 1$, (iii) H: (i) $\cup$ (ii), (iv) H: at least one ch $(\Sigma_1\Sigma_2^{-1}) > 1$ and (v) H: at least one ch $(\Sigma_1\Sigma_2^{-1}) < 1$. The associated confidence bounds are also obtained and interpreted, and finally, a partial generalization of these results are made to the case of k populations, with regard to both testing of hypotheses and confidence bounds.

20. **On the Monotonic Character of the Power Functions of Two Multivariate Tests.** S. N. Roy and W. F. Mikhail, University of North Carolina.

The power function of the largest root test of normal multivariate linear hypothesis on means or of independence between two sets of variates involves, in each case, aside from the degrees of freedom, certain non-negative, non-centrality parameters. This paper supplies a relatively simple and compact proof that the power function monotonically increases as each parameter, separately, increases—a result that was conjectured and proved (but not published) by one of the authors several years ago by a very lengthy and laborious method. It is believed that, with suitable and slight modifications, the method used here should prove useful in proving or disproving similar results in a wide variety of problems in testing of hypotheses involving multivariate normal distributions.

21. Confidence Bounds for an Integral Function of an Estimate with Applications to Reliability Theory. SAM C. SAUNDERS, Boeing Scientific Research Laboratories.

Let X and Y be independent random variables with distributions $F \varepsilon \mathfrak{F}$ and $G \varepsilon \mathcal{G}$, respectively, where $\mathfrak{F}$ and $\mathcal{G}$ are subsets of the class of continuous distributions on given positive sample spaces $\mathfrak{X}$ and $\mathcal{Y}$. Let ω be a homeomorphism from $\mathcal{Y}$ into $\mathfrak{X}$ and define $H(\omega) = \int F(\omega)\, dG$. From samples $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_m$ we form $\tilde{F}_n$ and $\tilde{G}_m$, estimates of F and G, respectively, and define $\tilde{H}$, the empirical estimate of H, by $\tilde{H}(\omega) = \int \tilde{F}_n(\omega)\, d\tilde{G}_m$ for $\omega \varepsilon \Omega$, a class of homeomorphisms linearly ordered by H.

We are interested in problems associated with this phenomenological interpretation. For some device: let $\omega(Y)$ be the taxation on life under usage ω and let X be the capacity for endurance. Then $H(\omega) = P[X < \omega(Y)]$ is the unreliability and $\tilde{H}(\omega)$ is an estimate of this unreliability. Using $\tilde{H}$ to determine a maximum usage $\tilde{\omega}$, what is the probability the unreliability $H(\tilde{\omega})$ is too large? We define $\tilde{\omega}$ so that $H(\tilde{\omega})$ is distribution-free re $\mathfrak{F} \times \mathcal{G}$ or obtain a stochastic bound majorizing $H(\tilde{\omega})$ for each $(F, G) \varepsilon \mathfrak{F} \times \mathcal{G}$ under the assumption $\tilde{F}(F^{-1})$ is distribution-free re $\mathfrak{F}$ and similarly for $\tilde{G}$, $\mathcal{G}$. This provides an answer in one important application and the theory is developed so that many such reliability problems can be treated.

22. A Rank Sum Test for Comparing all Pairs of Treatments. ROBERT G. D. STEEL, Cornell University.

Consider a permutation of n_1X_1's, $\cdots$, n_kX_k's with $n_1 \leqq \cdots \leqq n_k$ arising from ordering, from smallest to largest, observations on k treatments. Assign ranks $1, \cdots, n_i + n_j$ to the observations on all possible pairs of treatments and sum the ranks assigned to the observations on the treatment with lower subscript. This gives a test criterion denoted by $(T_{12}, \cdots, T_{1k}, T_{23}, \cdots, T_{k-1,k})$. A recursion formula is developed for computing probabilities and is used to show, by induction, that $\mu(T_{ij}) = n_i(n_i + n_j + 1)/2$, $\sigma^2(T_{ij}) = n_in_j(n_i + n_j + 1)/12$, $\sigma(T_{hi}T_{hj}) = n_hn_in_j/12 = \sigma(T_{hj}T_{ij})$, $\sigma(T_{hi}T_{ij}) = -n_hn_in_j/12$ and $\sigma(T_{gh}T_{ij}) = 0$. From the distribution of $(T_{12}, \cdots, T_{k-1,k})$, the distribution of $\min\{T_{ij}\}$ can be obtained. Several such distributions are computed for a common value of n. These provide critical values for a non-parametric multiple comparison rank sum test.

23. Asymptotic Expansions for the Mean and Variance of the Serial Correlation Coefficient. JOHN S. WHITE, Aero Division, Minneapolis Honeywell Regulator Co.

Following the procedure used by W. J. Dixon (*Ann. Math. Stat.*, 1944, pp. 119–144) series expansions are obtained for the first two moments of $\hat{\alpha} = \sum x_t x_{t-1} / \sum x_{t-1}^2$ where (x_t) is a first order auto-regressive Gaussian process with parameter α. The series expansions are carried out to terms of order T^{-3} and α^4 thus extending the asymptotic results of several authors.

The results are obtained for both the stationary and fixed initial variate case.