

# Discussion of “On the Birnbaum Argument for the Strong Likelihood Principle”

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*Abstract.* Deborah Mayo claims to have refuted Birnbaum’s argument that the Likelihood Principle is a logical consequence of the Sufficiency and Conditionality Principles. However, this claim fails because her interpretation of the Conditionality Principle is different from Birnbaum’s. Birnbaum’s proof cannot be so readily dismissed.

*Key words and phrases:* Conditionality principle, Birnbaum’s theorem, likelihood principle, sufficiency principle, weak conditionality principle.

Deborah Mayo (2014) is not the first devoutly to wish that the (strong) Likelihood Principle [principle L of Birnbaum (1962)] was *not* a logical consequence of the Sufficiency Principle (Birnbaum’s S) and the Conditionality Principle (Birnbaum’s C). This concern arises because much of frequentist inference is in clear violation of L, while at the same time purporting to abide by S and C. This constitutes a self-contradiction, which frequentists are, however, loth to admit. Birnbaum himself appears to have been quite distraught at his own finding, and in the half-century since publication of his argument there has been a constant trickle of attempts to come to terms with it, including one or two of my own (Dawid, 1977; Dawid, 1983; Dawid, 1987; Dawid, 2011); a detailed account that I consider displays the underlying logic clearly can be found in Chapter II “Principles of Inference” of Dawid (2013).

Those who feel disquiet at the destructive implications of Birnbaum’s theorem for their favored method of inference (be it frequentist or, for example, “objective Bayesian,” which also violates L) have a number of strategies to try and ease that disquiet. If they accept the validity of the theorem, they might argue [along with Fraser (1963); Durbin (1970); Kalbfleisch (1975)] that S or C should not be taken as universally applicable—thus evading the consequent of the theorem by denying

its antecedents. This is at least a logically sound ploy, although it reeks of adhocery. Also, the ploy may not be totally successful, since some of the “undesirable” implications of the theorem may survive weakening of its hypotheses: Dawid (1987) suggests that the principle of the irrelevance of the stopping rule is one such survivor.

A second possible strategy is to fully accept S and C and Birnbaum’s argument—and thereby come to accept L. This is the path of enlightenment followed by conversion.

The third strategy involves accepting S and C, but still rejecting L. If that is your motivation (and you care about self-consistency), you have no option but to try and find fault with the logic of Birnbaum’s theorem. This is Mayo’s strategy. The only problem is that Birnbaum’s theorem is indeed logically sound. That means that Mayo’s attempt to argue the contrary must itself be unsound. Although there are many points at which I am deeply critical of her argument, I will content myself with drawing attention to her principal misunderstanding, which vitiates her entire enterprise: she simply has not grasped Birnbaum’s conditionality principle C, conflating and confusing it with Cox’s WCP, which is quite different.

According to Mayo, WCP requires that “one should condition on the known experiment,” or (as she phrases it in Section 4.3) “eschew unconditional formulations.” But Birnbaum describes his principle C as the requirement that

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the evidential meaning of any outcome of any mixture experiment is the same as that of the corresponding outcome of the corresponding component experiment, ignoring the over-all structure of the mixture experiment.

That is, Birnbaum's principle C requires *identity* of the inferences to be drawn (from the same data) in different circumstances. This imposes an equivalence relationship across such circumstances. Principle C has nothing to say about the form or nature of the inferences, and—importantly—unlike WCP is entirely nondirectional. Mayo has misconstrued it as synonymous with WCP, which would require that we should discard whatever inference we might have been contemplating in the mixture experiment and replace it by our favored inference in the component experiment. However, an equally (in)valid reading of C would be the contrary: that we should discard a contemplated component-experiment inference in favor of an inference formed for the mixture experiment. In fact, neither of these interpretations has anything to do with principle C and, typically—as indeed follows from Birnbaum's theorem and the fact that frequentist inference violates C—neither of them can be implemented consistently within a frequentist framework.

In her Section 4.3.3 Mayo does consider the relationship between WCP and equivalence principles, and quite correctly decides that WCP is not one of these. In Section 7 she opines, “The problem stems from mistaking WCP as the equivalence. . . .” So at least she realises that WCP and Birnbaum's principle C are different. However the “problem” is just the contrary: she

has mistaken Birnbaum's equivalence requirement C as the “nonequivalence” principle WCP.

Mayo has attempted to argue that L does not follow from S and WCP. Notwithstanding the shortfalls in her arguments, I agree with that conclusion. The trouble is, it has nothing to do with Birnbaum's theorem. Mayo has been attacking a straw man, and Birnbaum's result,  $S \ \& \ C \Rightarrow L$ , remains entirely untouched by her criticisms.

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