THOMAS L. SAATY, *Elements of Queueing Theory, with Applications*. McGraw-Hill Book Company, New York, New York, 1961. $11.50 xv + 423 pp.

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Since around 1955, the literature of mathematical congestion theory has been emerging from the journal articles and appearing in books. One of the first of these books written in English is Dr. Saaty's, and it is an ambitious effort in that he tries to accomplish three tasks: (1) to give the reader most of the mathematical methods that are used in queueing theory, (2) to canvass the enornous literature of the subject, indicating who has written about what problem, and (3) to present systematic accounts of the mathematical models used in congestion theory. His book is divided into four parts. Part 1 introduces the topics of queueing theory, describes illustrative mathematical models of queueing situations, and gives a review of probability and Markov stochastic processes. Part 2 is brief, and deals with those queueing models which can be treated by use of birth-and-death processes. Part 3, entitled "Non-Poisson Queues" is about models in which either the interarrival times or the service times do not have a negative exponential distribution. The material covered is the by now well-known work of Pollaczek, Khinchin, and Takács. Throughout, the methods employed reduce to the solution of Kolmogorov's equations for a Markov process. Part 4, finally, concerns ramifications and applications of the preceding theory, together with an account of renewal theory. Each Part described above consists of several chapters (minimum two, maximum five), and each chapter ends in copious exercises and further results that the reader is invited to solve and verify, respectively.

Upon glancing at the book, it is apparent that the author has tried to give a wide survey of the literature and methods of queueing theory, one that will be accessible (that is, intelligible, and hence useful) to a large number of potential users: engineers, mathematicians, operations analysts, etc. For this reason he has held the mathematical level of his book down to a denominator consisting of elementary probability, calculus, Laplace-Stieltjes transforms, and in a few places, elementary complex variables. In making such a restriction he is justified also by the historical development of the field, for these topics form a kernel on the basis of which most of the literature of congestion can be understood.

How then does the theory of queues fare with the author when he has set his sights in this way? As a review of the subject, Dr. Saaty's text is superb in several respects, and lacking in others. In comprehensiveness he has outstripped all authors except possibly R. Syski, whose 700-page volume was nevertheless devoted largely to telephony problems. His is the second English text on the subject that is not explicitly a monograph of frankly limited scope. He gives an abundance of concrete formulas and actual results which the practitioner can use or adapt, and he surveys the principal methods of solving problems.

However, it is one thing to shun unnecessary rigor, and another to do a hurried job of writing. Even with the limitation to elementary methods, the book shows many signs of a kind of indigestion. For example, even though he gives a three-page list of essential symbols on pp. xiii–xv, the author sometimes adopts, and at other times eschews, the notation of the writer whose work he is reporting. The writing is adequately clear when engineering matters are at issue, but it is often misleading or inaccurate when mathematical niceties are involved. E.g., it is said (p. 74) that a Markov process is "described by the Chapman-Kolmogorov equation"; the nearest correct definition of the Markov property appears in a concluding afterthought on p. 77. On the other hand, what is the value of mentioning, to readers armed only with calculus and elementary probability, semi-groups of bounded linear operators, as is done on p. 76? There are a few egregious errors; e.g., the argument for equation (9-30), allegedly evaluating the waiting-time distribution of the nth customer in Lindley's model, is completely fallacious.

A book review is most informative to its readers only if it includes comparisons of the book under review to other recent books in the same field. At the risk of being odious, then, we take this opportunity to draw some comparisons among the following: Saaty's book, Riordan (1962), Takács (1962) and LeGall (1962).

All four volumes are about a portion of applied mathematics that still has a close affinity to its applied origins, and naturally, the intersection of their topics is large. All four are surveys of some of the principal works and results of the field. In Saaty's and LeGall's books, however, the "survey" aspect is inescapable, being built right into the organization of the books. In Riordan's it is more attenuated, and in Takács' it is virtually hidden by his having translated other authors' results into his own careful, uniform notation, while giving credit, usually, where due.

There is a further difference in emphasis between the books of Saaty and LeGall on one hand, and those of Takács and Riordan on the other. I think it is fair to state that Saaty's and LeGall's (resp.) books emphasize the practical aspect, the problem of "getting meaningful numbers" out of the analysis, and the very wide scope of applications that queueing theory has been made to have. For these reasons they resemble handbooks, rather than pearls, of queueing theory. It is equally true that Takács' and Riordan's books are shorter, and consist of material carefully selected more for unity, elegance, and completeness than for the actual possibility of easy numerical answers. One might even say that they are "author's books" as opposed to "user's books," in that they show clearly how their (resp.) authors conceived and executed an exposition of a limited coherent topic, writing for themselves at least as much as for their audience. None of the remarks just made is to be interpreted in a pejorative sense, for getting one's hands dirty with the real-life facts of stochastic behavior is just as difficult and important as constructing an elegant mathematical theory describing them.

REFERENCES

LeGall, P. (1962). *Les Systèmes Avec ou Sans Attente et les Processus Stochastiques*. Dunod, Paris.

Riordan, J. (1962). *Stochastic Service Systems*. SIAM Series in Applied Mathematics, Wiley, New York.

Takács, L. (1962). *Introduction to the Theory of Queues*. Oxford University Press.