BOOK REVIEW

Correspondence concerning reviews should be addressed to the Book Review Editor, Professor William Kruskal, Department of Statistics, University of Chicago, Chicago 37, Illinois.

KENNETH J. ARROW, SAMUEL KARLIN AND HERBERT SCARF (editors). *Studies in Applied Probability and Management Science.* Stanford Mathematical Studies in the Social Sciences VII. Stanford University Press, Stanford, California, 1962. $8.50. 287 pp.

Review By GEORGE HADLEY

University of Chicago

This work contains fifteen individual papers, a fair number of which are parts of Stanford Ph.D. theses. Each of the papers can be considered to fall into one of the following five categories:

I. Capital Theory
II. Inventory Theory
III. Theory of Dams
IV. Equipment Replacement, Maintenance, and Repair
V. Particle Counters

Categories II and III are both part of the theory of storage, the difference between them being that for inventory problems one has some control over replenishment of the system but no control over the demands on the system (efflux from the system), while for problems on the theory of dams, one normally has no control over replenishment but does have some control over the efflux from the system.

The following list indicates the way in which the various papers can be grouped into the above categories (the number preceding the paper is the chapter number in the book). It might be noted that the grouping is not always clear cut.

I.

1. K. J. Arrow, "Optimal Capital Adjustment."
9. S. Karlin, "Stochastic Models and Optimal Policy for Selling an Asset."

II.

5. A. J. Clark and H. Scarf, "Approximate Solutions to a Simple Multi-Echelon Inventory Problem."
8. D. Iglehart and S. Karlin, "Optimal Policy for Dynamic Inventory Processes with Non-Stationary Stochastic Demands."
10. S. Karlin and C. R. Carr, "Prices and Optimal Inventory Policy."
13. D. M. Roberts, "Approximations to Optimal Policies in a Dynamic Inventory Model."

14. R. C. Singleton, "Steady-State Properties of Selected Inventory Models."
15. P. W. Zelma, "Inventory Depletion Policies."

III.

6. J. Gani, "The Time-Dependent Solution for a Dam with Ordered Poisson Inputs."
7. D. P. Gaver, Jr. and R. G. Miller, Jr., "Limiting Distributions for Some Storage Problems."

IV

2. R. E. Barlow, "Repairmen Problems."
4. R. E. Barlow and F. Proschan, "Planned Replacement."
12. R. Radner and D. W. Jorgenson, "Optimal Replacement and Inspection of Stochastically Failing Equipment."

V.

3. R. E. Barlow, "Applications of Semi-Markov Processes to Counter Problems."
11. R. Pyke, "Markov Renewal Processes of Zero Order and Their Applications to Counter Theory."

In Group I, Arrow's paper discusses the problem of determining the optimal stock of capital goods $K(t)$ which should be possessed by a firm as a function of time, starting from an initial stock K_0. The function relating profit to the capital stock, rate of depreciation, etc., is assumed to be independent of time. The function $K(t)$ is determined by minimizing the present worth of the surplus of receipts over expenditures. It is shown, for example, that if the net profit has only one local maximum (as a function of K) at $\bar{K}$ then, provided that capital goods cannot be sold outright but can only depreciate, $K(t) = \bar{K}$, $t > 0$, if $K_0 \leqq \bar{K}$, and if $K_0 > \bar{K}$, $K(t)$ should decrease exponentially from K_0 to $\bar{K}$ and then remain constant at $\bar{K}$. No restrictions are placed on the firm's ability to accumulate capital if $\bar{K} > K_0$, i.e., it can do so instantaneously. Given the assumptions, these results are not too astonishing. No probabilistic considerations appear in this paper, and hence, to fit in with the title of the book, it is presumably to be considered to be a study in the management science area. Karlin's paper on optimal policies for selling an asset, studies a situation where an individual must sell some asset in a fixed length of time or after a fixed number of offers. Offers arrive at random points in time, with the offers being made also being random variables. Each time an offer is made, a decision must be made whether or not to accept it. The problem is to determine the decision rule which maximizes the expected selling price.

In category II, the paper by Clark and Scarf is concerned with extending their previously published multi-echelon model to the case where there can be fixed charges incurred when shipping units from one installation to the one at the next lowest level. The authors do not solve this problem. They merely suggest an approximate solution for a very special case consisting of only two installations, and provide bounds on the true optimal cost. The paper of Iglehart and Karlin is concerned with determining the form of optimal policies for a dynamic periodic review inventory system in which the density function for the demand can change from period to period. However, the density function is to be selected from one of a finite number of such, and if the density function for the demand x is $\varphi_i(x)$ in a given period, the probability that it will be $\varphi_j(x)$ in the next period is ρ_{ij}. Both the backorders and lost sales cases are studied under very restrictive assumptions concerning the lead times and the way costs are incurred. The paper of Karlin and Carr generalizes the single period and stationary periodic review inventory models to the case where the distribution of the random variable representing the demand in any period will depend on the price charged for the product. Two models, called multiplicative and additive models, are considered. In the multiplicative model the demand $x = g(p)w$ where $g(p)$ is a deterministic function of the price p and w is a random variable with a specified distribution. For the additive model $x = g(p) + u$, where u is a random variable with a specified distribution. The paper by Roberts discusses methods for computing approximate values of S, s, when a periodic review system in which all demands occurring when the system is out of stock are backordered is operated using an (S, s) policy. Attention is restricted to the case where the fixed ordering cost and stockout cost are large. Singleton's paper is concerned with determining the steady state distributions of the stock level at the beginning or end of a period for periodic review inventory systems with a stationary distribution of demand under assumptions concerning resupply which are different from those normally assumed. For example, one case he considers is that where the number of periods which elapse between time when orders can be placed is a random variable, and also the quantities available for delivery are also random variables. Zelma is concerned in his paper with how to issue items from a stockpile so as to maximize the total field life when the life of an individual item is a function of its age at the time of issue.

Gani's paper in category III is concerned with determining the probability of emptiness for a dam of infinite capacity having a steady release from the dam and fed by two unequal ordered inputs whose arrival times form a Poisson process. The work of Gaver and Miller uses the theory of continuous-time Markov processes to determine limiting distributions for the content of a storage system when the time between arrivals of inputs is Poisson distributed and the quantity arriving is an arbitrary random variable. Various release rules are considered.

In category IV, Barlow's paper is concerned with finding analytic expressions for the transient solutions to the queuing problem in which there are m machines, n spares and s service facilities for a number of special cases. The case which receives the greatest attention is, of course, that for exponential service and failure. The paper of Barlow and Proschan deals with three types of planned replacement policies for some item which is to operate continuously over some time interval. If the item actually fails it is imagined that it is detected at once and replaced instantaneously at a cost c, which is greater than the replacement cost for an item which has not failed. The optimal policy of a given type is selected by minimizing expected costs. Radner and Jorgenson study several models dealing with inspection and replacement of stochastically failing equipment. Their study was no doubt mainly motivated by problems arising with complex missile systems such as the Minuteman. Their first model deals with determining the optimal time between replacement or repair for a single part when replacements are made only at discrete equally spaced intervals of time. The criterion used is to maximize the time that the part is operating. The time required for replacement is assumed to be positive. Other models deal with inspection and replacement of a part in the presence of another part whose status is continually monitored.

In category V, Barlow's paper deals with the application of the theory of semi-Markov processes to particle counters to answer such questions as what is the probability of registering the first k out of n particles in a time interval of length t, and what is the probability that the counter is inoperative at time t. Pyke's paper deals with a generalization of renewal theory to sequences of random variables that are independent, but not necessarily identically distributed. The theory has applications to inventory problems as well as particle counters.

There seems to be almost no thread of continuity whatever between the papers, even within a given category. While it will no doubt be true that there is material in this book that will be of interest to workers in a variety of fields, it seems unlikely that there will be many readers who will care to study all the papers. The reviewer did not find this book to be as stimulating as some of the previous works which the same editors have produced, and is not able to provide more than a halfhearted recommendation for the current work. He feels that it would have been more suitable to have published the articles in appropriate journals.