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BOOK REVIEWS

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KAI LAI CHUNG. *A Course in Probability Theory*. Harcourt, Brace & World, Inc., New York, 1968. viii + 331 pp. $12.00.

Review by RAOUL LEPAGE
Columbia University

This tightly organized volume can have a salutory effect on the confidence with which the novice embraces probability theory. Yet while tending so carefully to his assimilation of modus operandi, the face of probability which it presents is unduly narrow. A remedy, of course, is to sample liberally the major references for probability theory. From the point of view of what the present volume has to offer, this would be desirable both for obtaining a balanced view in the direction of Feller's *An Introduction to Probability Theory and its Applications*, Vol. II, and for further discussion of generalizations and periphera in the direction of Loève's *Probability Theory*, though there are other references more appropriate than these for some topics. With this in mind, we can proceed to a discussion of particulars.

Building on real and complex analysis, this is exclusively a textbook for the initiate to probability theory. Evidence indicates the author intended that the book be relatively self-contained for a reader at least familiar with Lebesgue integration on the real line. Though the easy pace does not place excessive demands on the student's knowledge of integration, the latter remains the high-water mark for what is assumed about his mathematical preparations.

Some will welcome frequent opportunities taken throughout to clarify the precise role of each result needed from analysis, in a text especially conscious of its relations with the mathematics on which it draws. This continuing benefit is not to be confused with undistinguished "review" chapters which begin this book. Introduced in the preface as, respectively, a review of elementary real variables and a synopsis of required measure and integration, the short Chapters 1 and 2 are insufficient to those ends. Specifically, Chapter 1 discusses probability distribution functions on the real line, and their associated decompositions into continuous and discrete parts. Chapter 2, with but a half-dozen proofs, compiles numerous lists of properties and statements of fact concerning classes and measures. In addition to being rushed, such a presentation invites oversights, such as the error in the definition of outer measure (page 26) which oversimplifies and renders false a statement of the extension theorem implicit in that discourse. Problems 2.1.12, 2.2.2, and Theorem 2.2.1 also contain errors. Curiously, Chapter 2 contains no mention of integration, that being taken up in Chapter 3 along with random variables and independence.

Beginning with Chapter 3 the text becomes progressively more self-contained, displaying a persistent effort at strong organization which strips away side issues and is careful to remove obscurity. Clearly-written Chapters 4 and 5 bring us to page 130 with a sureness bred of eighteen years since Halmo's *Measure Theory* and thirty-one years since Cramér's *Random Variables and Probability Distributions.* The emphasis is on methodology, pure and not over-simplified. In this context it is an assured success, for these proofs of the Borel–Cantelli proposition, strong law, 3-series, and Helly's extraction principle, have had a lot of time to settle.

The refinement of earlier lines of development reaches a peak of effectiveness with Chapter 6, which is an especially readable introduction to characteristic functions, Bochner's theorem, inversion formulae, and the relations to weak convergence of probability measures on the real line. This phase of the book completes itself with Chapter 7 and the Central Limit Problem where all is in order for a straightforward proof of the Lindeberg–Feller theorem, with time to spare for the extension to standardized sums of m-dependent uniformly bounded random variables (when the variance of the sums tends to infinity faster than $n^{2/3}$). Unfortunately, the accompanying sojourn into a special case of the invariance principle for standardized sums of independent and identically distributed second order summands proceeds without benefit of the close relation of this problem to Brownian motion. As it is, the substitute four-page direct proof so lacks intuitive content that one questions the value of including it at all. The chapter also treats speed of convergence (Berry–Essen), the law of the iterated logarithm, and indefinite decomposability. The latter proceeds by way of Theorem 7.6.2 which removes unnecessary obscurities sometimes associated with taking the logarithm of a complex function.

With Chapter 8 (devoted exclusively to random walk) the book takes on a character more closely associated with probability today. Optional random variables and first-entrance times receive needed emphasis in the context of recurrence. This chapter could be loosened up a bit, for despite a sympathetic reading, individually interesting formulas tend to form an unremitting progression.

Surely even the novice suspects a price for his earlier serenity when on entering Chapter 9 only fifty pages remain for everything relating to conditioning. Stopping times and martingales might have been made a visible part of such matters as the strong law and the law of the iterated logarithm, but were not. Numerous arguments could have made outright use of conditional expectations, but did not. That this separation is artificial, there can be no doubt. The chapter title of "Conditioning and the Markov Property" is to be taken literally. Though the optional sampling theorem and convergence theorem are proved for submartingales, nothing is done for Markov sequences but to define them, relate superharmonic functions to supermartingales, and perfunctorally define a potential. The Chapman–Kolmogorov equations surface (!) in problem 9.2.6.

Many good problems (usually a dozen or more following each section of some eight pages) round out this otherwise narrowed presentation, and are aptly chosen. These are usually suitably enlarged upon, though there are lapses, as when the

Brownian motion makes its walk-on appearance in problem 5.3.8. as the limit of an a.e. uniformly convergent random series, or when the Lèvy concentration function is introduced in 6.1.17 without any hint as to the role it plays. Over all however, the problems get high marks.

A number of errors have been pointed out in the review. There are others, for the most part minor, for an aggregate total in excess of twenty-five. The author has circulated privately a list of corrections for most of these. Such a list should be enclosed with each copy sold, until a revision is forthcoming.

I anticipate that this book will be a friend to many students seeking a careful introduction to methods. The question confronting the teacher of probability is whether the book is not too strongly devoted to methodology, too narrowly defined, or inconveniently arranged in its treatment of conditioning.

HOWARD TUCKER, *A Graduate Course in Probability*. Academic Press, Inc, 1967. xiii + 273 pp. $12.00.

Review by NARESH C. JAIN
University of Minnesota

The author of this book explains his point of view in the Preface. The book is meant for a one-year graduate course in probability, as its title suggests. In the words of the author: "The selection of material reflects my taste for such a course. I have attempted here what I consider a proper balance between measure-theoretic aspects of probability . . . and distributional aspects . . . . The material presented does not wander along any scenic byways. Rather, I was interested in traveling the shortest route to certain theorems I wished to present." There are arguments for and against this point of view. The main argument against this approach is that the exposition may become too cold and uninspiring, as it does to some extent in this book. "Scenic byways" sometimes provide a good deal of insight into the theorems presented and can be a source of interest for a reader not working directly in probability as well as for a serious student of the subject. However, if one keeps wandering off into "Disneylands" every now and then, the main ideas may become lost.

The book is divided into eight chapters. Chapters 1 and 2 discuss probability spaces and distribution functions. The Kolmogorov–Daniell extension theorem is given in Chapter 2. Stochastic independence is introduced in Chapter 3 and various convergence concepts are discussed in Chapter 4. Strong limit theorems for independent variables are given in Chapter 5. Among these theorems are included the Three Series Theorem, Kolmogorov's strong law of large numbers, the Glivenko–Cantelli theorem, and a form of the law of the iterated logarithm. Chapter 6 deals with the central limit problem. The central limit theorem is proved in a very general form and most of the well-known theorems, such as the Lindeberg–Feller central limit theorem, are deduced from it as corollaries. It is not even mentioned or