M. C. Chakrabarti, *Mathematics of Design and Analysis of Experiments*. Asia Publishing House, Bombay, London and New York, (Distributed in U.S.A. by Taplinger Publishing Co., New York) 1962. \$5.50 120 pp.

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The book covers the substance of 12 lectures given by Professor Chakrabarti at Maharaja Sayajirao University of Baroda in the years 1954 to 1956, and is a very succinct presentation of the intermediate knowledge on the design of experiments from the point of view of normal distribution of errors.

The first chapter on Linear Estimation is the basis for the whole book. The subject is given entirely by matrix manipulation. The mode of presentation is very successful for the most part, but is forced in some cases. The proofs of theorems with $y = A\theta + e$, A being $n \times p$ of rank r, are based on picking out a set of r linearly independent columns of A, and writing a solution in terms of these. A better presentation would have been in terms of conditional inverses or projection operators. As regards tests of hypotheses the presentation is not clear in that Cochran's rule is used to obtain the usual F distribution, but then the reduction of the linear hypotheses to canonical form is done independently, from which the general non-null distribution of the criterion could have been derived very easily, but was not. Tests involving several linear functions of the parameters involve rather tedious matrix manipulations, using the general rule that if

$$\Lambda\theta = c,$$

where Λ is $k \times p$ of rank k, and the elements of $\Lambda\theta$ are estimable, then

$$(\Lambda\hat{\theta} - c)'[(1/\sigma^2)V(\Lambda\hat{\theta})]^{-1}(\Lambda\hat{\theta} - c)$$

is distributed as χ^2, but again there is essentially no discussion of the distribution under the non-null hypothesis. The relationship of test criteria in the linear hypothesis to minimum sums of squares under the null and alternate hypotheses is not given. The first chapter then is a very abbreviated discussion of the general linear hypothesis, but not really suitable for the student.

Chapter II, entitled General Structure of Analyis of Designs, gives a general treatment of the two-way and three-way classification (no interaction, uncorrelated errors with expectation zero, and the same variance) in terms of matrices, and the presentation is pedagogically useful. It also deals with a particular case of partial confounding, where treatments are arranged in replicates with different confounding.

Chapter III describes the standard designs, randomized block, Latin square, Graeco-Latin square, crossover, balanced incomplete block designs, Youden squares, lattice, and PBIB designs, all with errors which have expectation zero, are uncorrelated, and have the same variance. The formula for estimating what the error mean square would have been with randomized blocks, given the analysis of variance of a Latin square experiment, is given, but no rationale is presented.

Chapter IV describes various split-plot designs by means of transformations to produce uncorrelated linear functions of the observations.

Chapter V gives the basic aspects of Galois fields, the finite geometries $PG(k, p^n)$ and $EG(k, p^n)$, orthogonalized Latin squares, construction of BIB and PBIB designs, confounded designs, hypercubes of strength d, and balancing of confounding in prime power factorial systems.

Finally, Chapter VI deals with the following miscellaneous topics: missing plot technique, fractional replication, confounding in asymmetrical factorials, and weighing designs.

It would have been appropriate to include in a book of this nature, somewhat more on PBIB designs (in particular Shah's generalization done in Professor Chakrabarti's department) and on existence theorem work for BIB and PBIB designs. Also, would not aims and optimality be worth consideration? In general, the treatment of topics is skimpy, and increasingly so as soon as the non-elementary aspects are met.

The design of experiments is commonly regarded as based on linear models with normally distributed errors as in this book, but such a view would be regarded by some, including the reviewer, as very limited. It is true that with such assumptions one can give mathematically pretty answers to the "standard" questions of "statistical theory", but this is surely not the only criterion of value. What is "error"? What reference set should one use in computing expectations and variances? Questions of this sort are not discussed. Is it because the author does not regard them as part of the "mathematics of design and analysis of experiments"?

What will make this book valuable to readers are two aspects: (a) the presentation of the least squares theory of standard designs by means of matrices and (b) the collection of exercises at the end of each chapter. These exercises vary from rather simple ones to others which are abstracted from papers of the last two decades. They will be excellent for more advanced courses in the area.