BOOK REVIEW

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FRANK SPITZER, *Principles of Random Walk*. D. Van Nostrand, Princeton, 1964. xi + 406, $12.50, 97s.

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This book is devoted exclusively to the study of processes in discrete time with independent stationary increments on the lattice points of d-dimensional Euclidean space, which the author calls a *random walk*. In this setting it is nothing more than the study of partial sums of independent, identically distributed lattice random vectors.

The so-called classical theory of sums of independent random variables concerns the limiting distribution of these sums, suitably normed; everything can be satisfactorily reduced to Fourier transforms and the entire analysis depends only on a few moments or the asymptotic behavior of the tails of distribution of the parent random variables.

But a more profound and worthwhile analysis concerns itself with the full sequence of partial sums, leading to greater delicacies than in the classical case with correspondingly deeper results obtaining. By limiting himself to random walks the author has been able to give an exhaustive and complete account not attainable in a more general setting. In addition, since in this case all measure-theoretic difficulties are obviated, a direct attack can be, and is, made without the formidable structural complications inherent in more general framework. But even with this reduced scope quite deep things are done—e.g. the theory of the Martin boundary is quite non-trivial here, and Watanabe's probabilistic solution to the little (Hausdorff) moment problem is given as an application. The book is pleasing to an analytical probabilist in that it deals with theorems instead of theories, formulas rather than formulations, and the ratio of the number of conclusions to definitions is gratifyingly large.

The guiding motivation has been to establish for random walks a complete potential theoretic setting and its relationship with the underlying Fourier analysis (here Fourier series in several variables). The basic notions are the hitting probabilities and stopping times for sets. From these, and the notion of the Green's kernel, the theory of potentials, regular and excessive functions, capacity, charge, etc. is built, not only in the easier case of transient processes as is ordinarily done, but for the more delicate recurrent case. The point has been not to establish these notions for their own sake but to use them in proving numerous limit laws, asymptotic formulas, ratio limit theorems, etc., and in obtaining basic representation theorems.

The style of writing is unhurried, informal, and well motivated without being digressive or voluble. The author has taken pains to orient his material with the classical potential theory (Newtonian and logarithmic) and other relevant areas of analysis—indeed it now seems that some of the work undertaken by analysts outside probability theory has a more natural setting there.

There are numerous informative examples in the text and many exercises, essentially none of which is routine or contrived, and which provide a valuable complement to the material of the text.

It is true that the author could have widened the scope of his investigation in certain areas to include Markov chains, non-discrete random variables, etc., with the material actually presented appearing then as special cases. But the unity and completeness would have disappeared with a larger context. The extension of the author's material to processes in continuous time is currently undergoing intensive study but those interested in such things could do well to study the material given here, if for no other reason than to establish goals for the more general theory to reach. In short, for those whose taste runs more to generalization than innovation, there is a rich source of material here.

This book should be within the scope of a senior or beginning graduate student and in the reviewer's opinion would provide him an excellent foundation and training for later work in probability. For, however great the case is for immediate abstraction and generality in other areas of mathematics, the reviewer feels that in probability the technique and intuition afforded by many concrete problems and applications in a simple setting is invaluable.