tributions; there is no such decision function. The proof of Theorem 17.3.1 on the spectral distribution of a stationary time series is incomplete. It is not shown that the approximating distributions F_M have a unique limit.

To sum up, although the material selected by the author is not always up-to-date, it acquaints the reader with the probabilistic background, the main branches and the basic problems of mathematical statistics. The presentation of the material in the first printing suffers from many errors and some other shortcomings which are mainly concentrated in the chapters on parametric estimation and hypothesis testing.

REFERENCES

[1] Chapman, Douglas G. and Robbins, Herbert (1951). Minimum variance estimation without regularity assumptions. *Ann. Math. Statist.* **22** 581–586.

[2] Fraser, D. A. S. (1957). *Nonparametric Methods in Statistics*. Wiley, New York.

[3] Lehmann, E. L. (1959). *Testing Statistical Hypotheses*. Wiley, New York.

[4] Lehmann, E. L. and Stein, C. (1949). On the theory of some nonparametric hypotheses. *Ann. Math. Statist.* **20** 28–45.

[5] Smirnov, N. V. (1944). Approximation of distribution laws of random variables from empirical data. (Russian.) *Uspehi Mat. Nauk* **10** 179–206.

Review 2, by D. R. Cox

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Professor Wilks's new book in an important addition to the text-books on the mathematical aspects of the theory of statistics, giving an original and coherent account of a wide range of topics. The book is intended primarily as an introduction to the subject for those with a first degree in mathematics and the mathematical style, clear and unfussy, is nicely judged for this. In addition to the main group of readers for whom the book was written, it should be useful as a reference book on theory for applied statisticians with a good mathematical training.

The general plan of the book is as follows. The first 190 pages deal with probability, random variables and the special distributions of statistics. The next 90 pages cover sampling theory, both small sample and asymptotic. Then there are 200 pages on estimation and testing, in a way the core of the book. The final 130 pages are concerned with sequential analysis, decision theory, time series and multivariate analysis. Each of the 18 chapters has exercises, over 400 in all. These range from fairly simple problems based directly on material in the book to much more difficult exercises introducing important results not covered in the main text. The exercises are a very valuable part of the book.

This is a book on mathematical theory, not on statistical methodology and, in his Preface, Professor Wilks vigorously defends separating the two. Now it would be most unfair to criticize Professor Wilks for not producing a comprehensive treatise on all aspects of statistics. Nevertheless, the book would, I think, have been much strengthened by the inclusion both of more extended motivation of some of the topics and also of critical discussion of such ideas as sufficiency, unbiasedness, and so forth. The first point is perhaps not too important. Thus the detailed treatment of components of variance is a highly formal, and mathematically unattractive, exercise. Yet we can assume that the student will, elsewhere in his work, see in detail why these topics are of practical interest. The second matter, the weakness of the discussion of statistical concepts, is, however, more serious. We can reasonably expect a major book on the theory of statistics both to give the distributional calculations of statistics and also to examine fairly searchingly the main concepts of the subject, putting them into proper relation to one another. The present book does the first with distinction, but largely omits the second.

A particular criticism is of the treatment of sufficiency (pp. 351, 354–358). The definition is given in one sentence without comment or explanation (p. 351). It is then applied to problems of point estimation only. Indeed the reader is left with the impression that the sole value of sufficiency is in connexion with the elegant but not very important process of Rao-Blackwellization (pp. 357–358). There is no hint that sufficiency is relevant in all problems of inference and decision. Also the exponential family of distributions, which can be used to collect together neatly a range of results about important special distributions, appears only briefly, in the exercises.

Other general criticisms are that the conceptual and mathematical connexion between confidence intervals and tests is not established, that invariance ideas are not exploited, that the derivation of tests from conditional distributions is not discussed, and that Bayes's theorem is not mentioned, even as a result in probability theory; prior distributions do, however, appear in a very formal way on pp. 401, 508–511. Now the importance in statistics of arguments based on Bayes's theorem is, of course, controversial. I would myself accept what is probably Professor Wilks's view that, because of the non-availability of a prior frequency distribution, the method is not often applicable. Yet prior frequency distributions are sometimes known and, in any case, it is most stimulating to examine problems from this different point of view, so that there seems a strong case for at least teaching students how to compute a posterior distribution.

The mathematical arguments used in the book are usually pleasantly direct and avoid irrelevant complications. In some cases, however, methods of proof are used which give no explanation of why the results hold; some people may not feel this a disadvantage. For example, the joint distribution of mean and variance in samples from a normal distribution is obtained by computing the joint characteristic function (pp. 208–210). This is an important and powerful method which the student should certainly know about. But the introduction of a suitable orthogonal transformation, supplemented by geometrical considerations, gives real insight into just why the sample variance can be treated as the sum of squares of independent normal variables, all independent of the sample mean. Further, this prepares the way for a treatment of least squares theory and multivariate analysis that I feel would give much more insight than the account given by Professor Wilks.

The organization of a book of this size into a consistent whole is a most formidable job and Professor Wilks deserves warm congratulations on his success. The general plan of the book, as well as the details, are likely to influence the teaching of the subject.

Nevertheless, there are imperfections of detail. These seemed particularly noticeable in Chapter 9, on asymptotic sampling theory. The chapter starts with the weak law of large numbers for the sample mean in Khintchine's form, involving first the finiteness of the population mean. Then the simpler form with finite variance is derived from Chebyshev's inequality. In fact, however, a more general case of this has already been given in **4.3.1**, so that repetition of the argument is unnecessary. The next topic is the central limit theorem proved for identically distributed one-dimensional components and stated without proof in more general cases. Then an important theorem is given on the asymptotic distribution of a function of the sample mean, the so-called Δ method. The proof starts with an appeal to the strong law of large numbers when the more elementary weak law would have been enough. Later in the proof there is hidden an application of **4.3.6.a**, a theorem on convergence in distribution. The absence of a reference could confuse the student. An extension of the theorem to functions of a vector mean is given without proof. The inclusion of a form of the theorem for statistics other than the sample mean would have been very useful and is needed to solve one of the problems.

Next a result is given on the limiting chi-squared distribution of a quadratic form in asymptotically normal variables. The proof here very naturally hinges on the general limiting results proved earlier (Section 4.3); the crucial result is **4.3.5** asserting convergence of a continuous function of a convergent sequence of random variables. Now I found the account of this most confusing, because of a mixing up of the concepts of convergence in probability and convergence in distribution. Theorem **4.3.4** asserts that convergence in probability (to a possibly non-degenerate random variable) implies convergence in distribution. But the converse is obviously false. Yet Theorems **4.3.5** and **4.3.6**, formulated as theorems on convergence in probability, are applied in conjunction with the central limit theorem, which concerns convergence in distribution. The conclusion seems to be that Theorems **4.3.5** and **4.3.6** should be reformulated as theorems on convergence in distribution. It would also probably be an advantage, in an introductory account not concerned with stochastic processes in continuous time, to restrict the definition of convergence in probability to convergence to a constant.

Chapter 9 continues with a good discussion of the Edgeworth expansion for the distribution of the mean; at the top of p. 264 the student could be saved some work by a reference to **7.2.1**. The next section, 9.5, is a valuable sketch of the proof of an important theorem of Wald and Wolfowitz on the asymptotic distribution of linear functions in sampling a finite population. References might usefully have been given to later extensions of this result. The final section of Chapter 9 is concerned with asymptotic distributions connected with order statistics.

There is not space to attempt a similarly detailed criticism of the whole book. I would like, however, to comment briefly on Chapters 10 and 17, entitled respectively Linear Statistical Estimation and Time Series.

The central results of Chapter 10 are the least squares and associated theorems. The general approach used, which is strictly algebraic, has been criticized earlier in the review. Further, there are gaps in the treatment. Results proved for hypotheses of full rank are later applied without comment to hypotheses not of full rank. The vital ideas of orthogonality and of the sum of squares for one group of parameters adjusting for another group are not covered; neither is the closely connected topic of analysis of covariance. The general method for estimating components of variance given in **10.7.1** could lead to very inefficient estimates; this is one point where an appeal to sufficiency could be very useful in dealing with the standard balanced situations. Among the other sections in this chapter is a good account of Scheffé's and Tukey's methods for determining simultaneous confidence intervals and some discussion of stratified sampling.

The chapter on time series is a very good short introduction to the main sampling properties of estimates of the correlogram and spectrum. A number of other, topics such as prediction theory, are briefly touched on.

To sum up, Professor Wilks's book, while it is open to criticism in a number of respects, meets a real need. While not displacing H. Cramér's *Mathematical Methods of Statistics* (Princeton University Press), it does, of course, contain much material that has appeared since Cramér's book was written. M. G. Kendall and A. Stuart's *Advanced Theory of Statistics*, **1, 2** (London: Griffin) will probably be preferred by the working statistician looking for an introduction to and survey of a particular advanced topic. But Professor Wilks's book develops in a more connected way and its choice of topics is more in line with conventional interests. Therefore the present book is likely to be preferred as an introductory text-book for advanced students.

The physical appearance of the book is of the high standard to be expected of Wiley's books, although the arrangement of some of the more complex formulae could have been improved and there are many minor misprints.