

## GLOBAL ESTIMATION OF CHILD MORTALITY USING A BAYESIAN B-SPLINE BIAS-REDUCTION MODEL<sup>1</sup>

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Estimates of the under-five mortality rate (U5MR) are used to track progress in reducing child mortality and to evaluate countries' performance related to Millennium Development Goal 4. However, for the great majority of developing countries without well-functioning vital registration systems, estimating the U5MR is challenging due to limited data availability and data quality issues.

We describe a Bayesian penalized B-spline regression model for assessing levels and trends in the U5MR for all countries in the world, whereby biases in data series are estimated through the inclusion of a multilevel model to improve upon the limitations of current methods. B-spline smoothing parameters are also estimated through a multilevel model. Improved spline extrapolations are obtained through logarithmic pooling of the posterior predictive distribution of country-specific changes in spline coefficients with observed changes on the global level.

The proposed model is able to flexibly capture changes in U5MR over time, gives point estimates and credible intervals reflecting potential biases in data series and performs reasonably well in out-of-sample validation exercises. It has been accepted by the United Nations Inter-agency Group for Child Mortality Estimation to generate estimates for all member countries.

**1. Introduction.** The under-five mortality rate (U5MR) is a key barometer of the well-being of a country's children and, more broadly, an indicator of socioeconomic progress. The U5MR is strictly not a rate, but the probability that a child born in a given year will die before reaching the age of five if subject to current age-specific mortality rates (UN IGME 2013), often expressed as the number of deaths per 1000 live births. National estimates of the U5MR are used to track progress in reducing child mortality and to evaluate countries' performance with respect to the United Nations' Millennium Development Goal 4 (MDG 4), which calls for a two-thirds reduction in the U5MR between 1990 and 2015 (UN IGME 2013), corresponding to an annual rate of reduction of 4.4%.

For the great majority of developing countries without well-functioning vital registration systems, estimating levels and trends in U5MR is challenging, not only

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because of limited data availability but also because of issues with data quality. Every year, the United Nations Inter-agency Group for Child Mortality Estimation (UN IGME, including the United Nations Children's Fund, the World Health Organization, the World Bank, and the United Nations Population Division) produces and publishes estimates of child mortality comparable across countries and years for 194 countries. In 2012, a Loess regression model was used to estimate the U5MR (UN IGME 2012). For each country, the default setting for its smoothness parameter  $\alpha$  was determined by the type and availability of data in the country. A bootstrap method was used to assess the uncertainty in the U5MR estimates [Alkema and New (2012)]. A number of limitations with this approach were identified. The first limitation was that for a subset of countries, the fitted Loess curve was deemed to not fit the data well and post-hoc adjustments in the  $\alpha$  value were necessary. The second limitation was that all observations were weighted equally to obtain point estimates; standard errors, potential data biases and indicators of data quality were not accounted for. The calibration of the resulting point estimates and uncertainty intervals left room for improvement.

Alternative methods for estimating child mortality for all countries have been developed by the Institute for Health Metrics and Evaluation (IHME) [Rajaratnam et al. (2010), Wang et al. (2012)], which uses Gaussian process regression modeling to obtain U5MR estimates. A model validation exercise to check model performance based on the 2010 version of the IHME approach also indicated room for improvement [Alkema, Wong and Seah (2012)], possibly explained by the approach not fully accounting for potential data biases. To the best of our knowledge, the same exercise has not been repeated for the most recent iteration of the IHME model [Wang et al. (2012)]. We expect that issues with model calibration have not yet been fully addressed given that the data model has not been updated to incorporate the possibility of data biases.

In this paper we propose an alternative U5MR estimation approach to improve upon the limitations and lack of calibration of existing methods. The approach is given by a Bayesian B-spline Bias-reduction model, referred to as the B3 model. The UN IGME has decided to use the B3 model to assess countries' progress toward MDG 4 and B3 estimates are included in "A Promise Renewed Progress Report 2013" [United Nations Children's Fund, Division of Policy and Strategy (2013)] and the "Child Mortality Report 2013" (UN IGME 2013).

The paper is organized as follows. Section 2 provides background information on child mortality estimation. In Section 3 we present the B3 model specification, followed by validation results and resulting U5MR estimates in Section 4. We end with a discussion of the model and scope for future research.

**2. Background.** U5MR data series are constructed from information from vital registration (VR) and sample vital registration (SVR) systems, surveys and censuses. U5MR data for selected countries are shown in Figures 1 and 2. The selected

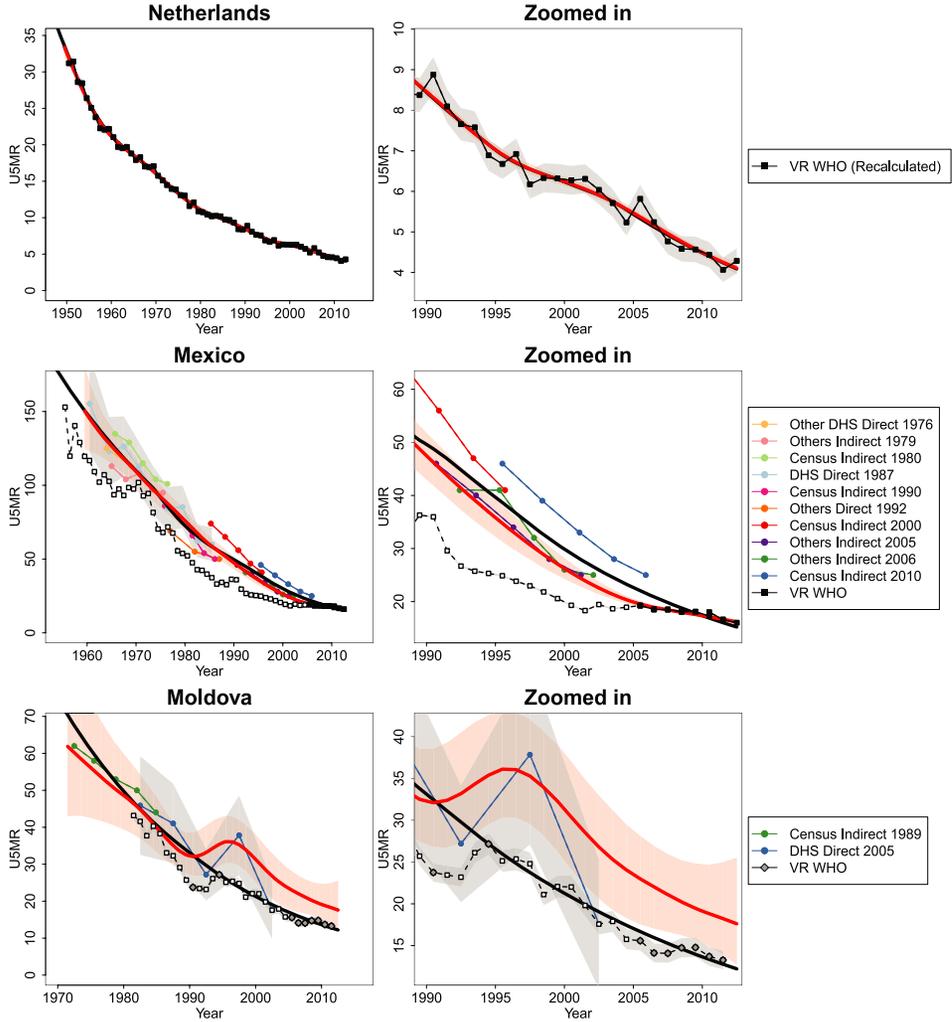


FIG. 1. *U5MR data series and estimates for the Netherlands, Mexico and Moldova. Connected dots represent data series from the same source, as explained in the legend. B3 estimates are illustrated by the solid red lines and 90% CIs are shown by the red shaded areas. The fitted Loess curve based on UN IGME 2012 methodology is illustrated with the solid black line. Shaded areas around series of observations represent the sampling variability in the series (quantified by two times the sampling standard errors).*

countries differ with respect to U5MR level and trend, as well as data availability and data quality.

In the Netherlands, data from the VR system capturing all births and deaths are available since 1940. Such data from well-functioning VR systems are the preferred data source for calculating U5MR. However, in 2013, 60 countries for which the UN IGME produces U5MR estimates did not have any data from VR sys-

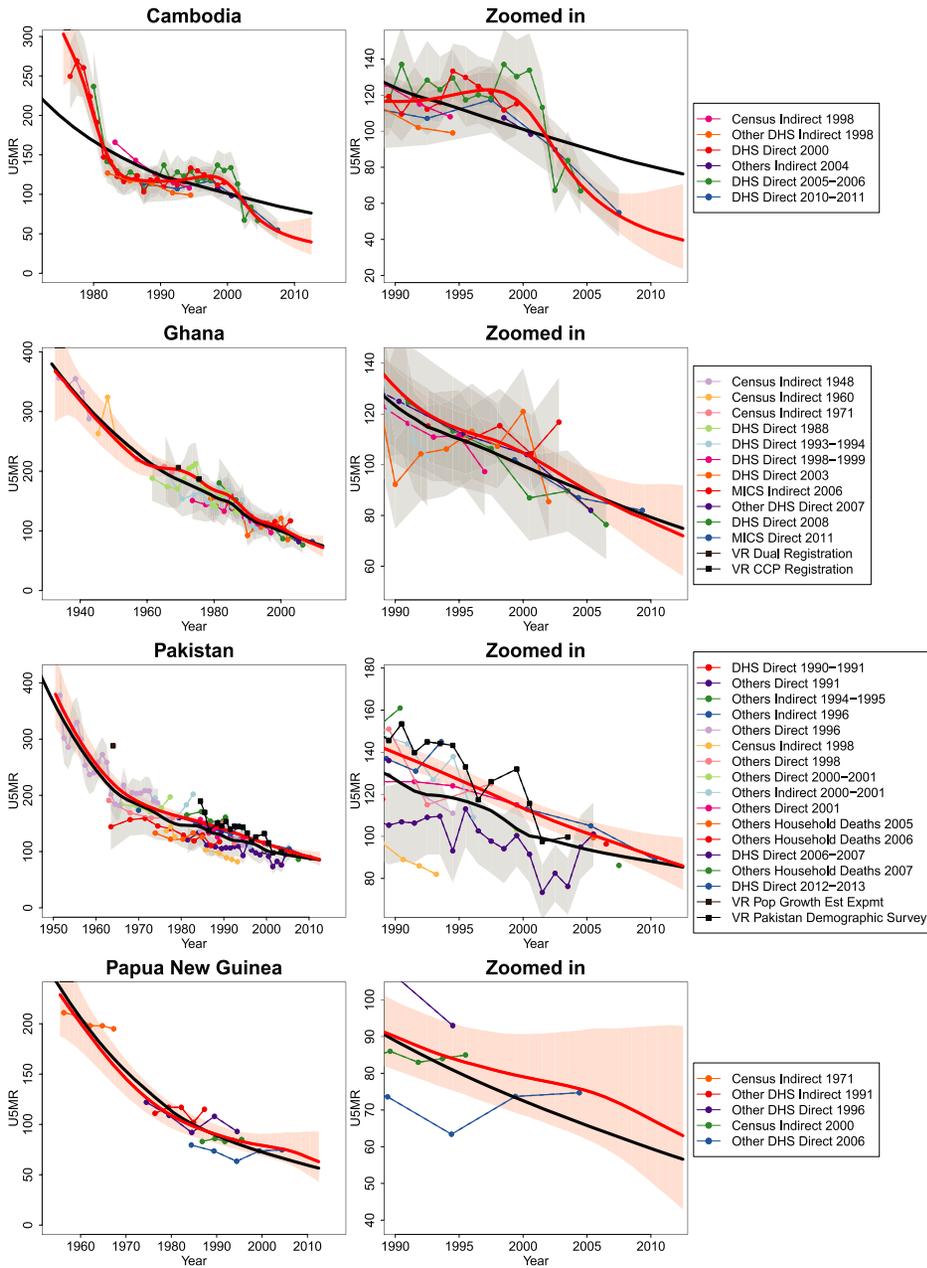


FIG. 2. U5MR data series and estimates for Cambodia, Ghana, Pakistan and Papua New Guinea. Connected dots represent data series from the same source, as explained in the legend. B3 estimates are illustrated by the solid red lines and 90% CIs are shown by the red shaded areas. The fitted Loess curve based on UN IGME 2012 methodology is illustrated with the solid black line. Shaded areas around series of observations represent the sampling variability in the series (quantified by two times the sampling standard errors).

tems. Among the 135 countries with VR or SVR systems, recording of birth and/or deaths is not necessarily complete; illustrations are given for Mexico and Moldova. In Mexico, VR data were deemed complete only since 2005. For Moldova, VR data are considered incomplete for all observation years.

For countries without (or with limited information from) well-functioning VR systems, complete or summary birth histories of women, collected in surveys and censuses, are often the main source of information on U5MR. A complete birth history lists all the live births a woman has had, including information on the date of birth of each child, whether the child is still alive, and if the child has died, the age at death. U5MR observations are calculated from such information through a synthetic cohort approach, whereby for a given period before the survey, survival probabilities are calculated for small age intervals and combined to obtain the U5MR for that period [Pedersen and Liu (2012)]. These observations are referred to as direct estimates of U5MR. Many of these direct series are obtained from complete birth histories that were collected as part of the international household survey program Demographic and Health Surveys (DHS). Other direct series are obtained from data from survey programs similar to the DHS [here referred to as Other DHS as opposed to (Standard) DHS], as well as other national surveys (referred to as Others Direct). Examples of direct series are shown in Figures 1 and 2. Because of the retrospective nature of the data, direct series can extend for up to decades before the survey. For example, the DHS in Cambodia that was carried out in 2005–2006 provides data from 1979 to 2004.

As the name suggests, summary birth histories provide a summary of complete birth histories: they list the number of live births a woman has had and the number of children that have died. These summarized histories are more commonly collected than complete birth histories because of the simplicity of data collection. For summary birth histories, demographic models are used to calculate the U5MR from the recorded proportion of dead children for different time references [Brass (1964), United Nations (1983)]. Because of the dependency on models, these estimates based on summary birth histories are referred to as indirect estimates. Indirect series are most commonly obtained using information from censuses and surveys such as the Multiple Indicator Cluster Survey (MICS), an international survey program that collects summary birth histories in many developing countries. Examples of indirect series are shown in Figures 1 and 2. As discussed for direct data series, indirect series also provide data points for a long retrospective period. For example, the Cambodian census from 1998 provides indirect estimates from 1983 to 1994.

The availability of nationally-representative surveys and censuses carried out in developing countries varies greatly. For instance, a large number of data series are available from various sources in Pakistan, but only five data series are available for Papua New Guinea. Moreover, data series do not necessarily tell a similar story about levels and/or trends in U5MR. For example, in Papua New Guinea, there are large differences between U5MR estimates from the various sources. In Pakistan,

the DHS 2006–2007 survey suggests lower levels of U5MR than data from its sample registration system. The spread in data points for countries without data from well-functioning VR systems is not specific to the selected countries in Figures 1 and 2, but is observed in many developing countries, as U5MR data are associated with a variety of data quality issues. Apart from sampling error, observations from non-VR sources may also be subject to bias and nonsampling error, for example, because of recall biases when collecting birth histories. Specific data series may be entirely biased upward or downward, for example, based on inaccuracies in the indirect estimation method that was used to translate the summary birth histories from a census or survey in U5MR observations.

Given issues with data quantity and quality, estimating the U5MR is challenging for many countries. A modeling approach needs to be flexible enough to capture short-term fluctuations in U5MR without being overly sensitive to erroneous data fluctuations.

**3. Constructing U5MR estimates.** We developed a modeling approach that combines a flexible curve fitting method with a comprehensive data model to account for data quality issues. In the model description, lowercase Greek letters refer to unknown parameters, uppercase Greek letters to functions of unknown parameters, and Roman letters to fixed variables, including data (lowercase).  $\Lambda_c(t)$  denotes the quantity of interest, the true U5MR in country  $c$  in year  $t$ . U5MR observations are combined across countries and indexed by  $i = 1, 2, \dots, N$ ;  $u_i$  denotes observed U5MR for observation  $i$  in country  $c[i]$  and year  $t[i]$ .

The complete model overview is given in Figure 3. In the center of the overview and the model is the description of the “Model fitting” for the true U5MR on the log-scale,  $\Psi_c(t) = \log(\Lambda_c(t))$  for country  $c$  at time  $t$ .  $\log(\text{U5MR})$  was modeled with a Bayesian penalized spline regression model, explained further in Section 3.1 and summarized in block 1 (spline coefficients) of Figure 3. For U5MR observations, we assumed

$$(1) \quad y_i = \Psi_{c[i]}(t[i]) + \delta_i,$$

where  $y_i = \log(u_i)$  and  $\delta_i$  is the error term on the log-scale. The data and specification of error term  $\delta_i$  are discussed further in Section 3.2 and summarized in blocks 2a and b (VR and non-VR data model) in Figure 3. Finally, short-term projections are discussed in Section 3.3 and summarized in block 3 (short-term projections).

Our analysis included 194 countries. For countries with high HIV prevalence, conflicts or natural disasters, we applied a modified estimation method based on the UN IGME 2012 estimation method, as explained in [Alkema and New \(2013\)](#).

**3.1. Bayesian penalized spline regression.** The regression spline model for log-transformed U5MR,  $\Psi_c(t)$  in equation (1), is given by

$$(2) \quad \Psi_c(t) = \sum_{k=1}^{K_c} b_{c,k}(t)\alpha_{c,k},$$

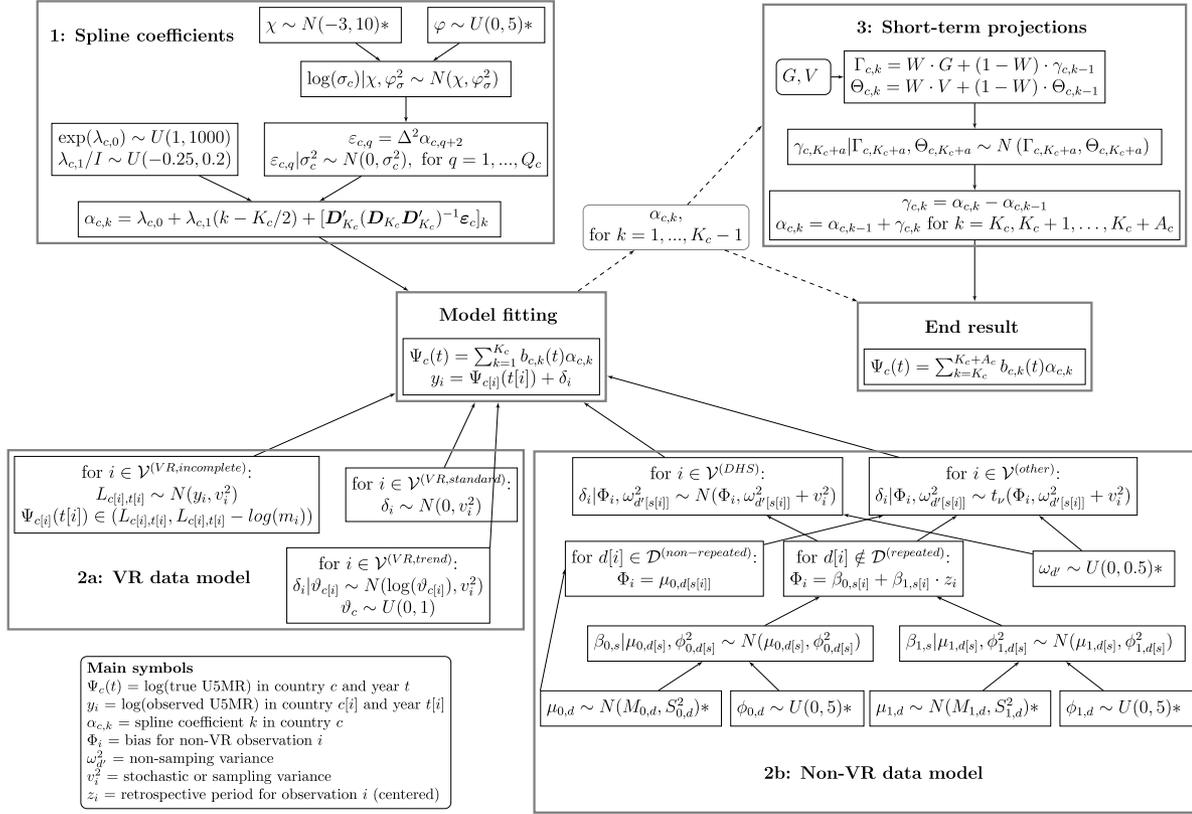


FIG. 3. *Model overview. This chart summarizes the model used to estimate the U5MR. In the center is the description of the “Model fitting” part, where  $\Psi_c(t)$  refers to the true U5MR on the log-scale, which was modeled with a Bayesian penalized spline regression model, as summarized in block 1 (see Section 3.1). The models for the error term  $\delta_i$  for observed log(U5MR) are described separately for VR and non-VR data in blocks 2a and 2b (see Section 3.2). Short-term projections are summarized in block 3 (see Section 3.3).*

where  $\alpha_{c,k}$  refers to spline coefficient  $k$  in country  $c$  and  $b_{c,k}(t)$  the  $k$ th B-spline in country  $c$ , evaluated in year  $t$ .  $K_c$  refers to the index of the most recent spline which is nonzero during the observation period.

In this application, B-splines, as discussed in Eilers and Marx (1996, 2010), were used, specifically third-degree (cubic) B-splines, illustrated for selected countries in (the bottom of) Figure 4. Equally spaced knots were used such that the resulting splines are nonzero for a total of  $4 \cdot I$  years, where  $I$  refers to the in-between-knots interval length. The same interval length of 2.5 years was used in each country regardless of the number/spacing of observations, to be able to exchange information across countries about the variability in changes between spline coefficients and assess the uncertainty in periods with limited data (further explained below).

When fitting the spline model from equation (2) to the observations, second-order differences in adjacent spline coefficients ( $\Delta^2\alpha_k = \alpha_k - 2\alpha_{k-1} + \alpha_{k-2}$ ) are penalized to guarantee smoothness of the resulting U5MR trajectory. To implement the smoothing, for each country  $c$ , spline coefficients  $\alpha_{c,k}$  for  $k = 1, 2, \dots, K_c$  were rewritten as follows [Currie and Durban (2002), Eilers (1999), Eilers and Marx (2010)]:

$$(3) \quad \alpha_{c,k} = \lambda_{c,0} + \lambda_{c,1}(k - K_c/2) + [\mathbf{D}'_{K_c}(\mathbf{D}_{K_c}\mathbf{D}'_{K_c})^{-1}\boldsymbol{\varepsilon}_c]_k,$$

where  $\lambda_{c,0}$  and  $\lambda_{c,1}$  are the unknown level and slope parameters for the spline coefficients in country  $c$  and parameter vector  $\boldsymbol{\varepsilon}_c = (\varepsilon_{c,1}, \dots, \varepsilon_{c,Q_c})'$  contains the  $Q_c = K_c - 2$  second-order differences in the spline coefficients,  $\varepsilon_{c,q} = \Delta^2\alpha_{c,q+2}$  for  $q = 1, \dots, Q_c$ ;  $[\mathbf{D}'_{K_c}(\mathbf{D}_{K_c}\mathbf{D}'_{K_c})^{-1}\boldsymbol{\varepsilon}_c]_k$  refers to the  $k$ th element of vector  $\mathbf{D}'_{K_c}(\mathbf{D}_{K_c}\mathbf{D}'_{K_c})^{-1}\boldsymbol{\varepsilon}_c$ , with known difference matrix  $\mathbf{D}_{K_c}$  (defined by  $D_{K_c,i,i} = D_{K_c,i,i+2} = 1$ ,  $D_{K_c,i,i+1} = -2$  and  $D_{K_c,i,j} = 0$  otherwise).

Second-order differences are penalized by imposing

$$(4) \quad \varepsilon_{c,q} | \sigma_c^2 \sim N(0, \sigma_c^2) \quad \text{for } q = 1, \dots, Q_c,$$

where variance  $\sigma_c^2$  determines the extent of smoothing; a smaller variance corresponds to smoother trajectories. In the limit when  $\sigma_c$  decreases to zero (as the penalty increases), a linear fit for  $\log(\text{U5MR})$  is obtained.

The model was fitted in the Bayesian framework. When estimating the spline coefficients, no information on levels or trends during the observation period was exchanged across countries to avoid the situation where estimates for a country A with little information are pooled downward because it is neighboring country B, where much progress has been made in reducing child mortality or vice versa. Diffuse priors were used for the  $\lambda_{c,0}$ 's and the  $\lambda_{c,1}$ 's (see Block 2 in Figure 3 and the Appendix).

Information on spline coefficients is exchanged across countries only through a multilevel model for the variance of the differences in the spline coefficients, that is, the standard deviation of  $\varepsilon_{c,q}$  was estimated hierarchically:

$$(5) \quad \log(\sigma_c) | \chi, \varphi_\sigma^2 \sim N(\chi, \varphi_\sigma^2),$$

where  $\chi$  and  $\varphi_\sigma^2$  refer to the mean and variance of the log-transformed standard deviations. Given the limited information on shorter term fluctuations in some countries, there was not sufficient information to estimate the variance parameter for each country separately. The hierarchical model allows for sharing of information across countries about the variability in changes between spline coefficients and for assessing the uncertainty in periods with limited data. Diffuse prior distributions were assigned to  $\chi$  and  $\varphi_\sigma^2$  (see the [Appendix](#)).

The in-between-knots interval length  $I = 2.5$  years was set by comparing U5MR estimates obtained using a range of  $I$ 's, for the full data set as well as for a subset of data in a validation exercise. U5MR estimates were found to be similar for interval lengths up to around 3 years, but for larger  $I$ , shorter term fluctuations were not captured, suggesting that the intervals of up to 3 years can be used. Here  $I = 2.5$  years was used such that each spline is nonzero for 10 years. At any time  $t$ , there are four nonzero B-splines  $b_{c,k}(t)$  such that  $\sum_k b_{c,k}(t) = 1$ . In each country, knot placement was fixed by setting  $T_{c,K_c} = t_{n_c} + 1.5 \cdot I$ , where  $t_{n_c}$  denotes the most recent observation year and  $T_{c,K_c}$  the knot for the  $K_c$ th spline in country  $c$  (motivated further in Section 3.3).

**3.2. Database and data model.** Under-five mortality data for all countries were taken from the UN IGME database. This database is publicly available on CME Info (<http://www.childmortality.org>).

Section 2 provided an introduction to U5MR data sources. A more detailed overview and explanation on data sources is given elsewhere [Hill et al. (2012)]. The breakdown of the U5MR observations by their main source types is given in Table 1. Based on potential differences in biases and nonsampling errors across data sources (explained further below), a distinction was made between series of observations from complete and summary birth histories (direct and indirect estimates, resp.), and observations based on different data sources and data collection methods (e.g., VR systems, records based on household deaths and life tables obtained from reports).

**3.2.1. Data model. VR data from complete registration systems.** The error distribution for observations from complete VR or SVR indexed by  $i \in \mathcal{V}^{(\text{VR standard})}$  is given by

$$\delta_i \sim N(0, v_i^2),$$

where  $v_i$  is the stochastic error variance. The stochastic error variance was calculated using a Poisson approximation and the delta method, assuming that

$$D_{c,t} | \Lambda_{c,t} \sim \text{Poisson}(B_{c,t} \cdot \Lambda_{c,t} / 1000),$$

where  $D_{c,t}$  is the number of under-five deaths and  $B_{c,t}$  is the number of live births for country  $c$  in year  $t$ .

TABLE 1

Summary of the U5MR data series and observations in the UN IGME 2013 database by source type

	Number of data series	Number of observations
VR (including SVR)	110	2968
(Standard) DHS Direct (with reported sampling errors)	203	2902
(Standard) DHS Direct (without reported sampling errors)	15	56
Other DHS Direct (with reported sampling errors)	49	634
Other DHS Direct (without reported sampling errors)	25	107
MICS Indirect (with reported sampling errors)	55	248
MICS Indirect (without reported sampling errors)	20	80
Census Indirect	228	1074
Others Direct	144	507
Others Indirect	168	793
Others Household Deaths	56	56
Others Life Table	56	56

Note: "Other DHS" refers to nonstandard demographic and health surveys, that is, Special, Interim and National DHS, Malaria Indicator Surveys, AIDS Indicator Surveys and World Fertility Surveys.

The number of births were obtained from the World Population Prospects [United Nations, Department of Economic and Social Affairs, Population Division (2011)] and stochastic errors were set to a minimum of 0.025 (i.e., 2.5%). For VR-type data from sample vital registration systems where the number of sampled live births was not available, it was set to 0.1 (i.e., 10%) based on the target standard error for the Indian sample registration system (Census of India, 2011).

VR observations were typically calculated for single-year periods but longer periods were used for smaller countries in instances where the coefficient of variation of the observation was larger than 10% (due to small numbers of births and deaths).

*VR data from incomplete registration systems.* For 10 countries in the regional grouping of the Central and Eastern Europe/Commonwealth of Independent States (CEE/CIS) (namely, Armenia, Azerbaijan, Georgia, Kazakhstan, Kyrgyzstan, Moldova, Tajikistan, Turkmenistan, Ukraine and Uzbekistan), VR data were incomplete with respect to the reporting of deaths (biased downward) and generally excluded from the estimation procedure in previous rounds of UN IGME estimation. However, although not informative about the level of U5MR, these observations were deemed to provide information on U5MR in the early 1990s and for recent years. During the early 1990s, in several CEE/CIS countries, data from the VR suggested a plateauing of or even an increase in U5MR. This is illustrated in Figure 1 for Moldova. This observed trend is assumed to reflect a true stagnation in progress in reducing U5MR. To use this information, we incorporated the option to include incomplete VR data into the model to inform trend estimates in

the country-specific B3 model. We also included the option to set upper and lower bounds for recent years. (These options were used in the country-specific models, as described in Section 3.5.)

To use the observed trend in VR data in the early 1990s to inform the U5MR estimates, the VR observation in 1990 and the maximum observed VR observation from 1991 to 1995 in each CEE/CIS country were selected, with indices denoted by index set  $V^{(VR,trend)}$ . For each selected observation  $i \in V^{(VR,trend)}$ , the distribution of the error term  $\delta_i$  was given by

$$\begin{aligned} \delta_i | \vartheta_{c[i]} &\sim N(\log(\vartheta_{c[i]}), v_i^2), \\ \vartheta_c &\sim U(0, 1), \end{aligned}$$

where country-specific bias parameter  $\vartheta_{c[i]}$  was added such that the two selected observations in country  $c$  could inform the trend in U5MR estimates but not the level.

For the most recent period starting from 2005, for a subset of CEE/CIS countries, U5MR extrapolations based on the global model either decreased below incomplete VR observations (where incomplete refers to incomplete reporting of deaths resulting in downward biased VR observations) or the extrapolation resulted in estimates far above VR observations for which an external assessment of VR data by the UN IGME suggested a minimum level of completeness ranging from 50% to 90%. We resolved the U5MR discrepancies between the B3 extrapolations and (assumed completeness of) VR data by including a subset of VR observations as a minimum U5MR value into the model (accounting for stochastic errors). More precisely, based on the most recent incomplete VR observation  $y_i$  (with  $i \in \mathcal{V}^{(VR,incomplete)}$ ), the lower bound  $L_{c[i],t[i]}$  for the  $\log(\text{U5MR})$  for country  $c[i]$  in year  $t[i]$  was obtained as follows:

$$L_{c[i],t[i]} \sim N(y_i, v_i^2).$$

For selected observations, where a minimum level of completeness  $m_i$  was set for incomplete VR observation  $y_i$ , we also included the upper bound  $L_{c[i],t[i]} - \log(m_i)$  for  $\log(\text{U5MR})$ . For example, if the minimum completeness for observation  $i$  is 80%, then  $m_i = 0.8$  and the upper bound for the U5MR is given by  $\exp(L_{c[i],t[i]})/m_i = \exp(L_{c[i],t[i]})/0.8$ . VR-based upper and lower bounds were incorporated into the model by excluding any  $\log(\text{U5MR})$  estimates which fell outside the interval  $(L_{c,t}, U_{c,t})$ .

*Non-VR data.* For non-VR data, the data model needs to account for (i) sampling and nonsampling errors, (ii) potential biases in trends and levels of U5MR data series, and (iii) possibility of outliers.

For observations from Standard and Other DHS Direct series, indexed by  $i \in \mathcal{V}^{(DHS)}$ , the error was assumed to be normally distributed

$$\delta_i | \Phi_i, \Omega_i^2 \sim N(\Phi_i, \Omega_i^2),$$

with mean bias  $\Phi_i$  and standard deviation  $\Omega_i$ . For observations from other source types, indexed by  $i \in \mathcal{V}^{(\text{other})}$ , posterior predictive checks suggested that more outliers were present, therefore, a  $t$ -distribution with unknown  $\nu$  degrees of freedoms was used:

$$\delta_i | \Phi_i, \Omega_i^2 \sim t_\nu(\Phi_i, \Omega_i^2),$$

$$\nu \sim U(2, 30),$$

where  $t_\nu(\Phi_i, \Omega_i^2)$  denoted a  $t$ -distribution with  $\nu$  degrees of freedom, centered at  $\Phi_i$  and rescaled by  $\Omega_i$ .

For observations from non-VR source types  $d$  with potentially multiple observations per series, mean biases were modeled as a linear function of the retrospective period of the observation in the survey (the difference between the observation reference date and the date of the survey/census). This setup was motivated by known problems with retrospective data, such as the occurrence of recall biases and violations of modeling assumptions when calculating indirect U5MR observations. The linear model for mean bias  $\Phi_i$  for observation  $i$  is given by

$$\Phi_i = \beta_{0,s[i]} + \beta_{1,s[i]} \cdot z_i,$$

where  $\beta_{0,s[i]} + \beta_{1,s[i]} \cdot z_i$  represents the bias in level and trend as a function of the retrospective period  $z_i$  for observation  $i$  (centered at 10 years) in data series  $s[i]$ . The bias in the level of the series  $\beta_{0,s}$  was estimated with a multilevel model:

$$\beta_{0,s} | \mu_{0,d[s]}, \phi_{0,d[s]}^2 \sim N(\mu_{0,d[s]}, \phi_{0,d[s]}^2),$$

where  $d[s]$  refers to the source type of series  $s$ . The set of source types with potentially multiple observations per series, indexed by  $\mathcal{D}^{(\text{repeated})}$ , is given by (Standard) DHS Direct, Other DHS Direct (including Special, Interim and National DHS, Malaria Indicator Surveys, AIDS Indicator Surveys and World Fertility Surveys), MICS Indirect, Census Indirect, Others Direct and Others Indirect.  $\mu_{0,d}$  and  $\phi_{0,d}^2$  represent source type-specific mean bias and between-series variance, respectively. These two hyperparameters were unknown and were assigned prior distributions, as illustrated in Figure 3.

A similar approach was used to estimate the slope  $\beta_{1,s}$ :

$$\beta_{1,s} | \mu_{1,d[s]}, \phi_{1,d[s]}^2 \sim N(\mu_{1,d[s]}, \phi_{1,d[s]}^2),$$

where  $\mu_{1,d}$  and  $\phi_{1,d}^2$  represent the mean slope and the between-series variance for source type  $d$ . For observations constructed from source types without repeated observations (reported household deaths and reported life tables,  $d \in \mathcal{D}^{(\text{nonrepeated})}$ ), we assumed that  $\Phi_i = \mu_{0,d[s[i]]}$ .

Scale parameter  $\Omega_i$  was modeled as a combination of sampling variance  $v_i^2$  and nonsampling variance  $\omega_{d'[s[i]]}^2$ :

$$\Omega_i^2 = \omega_{d'[s[i]]}^2 + v_i^2,$$

where source type  $d'[s]$  for series  $s$  refers to a further breakdown of source types to distinguish between DHS, Other DHS and MICS surveys with and without reported sampling errors for their observations (as indicated in Table 1). If the sampling standard errors were not reported, a sampling standard error of 2.5% was used for Census Indirect observations and 10% for all other observations. Non-sampling variance refers to variability because of random errors that arise through imperfections in the data collection process and is unknown.

Hyperparameters  $\mu_{0,d}, \phi_{0,d}^2, \mu_{1,d}, \phi_{1,d}^2, \omega_{d'}^2$  and  $\nu$  were assigned prior distributions, as listed in the Appendix. Diffuse priors were used for all hyperparameters, with the exception of the mean bias  $\mu_{0,d}$  for the DHS Direct series: an informative prior distribution was used, based on an analysis of these biases in the previous 2012 round of UN IGME estimates.

3.3. *Extrapolation using a logarithmic pooling approach.* The one-step-ahead projection of a future change in spline coefficients based on the penalized spline regression model is given by

$$(6) \quad \gamma_{c,k} | \gamma_{c,k-1}, \sigma_c^2 \sim N(\gamma_{c,k-1}, \sigma_c^2),$$

where  $\gamma_{c,k} = \Delta\alpha_{c,k} = \alpha_{c,k} - \alpha_{c,k-1}$ . This extrapolation can result in a high probability of unusually low or high projected rates of change in the spline coefficients for a specific U5MR trajectory if  $\sigma_c$  is large and/or if  $\gamma_{c,k-1}$  is unusually small or large. If projected changes in spline coefficients are unusually low or high over longer periods, so are the projected changes in the U5MR, potentially giving rise to unrealistic U5MR projections. To overcome this potential problem with the spline extrapolations, we implemented a logarithmic pooling procedure to combine country-specific posterior predictive distributions (PPDs) for changes in spline coefficients with a global PPD and verified whether this approach improved out-of-sample projections. This procedure was applied to modify the PPDs for  $\alpha_{c,k}$  for  $k = K_c, K_c + 1, \dots, P_c$ , where  $K_c$  and  $P_c$  refer to the indices of the most recent splines in the observation and projection periods, respectively. Spline coefficient  $\alpha_{c,K_c}$  was included in the set of “projected” coefficients to be pooled because it is based on very limited information only; the  $K_c$ th spline is placed such that it is nonzero only for 1.25 years during the observation period, from  $t_{n_c} - 1.25$  to  $t_{n_c}$ .

The approach is summarized as follows (see also block 3 in Figure 3): Let  $\alpha_{c,k}^{(j)}$  denote the  $j$ th posterior sample of spline coefficient  $k$  for country  $c$ ,  $j = 1, \dots, J$  and let  $\Gamma_{c,k+1}^{(j)} = \Delta\alpha_{c,k+1}^{(j)} = \alpha_{c,k+1}^{(j)} - \alpha_{c,k}^{(j)}$ , the  $j$ th posterior sample of the differences between two adjacent spline coefficients. After fitting the B3 model, we obtain  $\gamma_{c,k}^{(j)}$  for  $k = 1, 2, \dots, K_c - 1$ , while  $\gamma_{c,k}$ 's for  $k = K_c, K_c + 1, \dots, P_c$  are drawn from a pooled PPD (see Figure 4). The pooled PPD is a combination of the “model-induced” country-specific PPD for  $\gamma_{c,k}^{(j)}$ , defined by the penalized splines model and a global PPD for future changes in the spline coefficients. The global PPD was based on the set of posterior median estimates of the  $\gamma_{c,k}^{(j)}$ 's,  $\hat{\gamma}_{c,k}$  for

$c = 1, \dots, C$  and  $k = 2, \dots, K_c - 1$  (during the observation period for each country). We used country-projection-step-specific logarithmic pooling weights to obtain the same extent of pooling for all countries. The resulting pooled PPD for  $\gamma_{c,K_c+a}^{(j)}$  for  $a \geq 0$  is given by

$$\gamma_{c,K_c+a}^{(j)} | \Gamma_{c,K_c+a}^{(j)}, \Theta_{c,K_c+a}^{(j)} \sim N(\Gamma_{c,K_c+a}^{(j)}, \Theta_{c,K_c+a}^{(j)}),$$

where

$$\begin{aligned} \Gamma_{c,k}^{(j)} &= W \cdot G + (1 - W) \cdot \gamma_{c,k-1}^{(j)}, \\ \Theta_{c,k}^{(j)} &= W \cdot V + (1 - W) \cdot \Theta_{c,k-1}^{(j)}, \end{aligned}$$

with  $G$  and  $V$  equal to the median and variance of the  $\hat{\gamma}_{c,k}$ 's, respectively,  $\gamma_{c,K_c-1}^{(j)} = \alpha_{c,K_c-1}^{(j)} - \alpha_{c,K_c-2}^{(j)}$  and  $\Theta_{c,K_c-1}^{(j)} = \sigma_c^{(j)}$ .

The overall pooling weight  $0 \leq W \leq 1$  was chosen through an out-of-sample validation exercise (described in Section 3.6). Further details of the logarithmic pooling procedure are given in the [Appendix](#).

**3.4. Computation.** A Markov Chain Monte Carlo (MCMC) algorithm was employed to sample from the posterior distribution of the parameters with the use of the software JAGS [[Plummer \(2003\)](#)]. Six parallel chains with different starting points were run with a total of 50,000 iterations in each chain. Of these, the first 10,000 iterations in each chain were discarded as burn-in and every 20th iteration after was retained. The resulting chains contained 2000 samples each. Standard diagnostic checks (using trace plots, the Raftery and Lewis diagnostic [[Raftery and Lewis \(1992, 1996\)](#)] and the Gelman and Rubin diagnostic [[Gelman and Rubin \(1992\)](#)]) were used to check convergence.

Estimates of relevant quantities are given by the posterior medians, while 90% credible intervals (CIs) were constructed from the 5% and 95% percentiles of the posterior sample. Given the inherent uncertainty in U5MR estimates, 90% CIs are used by UN IGME instead of the more conventional 95% ones.

**3.5. Country-specific UN IGME model and adjustments.** The B3 model was accepted by UN IGME to evaluate countries' progress and performance in reducing U5MR. For this purpose, a computationally cheaper and more user-friendly country-specific model was implemented, with noncountry-specific parameters (marked with a star in Figure 3) fixed at the posterior medians from the global model run, which resulted in very similar estimates. For the country-specific runs, we ran 6 chains with a total of 35,000 iterations in each chain. Of these, the first 10,000 iterations in each chain were discarded as burn-in and every 20th iteration after was retained. The resulting chains contained 1250 samples each.

After reviewing the estimates, two model adjustments were incorporated in the country-specific models to consistently adjust the level of under- or over-smoothing in a subset of countries [see [Alkema and New \(2013\)](#) for details].

Adjustments were also applied to the Democratic Republic of Congo and Somalia, where the U5MR data are not deemed to be representative of the country's past. Specifically, B-splines corresponding to conflict periods where the U5MR is unlikely to have declined were combined such that only one spline coefficient was estimated for each conflict period. The resulting fit is constant during the conflict periods.

*3.6. Model validation.* Model performance was assessed through an out-of-sample validation exercise. Given the retrospective nature of U5MR data and the occurrence of data in series, the training set was not constructed by leaving out observations at random, but based on all available data in some year in the past [Alkema, Wong and Seah (2012)]; here 2006 was chosen. To construct the training set, all data that were collected in or after 2006 were removed. For example, if a DHS was carried out in 2006, all (retrospective) observations from that DHS were left out of the training set. Fitting the B3 model to the training set resulted in point estimates and CIs that would have been constructed in 2006 based on the proposed method.

To validate model performance, we calculated various validation measures based on the left-out observations and based on the estimates obtained from the full data set and the estimates obtained from the training data set. The validation measures considered were mean and median errors, coverage of prediction intervals (to quantify the calibration of the prediction intervals) and interval scores (to quantify calibration and sharpness of the prediction intervals).

For the left-out observations, errors are defined as  $e_i = u_i - \tilde{u}_i$ , where  $\tilde{u}_i$  denotes the posterior median of the predictive distribution for a left-out observation  $u_i$  based on the training set. Coverage is given by  $1/N \sum 1[u_i \geq l_{c[i]}(t[i])] \cdot 1[u_i \leq r_{c[i]}(t[i])]$ , where  $N$  denotes the total number of left-out observations considered and  $l_{c[i]}(t[i])$  and  $r_{c[i]}(t[i])$  the lower and upper bounds of the 90% prediction intervals for the  $i$ th observation. The (negatively oriented) interval score  $n_i$  for observation  $i$  is given by Gneiting and Raftery (2007)

$$n_i = (\log(r_{c[i]}) - \log(l_{c[i]})) + 2/x(\log(l_{c[i]}) - y_i) \cdot 1[u_i < l_{c[i]}] \\ + 2/x(y_i - \log(r_{c[i]})) \cdot 1[u_i > r_{c[i]}],$$

with significance level  $x = 0.1$ . This score combines the width of the prediction interval with a penalty for any intervals that do not contain the left-out observation. The validation measures were calculated for 100 sets of left-out observations, where each set consisted of a random sample of one left-out observation per country. Reported results include the median and standard deviation of the validation measures based on the outcomes in the 100 sets.

“Updated” estimates, denoted by  $\hat{\Lambda}_c(t)$  for country  $c$  in year  $t$ , refer to the median U5MR estimates obtained from the full data set. The error in the estimate based on the training sample is defined as  $e_{c,t} = \hat{\Lambda}_c(t) - \tilde{\Lambda}_c(t)$ , where  $\tilde{\Lambda}_c(t)$  refers

to the posterior median estimate based on the training sample, while relative error is defined as  $e_{c,t}/\widehat{\Lambda}_c(t) \cdot 100$ . Coverage and interval scores were calculated in a similar matter as for the left-out observations, based on the lower and upper bound of the 90% CIs for  $\log(\text{U5MR})$  obtained from the training set. Coverage, mean/median errors and interval scores were also evaluated for the annual rate of reduction (ARR) from 1990 to 2005.

Particular attention was paid to the performance of the B3 model for the group of high mortality countries, where high here refers to a U5MR of at least 40 deaths per 1000 births in 1990. This set was selected because of the importance of the UN IGME U5MR estimates for tracking progress in reducing child mortality. Crisis years and HIV adjustments were not considered in the out-of-sample model validation because the calculation of crisis-related and HIV-related U5MR is not included in the B3 method (so it is not possible to reconstruct these estimates).

## 4. Results.

4.1. *Model validation and choice of pooling weight.* To set the pooling weight  $W$  (to combine the PPD of country-specific changes in spline coefficients with the global PPD), validation measures were obtained for  $W = 0, 0.1, \dots, 0.6$ , where  $W = 0$  corresponds to the “no-pooling” (country-specific) variant. In general, differences between country-specific and pooling variants were small for the posterior median point estimates but more noticeable for projection intervals. An illustration of the default (unpooled) and pooled projections (using  $W = 0.5$ ) are shown in Figure 4 for Cambodia, Ghana and Papua New Guinea. The introduction

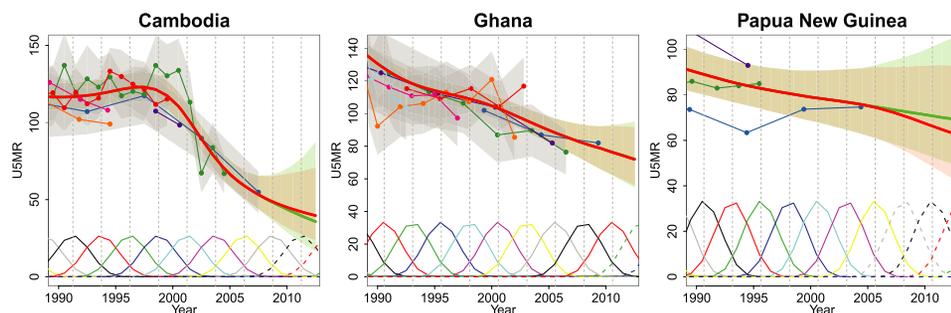


FIG. 4. Illustration of differences in estimates and projections for Cambodia, Ghana and Papua New Guinea between the unpooled (country-specific) and pooled B-spline model projection approach. U5MR estimates based on the unpooled (country-specific) projection approach are displayed in green, and results based on the pooling approach, using weight  $W = 0.5$ , are displayed in red. Solid lines denote posterior medians and shaded areas denote 90% CIs. Additionally, the placement of B-splines is shown in the bottom of the graph (scaled vertically for display purposes) in different colors. The dashed lines are used for B-splines for which the corresponding coefficient is included in the set of “projected” coefficients (with index  $k \geq K_C$ ). Gray dotted vertical lines indicate knot positions of the B-splines.

of the pooling procedure increased the U5MR projections in Cambodia and led to a decrease in Ghana and Papua New Guinea, but differences in point estimates were minor. Projection intervals varied more across countries; the bounds were similar for weights 0 and 0.5 for Ghana, but narrowed down in Cambodia and were lower for the pooled projections in Papua New Guinea.

Model validation results based on the left-out observations and the comparison between estimates based on the training and full data set are shown in Tables 2, 3 and 4 in the Appendix for the range of pooling weights. Differences in mean/median (absolute) errors were small. While for median errors the comparison across the different weights varied by indicator, mean errors generally decreased with increasing pooling weights. Coverage and interval width scores for left-out observations generally improved slightly with increasing pooling weight. For the estimated U5MR and ARR, findings on coverage of 90% credible intervals were mixed, but mean interval scores for U5MR decreased with increasing pooling weight.

Based on these findings, we chose to apply the pooling. Because differences in validation outcomes were small when comparing the results for  $W = 0.5$  to those with  $W = 0.6$ , and because of the convenient interpretation of  $W = 0.5$  (the projected mean and variance of the differences in the spline coefficients are the simple average of the country-specific and global estimates), we set  $W = 0.5$ . A comparison of estimates and short-term projections based on  $W = 0$  and  $W = 0.5$  for all countries is included in supplementary Figure S1 [Alkema and New (2014)].

With this choice of  $W$ , the model validation results for the B3 model showed an improvement over those for the UN IGME 2012 estimation approach. In a similar validation exercise carried out for the UN IGME 2012 estimation approach [Alkema and New (2012)], the updated estimate of ARR for 1990–2005 (based on the full data set) was above the training 90% CI for 16% of the high mortality countries (11 out of 70 countries) and below that for only 6% of those countries. This indicates that declines in U5MR were underestimated for a substantial proportion of high mortality countries. The same effect is observed in the validation results for the B3 model but to a much lesser extent, with only 9% of the updated upper bounds for the ARR being too low and 3% of the updated lower bounds being too high. Overall, the calibration measures are better with the B3 model. Specifically, the percentages of updated estimates falling below and above the 90% uncertainty intervals were 4% and 5%, respectively, for the U5MR in 2000 and 8% and 1% for the U5MR in 2005 in the B3 model. These percentages were 10% and 6% for the U5MR in 2000 and 17% and 7% for the U5MR in 2005 in the IGME 2012 estimation approach.

*4.2. Data model biases.* Mean biases in U5MR levels and trends, as well as 90% prediction intervals for the expected range of U5MR values, were calculated based on the posterior sample of data quality parameters and are visualized in Figure 5 for the different source types. Mean biases and prediction intervals are

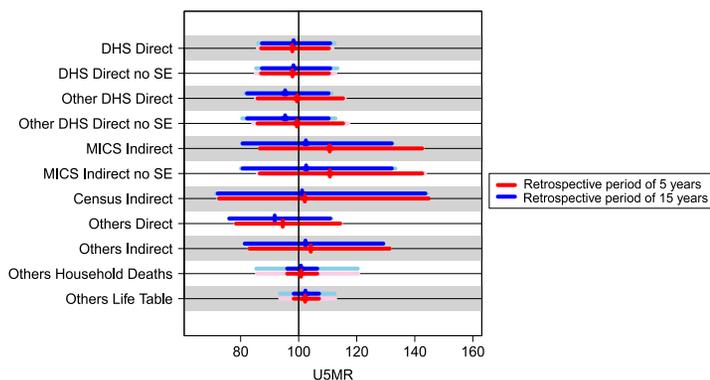


FIG. 5. Visualization of 90% prediction intervals for new data points by source type and retrospective period. For a “true” U5MR of 100 deaths per 1000 live births (represented by the black line), the 90% prediction interval for a U5MR observation with a retrospective period of 5/15 years is shown in light blue/pink (excluding the sampling variability) and the predicted mean observed U5MR is represented by the dark blue/red vertical line (the difference between the mean U5MR and 100 represents the mean bias). The dark blue/red horizontal line represents the 90% prediction interval for an observation based on uncertainty in the bias parameters only (excluding sampling and nonsampling variability).

relative to the unknown true U5MR level, which in the figure is assumed to be 100 deaths per 1000 live births for ease of interpretation. The prediction intervals thus illustrate the expected range of U5MR values for a “new” data series when the true U5MR is 100 deaths per 1000 births. Results are shown for retrospective periods of 5 and 15 years, thus for observations that refer to 5 and 15 years before the survey date.

For indirect series, the 90% prediction intervals based on uncertainty in biases alone (the dark blue horizontal lines) are wide, indicating substantial variability in biases across data series. For example, the prediction interval ranges from about 87 to about 143 deaths per 1000 live births for an observation from a MICS Indirect series, with a retrospective period of 5 years. The error variance tends to contribute less to the width of the 90% prediction intervals, implying that there is significant variability in data series that is not attributed to random error. For retrospective periods of 5 years, mean biases are slightly positive for indirect series, but almost zero or negative for direct series: observations from direct series tend to be below indirect series for these retrospective periods.

4.3. *U5MR estimates.* U5MR data series and B3 estimates for all 194 countries are shown in supplementary Figure S2 [Alkema and New (2014)]. For the selected illustrative countries in Figures 1 and 2, B3 estimates are displayed in the country-specific figures, together with the estimates that would have been obtained using the default Loess estimation approach used for constructing the IGME 2012 estimates.

Point estimates from the B3 model and default Loess are almost identical for the Netherlands during the entire observation period, but differ for all or a subset of observation years in the other countries. For Mexico, the trend in the Loess estimates for the late 2000s contradicts the observed trend in VR data. B3 estimates take into account the small stochastic error in the VR and follow the data points closely. For Moldova, the inclusion of the VR observations in the early 1990s with a VR bias parameter for those years results in U5MR estimates that capture the VR-indicated trend. The inclusion of VR data for recent years guarantees that the point estimates and credible intervals do not cross through the VR. In future revisions for Moldova, a further extension could be to include all incomplete VR observations as a minimum to avoid the situation in the early 1980s, when the lower bound of the CI is below the incomplete VR.

For Ghana, B3 estimates and Loess estimates are similar. Small differences are observed in the years with VR data, where the B3 estimates capture these points while the Loess does not. In more recent years, the extrapolated decline is slightly steeper for the B3 model, as indicated by the decline in the most recent observations. Differences between B3 and Loess estimates are much larger in the other countries in the figure. In Cambodia, the B3 estimates follow the trend as observed in the data series, including the stagnation of child mortality decline in the 1980s and 1990s and the more recent acceleration in the decline of child mortality. The default Loess fit does not capture these fluctuations. In the IGME 2012 method, this country would be a candidate for an expert-based adjustment of the Loess smoothing parameter to better capture the trend. In the B3 penalized spline model approach, such expert adjustments are not necessary.

In Pakistan, the B3 estimates follow the registration data. The DHS from 2006–2007 does not bias downward the estimates (as observed in the Loess estimates) because of the inclusion of bias parameters for survey data; we estimate that the DHS Direct series is biased downward. Last, in PNG, B3 estimates suggest a slightly flatter trend in U5MR than the Loess during the 1980s and 1990s based on the lack of downward trends in all individual series during that period.

**5. Discussion.** The estimation of child mortality is challenging for the great majority of developing countries without well-functioning VR systems due to issues with data quantity and quality. In this paper, we described a Bayesian penalized B-spline regression model to evaluate levels and trends in the U5MR for all countries in the world. This model estimates biases in data series for all non-VR source types using a multilevel model to improve upon the limitations of current methods. Improved spline extrapolations are obtained via logarithmic pooling of the posterior predictive distribution of country-specific changes in spline coefficients with observed changes on the global level. The proposed model can flexibly capture changes in U5MR over time, provides point estimates and credible intervals that take into consideration potential biases in data series and gives better model validation results than the UN IGME 2012 estimation approach.

The differences between the B3 estimates and the default Loess fits as discussed in Section 4.3 highlight the need for more attention for appropriate data models in U5MR estimation. When treating all observations equally, U5MR estimates can end up below (incomplete) VR observations or follow a trend in U5MR that is dictated by the (lack of) overlap of different data series with potentially different level biases.

While our data model overcomes the main limitations of the previous UN IGME estimation methods, there remains room for improvement. The primary issue with child mortality estimation is data quality. In the B3 data model, we incorporated source-specific bias parameters, that are drawn from a source type-specific distribution based on the assumption that biases are comparable across data series of the same source type. However, large variation exists across series; ideally, external information on data quality should be included to distinguish between the more or less reliable series in the database. In a residual analysis, (absolute) residuals were plotted against a number of data quality predictors (region that country belongs to, series source type, series year, observation year, retrospective period, level of U5MR in observation year, total fertility rate in the series year and change in the total fertility rate in the last 15 years before the series) to explore whether any of those covariates should be incorporated into the model for biases in direct and indirect series. Overall, the linear model without covariates seemed to work reasonably well except for some DHS Direct series, for which an additional negative bias for observations with retrospective periods shorter than 5 years may be present. This may be due to birth transference, whereby dates of birth are incorrectly reported to avoid answering more questions pertaining to those births in the DHS questionnaires [Sullivan (2008)]. Given the importance of the observations with short retrospective periods in driving recent estimates and short-term projections, this issue needs to be investigated more in future work.

Improved spline extrapolations were obtained via logarithmic pooling of the posterior predictive distribution of country-specific changes in spline coefficients with observed changes on the global level. While short-term projections that are based only on country-specific information may be preferred from a political/country-user point of view, the pooling procedure was used because it was found to improve out-of-sample model performance and deemed to lead to more plausible projection intervals in countries where differences between the pooled and unpooled predictions occurred. In summary, the pooling approach reduces the probability of unrealistically high or low rates of changes in extrapolations and also reduces the probability of sustained high or low rates of change over longer projection periods by pooling the predictive distribution for rates of change toward a global distribution. This procedure did not result in large differences in point estimates for the majority of countries (as illustrated in Figure S1); its main effect was a reduction of upper bounds for the U5MR by reducing the probability of very low or even negative rates of change. Alternative projection methods may be considered, for example, based on country-specific covariates which may be informative

of U5MR declines. However, given the limited availability of such covariates for recent years, we did not pursue this research direction.

Ultimately, the issues of data quality and availability of more recent data can only be resolved by implementing fully functioning VR systems that can provide accurate data on births and deaths in every country. However, currently only about 50 countries have such VR systems in place; the implementation of VR systems for all countries remains an ambitious and long-term goal [United Nations Children's Fund and USAID (2012)]. In the short term, the B3 model allows for inclusion of information from incomplete VR systems, as illustrated for Moldova. The inclusion of data from alternative data sources and the implementation of novel data collection methods, that can provide accurate and timely child mortality data [e.g., see Clark et al. (2012) and Amouzou (2011)], could further aid child mortality estimation. The advantage of the use of the Bayesian framework in the B3 model is that the model can be readily extended to incorporate such information into the estimation process.

To assess progress toward MDG 4, much focus is placed on the point estimates of the U5MR and ARR despite the large uncertainty in estimates because communication of uncertainty in U5MR estimates is challenging [Oestergaard, Alkema and Lawn (2013)]. To provide a straightforward inclusion of the uncertainty assessment into the MDG 4 progress assessment, countries could be categorized by whether the attainment of the MDG target of an ARR of 4.4% is considered to be unlikely, not clear or likely based on the uncertainty intervals of the ARR estimate [Alkema and New (2012)].

Moving beyond the MDGs, the issue of inequality is likely to feature prominently in the post-2015 development agenda. While the MDGs have focused much attention on national, regional and global averages of key indicators, they have also potentially masked growing disparities at the intra-national level [UN System Task Team on the Post-2015 UN Development Agenda (2012)]. In light of this, disaggregated estimates of child mortality (e.g., by state, wealth quintile, residence) will be increasingly important to evaluate progress for all population groups to better address inequalities. Further work can be carried out to extend the B3 model so that this growing body of disaggregated data can be fully utilized to produce disaggregated estimates in the future.

## APPENDIX

*Prior distributions.* Prior distributions for the spline model parameters are specified as follows:

$$\begin{aligned}\exp(\lambda_{c,0}) &\sim U(1, 1000), \\ \lambda_{c,1}/I &\sim U(-0.25, 0.2), \\ \chi &\sim N(-3, 10), \\ \varphi &\sim U(0, 5),\end{aligned}$$

where  $\exp(\lambda_{c,0})$  represents the level of U5MR in the approximate midyear of the observation period, and  $\lambda_{c,1}/I$  is approximately the average ARR over the observation period ( $I$  is the interval length between knots).

Diffuse prior distributions were assigned to all data model parameters, with the exception of the mean bias  $\mu_{0,d}$  for the DHS Direct series, which has an informative prior distribution:

$$\begin{aligned} \mu_{0,d} &\sim N(M_{0,d}, S_{0,d}^2), \\ \mu_{1,d} &\sim N(M_{1,d}, S_{1,d}^2), \\ \phi_{0,d} &\sim U(0, 5), \\ \phi_{1,d} &\sim U(0, 5), \\ \omega_{d'} &\sim U(0, 0.5), \end{aligned}$$

where  $M_{0,d} = -0.0123$  for  $d = \text{DHS direct}$  and 0 otherwise,  $M_{1,d} = 0$  for all  $d$ ,  $S_{0,d} = 0.00556$  for DHS Direct and 0.15 otherwise,  $S_{1,d} = 0.02$  for all  $d$ .

*Logarithmic pooling approach.* The penalized spline model-induced PPD for  $\gamma_{c,K_c}^{(j)} = \Delta\alpha_{c,K_c}^{(j)} = \alpha_{c,K_c}^{(j)} - \alpha_{c,K_c-1}^{(j)}$ , based on (4), is given by

$$(7) \quad \gamma_{c,K_c}^{(j)} | \gamma_{c,K_c-1}^{(j)}, \Theta_{c,K_c}^{(j)} \sim N(\gamma_{c,K_c-1}^{(j)}, \Theta_{c,K_c}^{(j)}),$$

where  $\Theta_{c,K_c}^{(j)} = (\sigma_c^2)^{(j)}$ . Its density function (leaving out superscripts to denote the posterior sample for notational convenience)  $p^*(\gamma_{c,K_c}) = f(\gamma_{c,K_c} | \gamma_{c,K_c-1}, \Theta_{c,K_c})$ , where  $f(\Gamma | \mu, \sigma^2)$  denotes the probability density function for a normal random variable with mean  $\mu$  and variance  $\sigma^2$ .

The model-induced PPD is pooled with a (direct) global PPD for future changes in the spline coefficients, which was based on the set of posterior median estimates of the  $\gamma_{c,k}^{(j)}$ 's,  $\hat{\gamma}_{c,k}$  for  $c = 1, \dots, C$  and  $k = 2, \dots, K_c - 1$  (during the observation period for each country):

$$(8) \quad p(\gamma) = f(\gamma | G, V),$$

where  $G$  and  $V$  were given by the median and variance of the  $\hat{\gamma}_{c,k}$ 's, respectively.

Logarithmic pooling is used to combine both density functions:

$$\tilde{p}(\Gamma_{c,K_c}) \propto p^*(\gamma_{c,K_c})^{1-w_{c,K_c}} \cdot p(\gamma_{c,K_c})^{w_{c,K_c}} = f(\gamma_{c,K_c} | \Gamma_{c,K_c}, \Theta_{c,K_c}),$$

where  $w_{c,K_c}$  is the country-projection-step specific logarithmic pooling weight that determines the extent of pooling,

$$w_{c,K_c} = \frac{W \cdot V}{W \cdot V + (1 - W)\Theta_{c,K_c-1}},$$

with overall weight  $0 \leq W \leq 1$  such that

$$\begin{aligned} \Gamma_{c,K_c} &= W \cdot G + (1 - W) \cdot \gamma_{c,K_c-1}, \\ \Theta_{c,K_c} &= W \cdot V + (1 - W) \cdot \Theta_{c,K_c-1}. \end{aligned}$$

TABLE 2

*Validation results based on left-out observations I. Results refer to the median (and standard deviation) of outcomes based on 100 sets of left-out observations, where each set contains one randomly selected observation per included country (before/including 2005, and after 2005). Included countries are given by high mortality countries (high means U5MR of at least 40 deaths per 1000 births in 1990) without crises or HIV adjustments, with data in both training and test set and left-out observations in the period of interest, 71 and 65 countries in total for the indicators left-out observations before and including 2005, and left-out observations after 2005, respectively. The outcome measures are as follows: % of observations below and above the 90% prediction interval based on the training set. The lowest value for each outcome measure is bolded*

W	% of observations outside 90% prediction interval			
	Year ≤ 2005		Year > 2005	
	% below	% above	% below	% above
0	8.5 (2.6)	<b>7.0 (1.9)</b>	9.2 (1.2)	4.6 (1.9)
0.1	<b>7.0 (2.6)</b>	<b>7.0 (2.0)</b>	<b>6.2 (1.2)</b>	3.1 (1.3)
0.2	<b>7.0 (2.6)</b>	<b>7.0 (2.0)</b>	<b>6.2 (1.2)</b>	3.1 (1.2)
0.3	<b>7.0 (2.5)</b>	<b>7.0 (2.0)</b>	<b>6.2 (1.2)</b>	<b>1.5 (1.0)</b>
0.4	<b>7.0 (2.5)</b>	<b>7.0 (1.9)</b>	<b>6.2 (1.3)</b>	<b>1.5 (1.0)</b>
0.5	<b>7.0 (2.4)</b>	<b>7.0 (1.8)</b>	<b>6.2 (1.5)</b>	<b>1.5 (1.0)</b>
0.6	<b>7.0 (2.5)</b>	<b>7.0 (1.7)</b>	<b>6.2 (1.5)</b>	<b>1.5 (1.0)</b>

For  $a \geq 1$ , the induced PPD is defined as

$$p^*(\gamma_{c,K_c+a}) = f(\gamma_{c,K_c+a} | \Gamma_{c,K_c+a-1}, \Theta_{c,K_c+a-1}).$$

With the global distribution from equation(8) and logarithmic pooling weights  $w_{c,K_c+a} = \frac{W \cdot V}{W \cdot V + (1-W)\Theta_{c,K_c+a-1}}$ , the pooled distribution for  $\gamma_{c,K_c+a}$  is given by

$$\begin{aligned} \tilde{p}(\gamma_{c,K_c+a}) &\propto p^*(\gamma_{c,K_c+a})^{1-w_{c,K_c+a}} \cdot p(\gamma_{c,K_c+a})^{w_{c,K_c+a}} \\ &= f(\gamma_{c,K_c+a} | \Gamma_{c,K_c+a}, \Theta_{c,K_c+a}), \\ \Gamma_{c,K_c+a} &= W \cdot G + (1 - W) \cdot \gamma_{c,K_c+a-1}, \\ \Theta_{c,K_c+a} &= W \cdot V + (1 - W) \cdot \Theta_{c,K_c+a-1}. \end{aligned}$$

*Validation results.* Validation results are described in Tables 2, 3 and 4.

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TABLE 3

Validation results based on left-out observations II. Results refer to the median (and standard deviation) of outcomes based on 100 sets of left-out observations, where each set contains one randomly selected observation per included country (before/including 2005, and after 2005). Included countries are given by high mortality countries (high means U5MR of at least 40 deaths per 1000 births in 1990) without crises or HIV adjustments, with data in both training and test set and left-out observations in the period of interest, 71 and 65 countries in total for the indicators left-out observations before and including 2005, and left-out observations after 2005, respectively. The outcome measures are as follows: median or mean relative error (MRE), median or mean absolute relative error (MARE), median or mean interval score (Score) based on the training set. The lowest value for each outcome measure is bolded

W	ME	MAE	MRE	MARE	Score
Year $\leq$ 2005					
Median					
0	-2.2 (1.4)	11.3 (1.2)	-2.2 (1.8)	13.2 (1.3)	<b>0.64 (0.01)</b>
0.1	-2.2 (1.3)	11.2 (1.2)	-2.2 (1.7)	13.2 (1.3)	<b>0.64 (0.01)</b>
0.2	-1.9 (1.3)	10.9 (1.3)	-2.1 (1.7)	12.9 (1.4)	0.64 (0.01)
0.3	-1.9 (1.4)	10.8 (1.3)	-2.1 (1.7)	12.9 (1.4)	0.64 (0.01)
0.4	-1.9 (1.3)	10.8 (1.3)	-2.0 (1.7)	12.9 (1.4)	0.64 (0.01)
0.5	-1.7 (1.3)	10.7 (1.3)	-1.9 (1.7)	12.9 (1.4)	0.64 (0.01)
0.6	<b>-1.5 (1.3)</b>	<b>10.6 (1.3)</b>	<b>-1.9 (1.7)</b>	<b>12.9 (1.5)</b>	0.64 (0.01)
Mean					
0	-2.9 (1.6)	16.6 (1.2)	-4.4 (1.8)	18.6 (1.5)	1.03 (0.11)
0.1	-2.5 (1.5)	16.4 (1.2)	-4.2 (1.7)	18.3 (1.4)	1.02 (0.11)
0.2	-2.3 (1.5)	16.2 (1.2)	-4.0 (1.7)	18.1 (1.4)	1.01 (0.10)
0.3	-2.2 (1.4)	16.1 (1.2)	-3.9 (1.7)	17.9 (1.4)	1.00 (0.10)
0.4	-2.0 (1.4)	16.0 (1.2)	-3.9 (1.6)	17.7 (1.4)	0.99 (0.10)
0.5	-1.9 (1.4)	15.8 (1.1)	-3.8 (1.6)	17.6 (1.3)	0.98 (0.10)
0.6	<b>-1.9 (1.4)</b>	<b>15.6 (1.1)</b>	<b>-3.7 (1.6)</b>	<b>17.4 (1.3)</b>	<b>0.98 (0.10)</b>
Year $>$ 2005					
Median					
0	-3.6 (0.4)	10.4 (0.6)	-10.2 (1.1)	17.6 (0.7)	0.88 (0.03)
0.1	-3.6 (0.3)	9.1 (1.0)	-9.6 (0.7)	17.9 (0.8)	0.91 (0.02)
0.2	-3.7 (0.2)	8.4 (1.5)	<b>-8.8 (1.2)</b>	17.3 (0.7)	0.89 (0.02)
0.3	<b>-3.5 (0.2)</b>	7.6 (1.6)	-9.2 (1.3)	<b>17.1 (0.9)</b>	0.88 (0.01)
0.4	-3.6 (0.1)	<b>7.3 (1.4)</b>	-10.0 (0.9)	17.2 (1.3)	0.86 (0.01)
0.5	-3.7 (0.1)	7.6 (1.2)	-10.7 (1.1)	17.5 (1.5)	0.86 (0.01)
0.6	-3.8 (0.2)	7.9 (1.0)	-10.3 (1.0)	18.2 (1.3)	<b>0.84 (0.01)</b>
Mean					
0	-8.1 (0.5)	18.3 (0.4)	-15.7 (1.7)	30.0 (1.6)	1.38 (0.09)
0.1	-7.1 (0.5)	17.0 (0.5)	-14.8 (1.5)	28.3 (1.4)	1.35 (0.08)
0.2	-6.6 (0.5)	16.0 (0.5)	-14.6 (1.4)	27.2 (1.3)	1.32 (0.08)
0.3	-6.2 (0.5)	15.2 (0.5)	<b>-14.6 (1.3)</b>	26.6 (1.2)	1.30 (0.08)
0.4	-6.1 (0.5)	14.7 (0.5)	-14.7 (1.3)	26.1 (1.2)	1.28 (0.08)
0.5	-6.0 (0.5)	14.2 (0.5)	-14.8 (1.2)	25.8 (1.1)	1.27 (0.08)
0.6	<b>-5.9 (0.5)</b>	<b>13.9 (0.5)</b>	-14.8 (1.2)	<b>25.6 (1.1)</b>	<b>1.25 (0.08)</b>

TABLE 4

Validation results for U5MR and ARR estimates. Results refer to high mortality countries (high means U5MR of at least 40 deaths per 1000 births in 1990) without crises or HIV adjustments, with data in both training and test set, 78 countries in total. Median and mean outcome measures are reported for the U5MR in 2000 and 2005, and the annual rate of reduction (ARR) from 1990 to 2005. Outcome measures are given by the following: median/mean relative error (MRE) and median/mean absolute relative error (MARE) for the U5MR, median or mean error (ME) and median/mean absolute error (MAE) for the ARR, and median/mean interval score (Score) as well as % of countries below and above the 90% uncertainty intervals based on the training set. The lowest value for each outcome measure is bolded

U5MR 2000								
W	Median			Mean			% of countries outside 90% UI	
	MRE	MARE	Score	MRE	MARE	Score	% Below	% Above
0	-2.4	4.5	0.30	-4.8	9.9	0.56	<b>3.8</b>	<b>5.1</b>
0.1	-2.4	4.5	0.30	-4.6	9.7	0.56	<b>3.8</b>	<b>5.1</b>
0.2	-2.4	4.5	0.30	-4.5	9.6	0.55	<b>3.8</b>	<b>5.1</b>
0.3	<b>-2.4</b>	4.5	0.29	-4.3	9.3	0.55	<b>3.8</b>	<b>5.1</b>
0.4	-2.5	4.5	0.29	-4.2	9.2	0.54	<b>3.8</b>	<b>5.1</b>
0.5	-2.4	<b>4.4</b>	0.29	-4.1	9.0	0.53	<b>3.8</b>	<b>5.1</b>
0.6	-2.5	4.5	<b>0.29</b>	<b>-4.0</b>	<b>8.8</b>	<b>0.52</b>	<b>3.8</b>	<b>5.1</b>

U5MR 2005								
W	Median			Mean			% of countries outside 90% UI	
	MRE	MARE	Score	MRE	MARE	Score	% Below	% Above
0	-5.0	10.4	0.51	-11.0	18.9	0.95	<b>6.4</b>	5.1
0.1	<b>-4.7</b>	9.4	<b>0.35</b>	-10.2	17.5	0.92	<b>6.4</b>	3.8
0.2	-4.8	8.9	0.53	-9.6	16.4	0.89	<b>6.4</b>	2.6
0.3	-5.3	8.1	0.53	-9.3	15.7	0.88	7.7	2.6
0.4	-5.3	<b>8.1</b>	0.54	-9.0	15.2	0.86	7.7	2.6
0.5	-6.1	8.1	0.53	-8.9	14.7	0.85	7.7	<b>1.3</b>
0.6	-6.0	8.5	0.53	<b>-8.7</b>	<b>14.4</b>	<b>0.83</b>	7.7	2.6

ARR 1990–2005								
W	Median			Mean			% of countries outside 90% UI	
	ME	MAE	Score	ME	MAE	Score	% Below	% Above
0	0.2	0.7	<b>3.5</b>	0.3	1.0	5.7	5.1	7.7
0.1	0.2	0.6	3.7	0.3	1.0	5.5	5.1	<b>6.4</b>
0.2	0.2	0.5	3.7	0.3	0.9	5.3	3.8	7.7
0.3	<b>0.1</b>	0.6	3.7	0.3	0.9	5.2	3.8	7.7
0.4	0.1	0.5	3.8	0.3	0.8	5.1	<b>2.6</b>	9.0
0.5	0.2	0.5	3.7	<b>0.3</b>	0.8	5.0	<b>2.6</b>	9.0
0.6	0.2	<b>0.5</b>	3.7	0.3	<b>0.8</b>	<b>5.0</b>	<b>2.6</b>	9.0

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### SUPPLEMENTARY MATERIAL

**Figure S1: Illustration of differences in estimates and projections for all 194 countries between the unpooled (country-specific) and pooled B-spline model projection approach** (DOI: [10.1214/14-AOAS768SUPPA](https://doi.org/10.1214/14-AOAS768SUPPA); .pdf). Country-specific graphs to illustrate the effect of the pooling, as in Figure 4, for all 194 countries.

**Figure S2: U5MR data series and estimates for all 194 countries** (DOI: [10.1214/14-AOAS768SUPPB](https://doi.org/10.1214/14-AOAS768SUPPB); .pdf). Country-specific graphs, as in Figures 1 and 2, for all 194 countries.

### REFERENCES

- ALKEMA, L. and NEW, J. R. (2012). Progress toward global reduction in under-five mortality: A bootstrap analysis of uncertainty in Millennium Development Goal 4 estimates. *PLoS Med.* **9** e1001355.
- ALKEMA, L. and NEW, J. R. (2013). Global estimation of child mortality using a Bayesian B-spline bias-reduction model. Technical report. Available at <http://arxiv.org/abs/1309.1602>.
- ALKEMA, L. and NEW, J. (2014). Supplement to "Global estimation of child mortality using a Bayesian B-spline Bias-reduction model." DOI:10.1214/14-AOAS768SUPPA, DOI:10.1214/14-AOAS768SUPPB.
- ALKEMA, L., WONG, M. B. and SEAH, P. R. (2012). Monitoring progress towards Millennium Development Goal 4: A call for improved validation of under-5 mortality rate estimates. *Statistics, Politics and Policy* **3** Article ID 2.
- AMOUZOU, A. (2011). Real-time results tracking. Technical report, Institute for International Programs, Johns Hopkins Bloomberg School of Public Health, Baltimore. Available at [http://www.jhsph.edu/departments/international-health/centers-and-institutes/institute-for-international-programs/\\_documents/RRT\\_Technical\\_Note.pdf](http://www.jhsph.edu/departments/international-health/centers-and-institutes/institute-for-international-programs/_documents/RRT_Technical_Note.pdf).
- BRASS, W. (1964). Uses of census or survey data for the estimation of vital rates. In *African Seminar on Vital Statistics, 14–19 December 1964*, United Nations, New York.
- CENSUS OF INDIA (2011). Sample registration. Available at [http://censusindia.gov.in/Vital\\_Statistics/SRS/Sample\\_Registration\\_System.aspx](http://censusindia.gov.in/Vital_Statistics/SRS/Sample_Registration_System.aspx).
- CLARK, S. J., WAKEFIELD, J., MCCORMICK, T. and ROSS, M. (2012). Hyak mortality monitoring system: Innovative sampling and estimation methods. Working Paper 118. Available at <http://www.csss.washington.edu/Papers/wp118.pdf>.
- CURRIE, I. D. and DURBAN, M. (2002). Flexible smoothing with  $P$ -splines: A unified approach. *Stat. Model.* **2** 333–349. MR1951589
- EILERS, P. H. C. (1999). Discussion of "The analysis of designed experiments and longitudinal data using smoothing splines" by A. P. Verbyla, B. R. Cullis, M. G. Kenward and S. J. Welham. *J. R. Stat. Soc. Ser. C. Appl. Stat.* **48** 300–311.

- EILERS, P. H. C. and MARX, B. D. (1996). Flexible smoothing with  $B$ -splines and penalties. *Statist. Sci.* **11** 89–121. [MR1435485](#)
- EILERS, P. H. C. and MARX, B. D. (2010). Splines, knots, and penalties. *Wiley Interdisciplinary Reviews: Computational Statistics* **2** 637–653.
- GELMAN, A. and RUBIN, D. (1992). Inference from iterative simulation using multiple sequences. *Statist. Sci.* **7** 457–511.
- GNEITING, T. and RAFTERY, A. E. (2007). Strictly proper scoring rules, prediction, and estimation. *J. Amer. Statist. Assoc.* **102** 359–378. [MR2345548](#)
- HILL, K., YOU, D., INOUE, M. and OESTERGAARD, M. Z. (2012). Child mortality estimation: Accelerated progress in reducing global child mortality *PLoS Med.* **9** e1001303.
- OESTERGAARD, M. Z., ALKEMA, L. and LAWN, J. E. (2013). Millennium Development Goals national targets are moving targets and the results will not be known until well after the deadline of 2015. *International Journal of Epidemiology* **42** 645–647.
- PEDERSEN, J. and LIU, J. (2012). Child mortality estimation: Appropriate time periods for child mortality estimates from full birth histories. *PLoS Med.* **9** e1001289.
- PLUMMER, M. (2003). JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling. In *Proceedings of the 3rd International Workshop on Distributed Statistical Computing (DSC 2003)*, Vienna. Available at <http://mcmc-jags.sourceforge.net/>.
- RAFTERY, A. E. and LEWIS, S. M. (1992). How many iterations in the Gibbs sampler? In *Bayesian Statistics 4* (J. M. Bernardo et al., eds.) 763–773. Oxford Univ. Press, Oxford.
- RAFTERY, A. E. and LEWIS, S. M. (1996). Implementing MCMC. In *Markov Chain Monte Carlo in Practice* (W. R. Gilks, D. J. Spiegelhalter and S. Richardson, eds.) 115–130. Chapman & Hall, London. [MR1397966](#)
- RAJARATNAM, J. K., MARCUS, J., FLAXMAN, A., WANG, H., LEVIN-RECTOR, A., DWYER, L., COSTA, M., LOPEZ, A. and MURRAY, C. (2010). Neonatal, postneonatal, childhood, and under-5 mortality for 187 countries, 1970–2010: A systematic analysis of progress towards Millennium Development Goal 4. *The Lancet* **375** 1988–2008.
- SULLIVAN, J. M. (2008). An assessment of the credibility of child mortality declines estimated from DHS mortality rates. Working Paper 1, United Nations Children’s Fund, New York. Available at [http://www.childinfo.org/files/Overall\\_Results\\_of\\_Analysis.pdf](http://www.childinfo.org/files/Overall_Results_of_Analysis.pdf).
- UN SYSTEM TASK TEAM ON THE POST-2015 UN DEVELOPMENT AGENDA (2012). Addressing inequalities: The heart of the post-2015 agenda and the future we want for all. Available at [http://www.un.org/millenniumgoals/pdf/10\\_inequalities\\_20July.pdf](http://www.un.org/millenniumgoals/pdf/10_inequalities_20July.pdf).
- UNITED NATIONS (1983). *Manual X: Indirect Techniques for Demographic Estimation*. United Nations, New York.
- UNITED NATIONS CHILDREN’S FUND AND USAID (2012). Real-time child mortality monitoring meeting. United Nations Children’s Fund, New York. Available at <http://newsletter.childrenandaids.org/real-time-child-mortality-monitoring-meeting-december-19-2012/>.
- UNITED NATIONS CHILDREN’S FUND, DIVISION OF POLICY AND STRATEGY (2013). Committing to child survival: A promise renewed progress report 2013. United Nations Children’s Fund, New York.
- UNITED NATIONS, DEPARTMENT OF ECONOMIC AND SOCIAL AFFAIRS, POPULATION DIVISION (2011). World population prospects. The 2010 Revision.
- WANG, H., DWYER-LINDGREN, L., LOFGREN, K. T., RAJARATNAM, J. K., MARCUS, J. R., LEVIN-RECTOR, A., LEVITZ, C. E., LOPEZ, A. D. and MURRAY, C. J. (2012). Age-specific and sex-specific mortality in 187 countries, 1970–2010: A systematic analysis for the global burden of disease study 2010. *The Lancet* **380** 2071–2094.
- UNITED NATIONS INTER-AGENCY GROUP FOR CHILD MORTALITY ESTIMATION (2012). Levels & trends in child mortality: Report 2012. United Nations Children’s Fund, New York.

UNITED NATIONS INTER-AGENCY GROUP FOR CHILD MORTALITY ESTIMATION (2013). Levels & trends in child mortality: Report 2013. United Nations Children's Fund, New York.

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