(1) M. O. Locks, M. J. Alexander and B. J. Byars, *New Tables of the Non-central t Distribution.* ARL 63-19, Aeronautical Research Laboratories, Wright-Patterson Air Force Base, January, 1963. For sale by OTS at \$6.00 v + 463 pp. of which 11 pp. are nontabular.

(2) D. B. Owen, *Factors for One-Sided Tolerance Limits and for Variables Sampling Plans.* SCR-607, Sandia Corporation, March, 1963. For sale by OTS at \$5.00 412 pp. of which 64 pp. are nontabular.

Review by B. L. Welch

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The above two volumes provide new tabulations for the non-central t-distribution. The non-central t-statistic is defined by the ratio of $(z + \delta)$ to $\sqrt{w}$ where z is a unit normal deviate, w is distributed as χ^2/f and δ is the non-centrality parameter. The distribution of t is required in the solution of a wide variety of problems which arise in sampling from a normal distribution. Early tables of the cumulative distribution of t were calculated for one restricted purpose by J. Neyman (1935) and J. Neyman and B. Tokarska (1936), viz. to facilitate the calculation of the power function of the standard "Student" t-test. For other purposes a wider coverage of the parameters involved is necessary and an approximation useful for all values, excluding only small f, was given by W. J. Jennett and B. L. Welch (1939). This approximation stems from the fact that $\sqrt{w}$ is nearly normally distributed and that the mean and standard deviation of $\sqrt{w}$ are a and $b(2f)^{-\frac{1}{2}}$, respectively, where a and b differ from unity by quantities of order f^{-1}. Taking $\sqrt{w}$ to be *exactly* normally distributed with mean unity and standard deviation $(2f)^{-\frac{1}{2}}$ we can indeed deduce the following approximate formulae connecting δ with the percentage points t_ϵ of the non-central t-distribution:

$$\delta = t_\epsilon - K_\epsilon(1 + t_\epsilon^2/2f)^{\frac{1}{2}},$$

$$t_\epsilon = [\delta + K_\epsilon(1 + \delta^2/2f - K_\epsilon^2/2f)^{\frac{1}{2}}]/(1 - K_\epsilon^2/2f).$$

(In these formulae K_ϵ is the normal deviate exceeded with probability ϵ, and t_ϵ is defined by the relation $\Pr(t > t_\epsilon \mid \delta) = \epsilon$).

Using this approximation as a starting point N. L. Johnson and B. L. Welch (1940) developed series expansions permitting a more accurate evaluation of the relation between δ and t_ϵ, and a tabulation of the relationship for certain values of f and ϵ was carried out by N. L. Johnson. These tables, however, ran only to about 10 *Biometrika* pages and their use involved a fair amount of subsidiary calculation.

The advent of electronic computing has brought about the production of much more extensive tables, the first of these, due to G. J. Resnikoff and G. J. Lieberman (1957), occupying a volume of about 400 pages and being constructed by methods differing completely from those employed in earlier tabulations.

The two volumes at present under review are of similar extent and in discussing them it will be convenient to refer back from time to time to the R. and L. publication (substituting for brevity initial letters for the authors' names).

R. and L. tabulated the probability density and the probability integral of the t-distribution against the argument $x = tf^{-\frac{1}{2}}$ at intervals of 0.05 for each of 280 combinations of f and δ. The chosen values of f were 2(1)24(5)49 and, in association with these, 10 values of $\delta(f + 1)^{-\frac{1}{2}}$ were chosen, so that the values of δ covered were not the same for all f. This rather special choice of values of δ was motivated by the desire to make the tables readily applicable to quality control problems where the fraction of the distribution of a normal variable falling beyond a specified point is the quantity to be controlled. The 10 values of $\delta(f + 1)^{-\frac{1}{2}}$ were not equidistant but were dictated by the requirements of a particular specification.

The new volume by M. O. Locks, M. J. Alexander and B. J. Byars under review consists for the most part of tables of the probability density and probility integral of t against the argument t at intervals of 0.2. Again there is a wide range of combinations of f and δ. The values of f are 1(1)20(5)40 and with these are associated, in the authors' Table 1, values of δ satisfying $\delta(f + 1)^{-\frac{1}{2}} = 0(.25)3$ and, in their Table 2, values of δ satisfying $\delta(f + 2)^{-\frac{1}{2}} = 0(.25)3$. The motivation of Table 1 is the same as that of R. and L. except that now, for given f, the values of δ are equidistant and the table therefore easier for interpolation. The values of δ chosen in Table 2 are designed to facilitate application of the table to certain problems arising in the situation where there is regression of one variable upon another and the number of degrees of freedom f is related to sample size n through $n = (f + 2)$. This provision of extensive tables for very special purposes is something made possible by the excellence of modern machines and the use of photographic reproduction of machine output. One wonders, however, just how far this tendency should be followed. One could, for instance, in the present case, proceed to the provision of an extensive table where the values of δ chosen correspond to a fixed set of values of $\delta(f + 3)^{-\frac{1}{2}}$. This might seem appropriate in certain problems associated with regression on two independent variables. Most statisticians, however, would, I believe, say that this was not worth doing and indeed it is arguable that a basic table with a fixed set of values for $f^{-\frac{1}{2}}\delta$ might be preferable to the Tables 1 and 2 already given. This would recognise the fact that in a wide range of problems δ is in magnitude of order $\sqrt{f}$. In the special applications very easy calculations would be needed to enter this basic table.

R. and L. and also L., A. and B. both tabulate percentage points in addition to the probability densities and probability integrals already noted. The second volume at present under review, due to D. B. Owen, is concerned in the main with tables of the percentage point form. The abstract prefaced to the volume states that:

"Tables are given of a quantity k which is used to define single-sample variables sampling plans and one-sided tolerance limits for a normal distribution.

The probability is γ that at least a proportion P of a normal population is below $\bar{x} + ks$ where $\bar{x}$ has a normal distribution with mean μ and variance σ^2/n and fs^2/σ^2 has a chi-square distribution with f degrees of freedom. The quantity k just described corresponds to a percentage point of the non-central t-distribution and is extensively tabulated. Tabulations of other functions computed from the noncentral t-distribution and various expected values are also given. Many other applications are discussed and various approximations compared. One section gives the mathematical derivations and there is an extensive bibliography which has been cross referenced to several indices of mathematical and statistical literature."

It is scarcely possible to give a better brief description of the contents than is indicated in this abstract and we shall here simply draw attention to four of the items: (i) In Section 2, extending to about 80 pages, tables of percentage points of $t(f + 1)^{-\frac{1}{2}}$ are given for values of $f = 1(1)199(5)399(25)999(500)1999$-$(1000)4999(5000)9999$, and for a similar set of values of δ to those used by R. and L.; (ii) In Sections 5 and 6 a return is made to the method of computing percentage points of t and confidence points of δ used by Johnson and Welch. The tables given in this section cover about 60 pages and are much more extensive than those given in *Biometrika*; (iii) About 60 pages of non-tabular material is included containing much useful information about the non-central t-distribution and the various approximations which have been suggested for it. This is completed by a very full list of references; (iv) In Section 4 there is a further tabulation of percentage points, covering 90 pages, in which a wide variety of pairs of values n and f appear as parameters (as distinct from the tables in Section 2 where $n = (f + 1)$.)

In respect of the first two of these items there is here considerable advance over the corresponding previous tabulations by other authors. The tables are much fuller and involve less labour in interpolation in certain directions. The general discussion of item (iii) is also of great interest for workers in this particular field. Coming, however, to item (iv) I feel some reservation in commending the tables given in Section 4. The four quantities γ, P, n and f do, of course, occur in the specification of many practical problems, but n and P can almost invariably be very simply amalgamated to produce one quantity for entry into a basic table. Such a basic table requires, however, to be one where δ proceeds at equal intervals so that interpolation can easily be made. It is Owen's very special choice of values of δ (corresponding to specific values of P) which in fact necessitates the consideration of n and f as separate parameters. These values of P are related to certain MIL STD sampling inspection plans. Accepting this practical consideration, however, I still feel that from the purely scientific view, the inclusion of the 90 pages of tables in Section 4, has produced an unduly bulky volume. Here, as is also the case with the L., A. and B. compilation, the real merits of the final work may to some extent be obscured by the inclusion of too much. But perhaps this is a danger which we shall more and more often encounter as availability of electronic computation facilities increases.

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