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Acceptance sampling reliability test plans for alpha distributed lifetime

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Abstract. Determining the acceptability of any product, reliability sampling plans are used. In this paper, reliability sampling plans for truncated life test are developed when the lifetimes of a test follow an alpha distribution. The sampling plan proposed here can save the test time in practical situations. Sampling plans are established through an algorithm. Moreover, some tables are provided for the proposed sampling plans so that proposed method can be used conveniently for the practitioner. Operating characteristic values of the sampling plans are also presented. Examples are provided to illustrate its use.

1 Introduction

With the globalization of business, the dimensions of business are changing rapidly. With the emergence of competition at the global level, the customer is the one who is benefited the most. He now has options, as he can choose what he would like to buy from various alternatives. Due to the highly competitive global business market, a product's quality plays one of the most important roles for industry growth in today's competitive business arena. Ensuring for a quality level, there are two important tools: statistical quality control and acceptance sampling. To assure the quality of products, inspections are performed, but in many situations, it may not be possible to perform hundred percent inspections, and if no inspections are performed, we cannot assure the quality of the lot or products. In this situation, acceptance sampling plans (ASPs) play the role of 'bridge' between hundred percent inspections and no inspection.

An acceptance sampling plan in statistical quality control refers to the process of randomly inspecting a certain number of items from a lot or batch in order to decide whether to accept or reject the entire batch or lot of product. It is used when inspecting every item is not physically possible or would be overly expensive, or when inspecting a large number of items would lead to errors due to worker fatigue. This last concern is especially important when a large number of items are processed in a short period of time. For example, when an acceptance sampling plan would be used is in destructive testing, such as testing eggs for salmonella

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or vehicles for crash testing. Obviously, in these cases it would not be helpful to test every item! However, 100 percent inspection does make sense if the cost of inspecting an item is less than the cost of passing on a defective item. A decision to accept or reject the lot subject to the risk associated with the two types of errors (rejecting a good lot or accepting a bad lot) is possible, such a procedure is called a 'Reliability Test Plan' or 'Acceptance Sampling based on life test plans.' For a given acceptance sampling plan, the consumer's and producer's risks are the probabilities that a bad lot is accepted and a good lot is rejected, respectively, and the goal of an acceptance sampling plan is to determine the criteria for acceptance or rejection based on the size of the lot, the size of the sample, and the level of confidence we wish to attain.

Sometimes, it might be time consuming to wait until all test items fail in a life test if the lifetimes of test items are high. One can use the truncated life test for saving time and money. The test can be performed without waiting until all test items fail, and then the test time can be reduced significantly. The problem considered here is that of finding the smallest sample size to ensure a certain mean lifetime when the life test is terminated at a preassigned time t, and when the number of failures observed does not exceed a given acceptance number c. The decision is to accept the lot only if the specified mean lifetime can be established with a preassigned high probability P^*, which provides protection to consumers. The life test is finished at the time at which the $(c + 1)$th failure is observed or at time t, whichever is earlier.

In the literature, Sobel and Tischendrof (1959) developed a truncated life test of this type for the exponential distribution. Goode and Kao (1961) developed the sampling plans based on a truncated life test for the Weibull distribution with known shape parameter. Gupta and Groll (1961) developed the sampling plans based on the truncated life test for the gamma distribution with known shape parameter. Kantam and Rosaiah (1998) developed the sampling plans based on the truncated life test for the half-logistic distribution. Kantam et al. (2001) developed the sampling plans based on the truncated life test for the log-logistic distribution with known shape parameter. For more insight in this regard, we refer to Baklizi (2003) for Pareto distribution, to Baklizi and El Masri (2004) for Birnbaum–Saunders distribution, to Rosaiah and Kantam (2005) for inverse Rayleigh distribution, to Tsai and Wu (2006) for generalized Rayleigh distribution, and to Rosaiah et al. (2006) for exponentiated log-logistic distribution. Balakrishnan et al. (2007) developed the ASPs for generalized Birnbaum–Saunders distribution. Aslam (2007) developed grouped ASPs for the Rayleigh distribution, and Aslam and Shabaz (2007) developed grouped ASPs for the generalized exponential distribution. Tsai and Wu (2008) developed ASPs for the Inverse Gaussian distribution, and Srinivasa et al. (2009) developed ASPs for the Marshall–Olkin extended distribution, etc.

In acceptance sampling based on truncated life tests, Balakrisnan et al. (2007) have the following assumptions: (i) the units are destructible or are degraded after the life test, and (ii) there are several distributions that model the product life reasonably well. Thus, considering similar risk and operating conditions and the assumptions (i) and (ii), the consumer will benefit with a smaller number of units required to test. For this reason, we could use a distribution that gives the smallest sample size. In this paper, alpha distribution is considered and we present a methodology to find the minimum sample size necessary to ensure a specified mean life. In Section 2 we describe the alpha distribution. In Section 3 we develop the proposed acceptance sampling plan. In Section 4, illustrative examples are presented and, finally, conclusions are given in Section 5.

2 Alpha distribution

The alpha distribution was first used as a result of wear analysis of broad nosed cutting tools and modeling the life characteristic of machine components which deteriorate according to a scheme of the nonstationary linear random wear process (Vysokovskii, 1970). Also, in many situations it was successfully used, such as modeling the cutting tool life (Kendall and Sheikh, 1979 and Pandit and Sheikh, 1980), monitoring the dimensions of machine parts for statistical quality control (Pronikov, 1973), and in size modelling (Ahmad and Chaudhary, 1992). Kattan (1996) obtained the strength-reliability of a product considering alpha distribution as strength as well as stress distribution. Recently, Khan and Islam (2007) obtained the strength-reliability for alpha distributed stress with respect to finite strength. Later, Khan and Islam (2010) discussed the robustness of the reliability characteristic of alpha distributed lifetimes. The density function of the alpha distribution with parameters α and β is

$$f(t) = \frac{1}{\Phi(\alpha)} \frac{\beta}{\sqrt{2\pi}} \frac{1}{t^2} \exp\left[-\left\{\frac{1}{2}\left(\alpha - \frac{\beta}{t}\right)^2\right\}\right]; \qquad t > 0, \alpha, \beta > 0. \tag{2.1}$$

where $\Phi(\alpha)$ is given as

$$\Phi(\alpha) = \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-(1/2)u^2}\, du$$

(the distribution function of the standard normal variate $N(0, 1)$ calculated for $t = \alpha$).

Now, the cumulative density function of alpha distribution is

$$F(t) = \frac{\Phi(\alpha - \beta/t)}{\Phi(\alpha)}, \qquad t, \alpha, \beta > 0. \tag{2.2}$$

Družinin (1977) described the time needed to perform some operations and applied by Katzev (1974) in cutting-tools durability analysis (Dorin & Vodă, 1973), this variable is in fact the inverse of a left truncated normal variable $N(\mu, \sigma^2)$ in the origin.

Indeed, if X is a measurable characteristic with $f(x, \theta)$ as its density and x_t is a left truncated point (i.e., $x \geq x_t$), then, the density of the truncated variable X_t is

$$X_t: \qquad f_t(x; \theta) = \frac{1}{1 - F_X(x; \theta)} f(x, \theta), \qquad x \geq x_T, \theta \in R, \tag{2.3}$$

where $F_X(x; \theta)$ is the distribution function of the initial variable $X(F'_X(x; \theta) = f(x; \theta))$. If we take $X \sim N(\mu, \sigma^2)$, $x_t = 0$ and compute the density of X_0^{-1}, denoting $\mu = \alpha/\beta$ and $\sigma = 1/\beta$, we get easily the form (2.1) (Dorin et al., 1994). Suppose the mean of the truncated normal distribution is zero, then the cumulative distribution function of the alpha distribution is

$$F(t, 1/\beta) = 2\Phi\left(\frac{t}{(1/\beta)}\right) - 1. \tag{2.4}$$

Now, the average life of the distribution is $\mu = (1/\beta)$. If $\mu = \mu_0$ is a specified mean, then a specified mean $\mu_0 = (1/\beta_0)$. The distribution function of t is

$$F(t, 1/\beta_0) = 2\Phi\left(\frac{t}{1/\beta_0}\right) - 1. \tag{2.5}$$

3 Development of sampling plan

Assume that a truncated life test is conducted when the lifetimes of test items follow the alpha distribution defined by equations (2.1) and (2.5). One objective of this experiment is to set a lower confidence limit on the mean lifetime, and we want to test whether the mean lifetime of items is longer than our expectation. Assume that $\mu = \mu_0$, where μ_0 is the specified mean lifetime for items. The decision is to accept the lot if and only if the number of observed failures at the end of the fixed time t does not exceed a given number c; or to terminate the test and reject the lot if there are more than c failures occurring before time t, which implies that the true mean lifetime of items is below the specified one. The sampling plan contains:

- the number of units, n, required on the test,
- an acceptance number, c, and a ratio t/μ_0.

If c or fewer failures occur during the test time t, the lot is accepted; otherwise, the lot is rejected.

Fix the consumer risk first, then the probability of accepting a bad lot is not to exceed $1 - P^*$. A bad lot means that the lot with the true mean lifetime is below the specified mean lifetime μ_0. Thus, the probability P^* is a confidence level in the sense that the chance of rejecting a lot with $\mu < \mu_0$ is at least P^*. For a predetermined value of P^*, our sampling plan is characterized by $(n, c, t/\mu_0)$.

Here, we consider a lot of infinitely large size so that the theory of binomial distribution can be applied, and the acceptance or rejection of the lot is equivalent to the acceptance or rejection of the hypothesis $\mu \geq \mu_0$. Mathematically, given

$P^*(0 < P^* < 1)$, the ratio of t/μ_0, and an acceptance number c, we need to find the smallest positive integer n so that we can assert that $\mu \geq \mu_0$ with confidence level of P^* if the number of failures observed in time t does not exceed c.

In accordance with the design of the proposed sampling plans, the required sample size, n, is the smallest positive integer which satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} P^i (1-P)^{n-i} \leq 1 - P^*, \tag{3.1}$$

where $F(t, 1/\beta_0)$ is the probability of a failure observed during the time t when the true mean lifetime of items is μ_0. It follows from the fact that the chance of observing x failures during the time t is a binomial distribution with density

$$f(x) = \binom{n}{x} P^x (1-P)^{n-x}. \tag{3.2}$$

Equation (2.5) shows that $F(t, 1/\beta_0)$ depends only on the ratio $t/\beta_0 = t/\mu_0$. Thus, experiment needs only to specify this ratio. The minimum values of n satisfying (2.4) are obtained for $P^* = 0.75, 0.90, 0.95, 0.99$ and $t/\mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712$. Following Gupta and Groll (1961), Kantam and Rosaiah (1998) and Srinivasa et al. (2009), the results are displayed in Table 1.

The operating characteristic (OC) curve of the sampling plan $(n, c, t/\mu_0)$ gives the probability of accepting a lot; it is given by

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} P^i (1-P)^{n-i}, \tag{3.3}$$

where $p = F(t, \mu)$ is treated as a function of lot quality parameter μ.

The producer risk is the probability of rejecting a good lot. For a given value of the producer risk, say, 0.05, one may be interested in knowing what value of μ/μ_0 will ensure the producer risk less than or equal to 0.05 if a sampling plan $(n, c, t/\mu_0)$ is adopted. The value of μ/μ_0 can be taken as the smallest number of μ/μ_0 (> 1) so that p satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} P^i (1-P)^{n-i} \geq 0.95. \tag{3.4}$$

For a given sampling plan $(n, c, t/\mu_0)$ at a specified confidence level P^*, we have computed the smallest values of μ/μ_0 satisfying the inequality (3.4). The OC values are given in Table 3 below. An algorithm is provided here to construct the tables of the proposed sampling plans for practitioners in three stages as follows:

- Set a given probability of accepting a bad lot $(1 - P^*)$.

Table 1 *Minimum sample size for the specified ratio* t/μ_0

		t/μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	2	1	1	1	1	1	1	1
0.75	1	4	3	2	2	2	2	2	2
0.75	2	6	4	3	3	3	3	3	3
0.75	3	7	5	5	4	4	4	4	4
0.75	4	9	7	6	5	5	5	5	5
0.75	5	11	8	7	6	6	6	6	6
0.75	6	13	9	8	7	7	7	7	7
0.75	7	14	11	9	8	8	8	8	8
0.75	8	16	12	10	9	9	9	9	9
0.75	9	18	13	11	10	10	10	10	10
0.75	10	20	14	12	12	11	11	11	11
0.90	0	3	2	1	1	1	1	1	1
0.90	1	5	3	3	2	2	2	2	2
0.90	2	7	5	4	3	3	3	3	3
0.90	3	9	6	5	4	4	4	4	4
0.90	4	11	8	6	5	5	5	5	5
0.90	5	13	9	7	6	6	6	6	6
0.90	6	15	10	9	7	7	7	7	7
0.90	7	16	12	10	8	8	8	8	8
0.90	8	18	13	11	9	9	9	9	9
0.90	9	20	14	12	10	10	10	10	10
0.90	10	22	16	13	12	11	11	11	11
0.95	0	4	2	2	1	1	1	1	1
0.95	1	6	4	3	3	2	2	2	2
0.95	2	8	5	4	4	3	3	3	3
0.95	3	10	7	5	5	4	4	4	4
0.95	4	12	8	7	6	5	5	5	5
0.95	5	14	10	8	7	6	6	6	6
0.95	6	16	11	9	8	7	7	7	7
0.95	7	18	13	10	9	8	8	8	8
0.95	8	20	14	11	10	9	9	9	9
0.95	9	22	15	13	11	10	10	10	10
0.95	10	23	17	14	12	11	11	11	11
0.99	0	5	3	2	2	1	1	1	1
0.99	1	8	5	4	3	2	2	2	2
0.99	2	10	7	5	4	3	3	3	3
0.99	3	12	8	6	5	4	4	4	4
0.99	4	15	10	8	6	5	5	5	5
0.99	5	17	11	9	8	6	6	6	6
0.99	6	19	13	10	9	7	7	7	7
0.99	7	21	14	11	10	8	8	8	8
0.99	8	23	16	12	11	9	9	9	9
0.99	9	25	17	14	12	10	10	10	10
0.99	10	27	18	15	13	11	11	11	11

- Find the smallest sample size n for each predetermined value c satisfying the inequality

$$\sum_{i=0}^{c} \binom{n}{i} P^{i}(1-P)^{n-i} \leq 1-P^{*}, \tag{3.5}$$

where $p = F(t, \mu_0)$. The sampling plan $(n, c, t/\mu_0)$ can be obtained after the value of t/μ_0 is specified.
- For a given producer risk α, find the smallest value of μ/μ_0 which satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} P^{i}(1-P)^{n-i} \geq 1-\alpha. \tag{3.6}$$

4 Illustrative examples

Example 1

Assume that the lifetimes of electrical devices follow an alpha distribution and an experimenter is interested in establishing a sampling plan to ensure that the mean lifetime is at least 1000 hours with confidence level $P^* = 0.90$. The experimenter wishes to stop the experiment at $t = 628$ hours. Then for an acceptance number $c = 2$, from Table 1, the required sample size corresponding to the values of $P^* = 0.90$, $t/\mu_0 = 0.628$ and $c = 2$ is $n = 7$. Thus, 7 units have to be put on test. If no more than 2 failures out of 7 are observed during 628 hours, then the experimenter can assert that the mean lifetime is at least 1000 hours with a confidence level 0.90 for the sampling plan ($n = 7$, $c = 2$, $t/\mu_0 = 0.628$) and confidence level $P^* = 0.90$. Under alpha distribution the values of the Operating Characteristic (OC) function from Table 2 are as follows:

μ/μ_0	2	4	6	8	10	12
OC	0.548	0.891	0.960	0.982	0.990	0.994

If the true mean lifetime average life is double the specified lifetime ($\mu/\mu_0 = 2$), the producer's risk is $(1 - 0.548 = 0.452)$, while it is about 0.01 when the true mean lifetime is ten times of the specified mean life.

In this example, we can get the smallest values of μ/μ_0 for various choices of c and t/μ_0 from Table 3 in order to assert the producer risk is less than or equal to 0.05. In the example, the smallest value of μ/μ_0 is 5.51 for $c = 2, t/\mu_0 = 0.628$ and $P^* = 0.90$, that is, the item should have a mean lifetime of at least 5.51 times of the specified mean lifetime of 1000 hours in order that the lot will be accepted with the probability 0.95 under the design. The actual mean lifetime necessary to transship 95 percent of the lot is provided in Table 3.

Table 2 *Values of the operating characteristic function of the sampling plans* $(n, c, t/\mu_0)$

				μ/μ_0					
P^*	n	c	t/μ_0	2	4	6	8	10	12
0.75	6	2	0.628	0.662	0.930	0.975	0.988	0.993	0.997
0.75	4	2	0.942	0.694	0.944	0.980	0.993	0.996	0.997
0.75	3	2	1.257	0.755	0.959	0.986	0.995	0.997	0.998
0.75	3	2	1.571	0.606	0.925	0.977	0.989	0.995	0.998
0.75	3	2	2.356	0.262	0.788	0.924	0.967	0.982	0.992
0.75	3	2	3.141	0.077	0.606	0.842	0.925	0.962	0.976
0.75	3	2	3.972	0.015	0.408	0.724	0.862	0.922	0.954
0.75	3	2	4.712	0.004	0.262	0.606	0.789	0.880	0.924
0.90	7	2	0.628	0.549	0.890	0.961	0.982	0.989	0.993
0.90	5	2	0.942	0.510	0.884	0.960	0.981	0.990	0.994
0.90	4	2	1.257	0.480	0.870	0.958	0.981	0.990	0.994
0.90	3	2	1.571	0.606	0.925	0.976	0.990	0.995	0.996
0.90	3	2	2.356	0.262	0.789	0.926	0.967	0.982	0.989
0.90	3	2	3.141	0.077	0.606	0.842	0.925	0.960	0.976
0.90	3	2	3.972	0,015	0.408	0.724	0.861	0.923	0.954
0.90	3	2	4.712	0.003	0.262	0.607	0.790	0.879	0.926
0.95	8	2	0.628	0.445	0.848	0.942	0.972	0.985	0.991
0.95	5	2	0.942	0.510	0.885	0.959	0.981	0.991	0.994
0.95	4	2	1.257	0.482	0.881	0.958	0.981	0.990	0.994
0.95	4	2	1.571	0.290	0.795	0.924	0.965	0.980	0.989
0.95	3	2	2.356	0.261	0.789	0.925	0.966	0.982	0.990
0.95	3	2	3.141	0.078	0.607	0.841	0.926	0.961	0.976
0.95	3	2	3.972	0.015	0.408	0.724	0.862	0.924	0.954
0.95	3	2	4.712	0.003	0.262	0.607	0.790	0880	0.926
0.99	10	2	0.628	0.277	0.750	0.896	0.949	0.972	0.983
0.99	7	2	0.942	0.236	0.734	0.890	0.947	0.972	0.981
0.99	5	2	1.257	0.275	0.776	0.915	0.960	0.979	0.987
0.99	4	2	1.571	0.291	0.795	0.924	0.965	0.981	0.988
0.99	3	2	2.356	0.262	0.790	0.924	0.967	0.983	0.990
0.99	3	2	3.141	0.077	0.607	0.843	0.925	0.961	0.977
0.99	3	2	3.972	0.015	0.409	0.724	0.862	0.922	0.954
0.99	3	2	4.712	0.003	0.262	0.607	0.790	0.880	0.926

Alternatively, we can get the sampling plans as follows: Assume that the lifetimes of the items follow an alpha distribution and the consumers require a probability of rejecting a bad lot, $P^* = 0.95$, and the sampling plan is based on an acceptance number $c = 2$ and $t/\mu_0 = 0.628$. What should the true mean lifetime of items fulfill so that the producer risk will be 5 percent? From Table 3, we find that $\mu/\mu_0 = 6.36$. Thus, the manufactured items should have a mean lifetime at least 6.36 times of the specified mean one in order that the items can be accepted with probability 0.95 under the design. From Table 1 we can find

Table 3 *Minimum ratio of true value* μ *to required* μ_0 *for the acceptability of a lot with producer's risk* 0.05

		t/μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	28.08	21.66	28.91	36.13	54.19	72.25	91.36	108.38
0.75	1	7.29	7.834	6.28	7.84	11.76	15.68	19.83	23.53
0.75	2	4.60	4.22	3.71	4.64	6.96	9.28	11.73	13.92
0.75	3	3.11	3.01	4.02	3.52	5.28	7.04	8.90	10.56
0.75	4	2.78	3.02	3.23	2.95	4.42	5.89	7.45	8.84
0.75	5	2.56	2.54	2.77	2.61	3.91	5.21	6.59	7.82
0.75	6	2.41	2.23	2.47	2.37	3.56	4.74	6.01	7.12
0.75	7	2.12	2.32	2.25	2.20	3.30	4.40	5.57	6.61
0.75	8	2.07	2.11	2.09	2.07	3.11	4.14	5.25	6.22
0.75	9	2.01	1.95	1.96	1.97	2.95	3.94	4.97	5.91
0.75	10	1.98	1.83	1.86	1.89	2.83	3.77	4.77	5.66
0.90	0	43.34	42.13	28.92	36.14	54.19	72.25	91.37	108.39
0.90	1	9.30	7.84	10.45	7.84	11.76	15.68	19.83	23.53
0.90	2	5.51	5.57	5.63	4.64	6.96	9.28	11.73	13.93
0.90	3	4.17	3.85	4.02	5.02	5.28	7.04	8.90	10.56
0.90	4	3.52	3.61	3.23	4.04	4.42	5.89	7.45	8.84
0.90	5	3.14	2.99	2.77	3.46	3.91	5.21	6.59	7.82
0.90	6	2.87	2.59	2.97	3.08	3.56	4.74	5.99	7.12
0.90	7	2.50	2.61	2.68	2.82	3.30	4.40	5.57	6.61
0.90	8	2.38	2.36	2.47	2.61	3.11	4.15	5.24	6.22
0.90	9	2.29	2.18	2.29	2.45	2.95	3.94	4.98	5.91
0.90	10	2.22	2.23	2.16	2.33	2.83	3.77	4.77	5.66
0.95	0	57.32	42.13	56.22	36.12	54.19	72.25	91.37	108.39
0.95	1	11.41	10.93	10.45	13.07	11.76	15.68	19.83	23.53
0.95	2	6.36	5.57	5.63	7.03	6.96	9.28	11.73	13.92
0.95	3	4.70	4.66	4.02	5.02	5.28	7.04	8.90	10.56
0.95	4	3.88	3.60	4.03	4.04	4.42	5.89	7.45	8.84
0.95	5	3.40	3.42	3.39	3.46	3.91	5.21	6.59	7.82
0.95	6	3.09	2.95	2.97	3.08	3.56	4.74	5.60	7.12
0.95	7	2.87	2.90	2.68	2.82	3.30	4.40	5.57	6.61
0.95	8	2.70	2.61	2.47	2.61	3.11	4.12	5.24	6.22
0.95	9	2.56	2.40	2.61	2.45	2.95	3.94	4.98	5.91
0.95	10	2.34	2.42	2.44	2.33	2.83	3.77	4.78	5.66
0.99	0	70.22	65.01	56.22	70.26	54.19	72.25	91.36	108.39
0.99	1	15.37	13.95	14.59	13.07	11.76	15.65	19.82	23.53
0.99	2	8.13	8.27	7.43	7.03	6.96	9.28	11.73	13.90
0.99	3	5.78	5.48	5.13	5.02	5.28	7.04	8.90	10.56
0.99	4	5.00	4.73	4.80	4.04	4.42	5.89	7.45	8.84
0.99	5	4.24	3.84	3.99	4.24	3.91	5.22	6.59	7.82
0.99	6	3.75	3.63	3.46	3.72	3.56	4.74	6.00	7.12
0.99	7	3.42	3.18	3.09	3.35	3.30	4.40	5.57	6.61
0.99	8	3.17	3.40	2.81	3.08	3.11	4.15	5.25	6.22
0.99	9	2.97	2.82	2.91	2.87	2.95	3.94	4.98	5.91
0.99	10	2.83	2.60	2.71	2.70	2.84	3.77	4.77	5.68

that the number of items required to be tested is $n = 8$ and the sampling plan is $(n = 8, c = 2, t/\mu_0 = 0.628)$.

Example 2

Consider a problem associated with software reliability provided by Wood (1996) and analyzed from the acceptance sampling viewpoint by Rosaiah and Kantam (2005), Balakrishnan et al. (2007), Srinivasa et al. (2009), and Srinivasa et al. (2009). We consider the failure times in hours of the release of software, which times correspond to the lifetimes from the starting of the execution of the software until which the failure of the software is experienced. We assume that while the software is operating, the development of intangible cumulative degradation deteriorates with the nonstationary linear random wear process of this software. Then it is reasonable to suppose that the random variable follows the alpha distribution. Let the specified mean life be 1000 hours and the testing time $t = 1257$ hours which leads to the ratio $t/\mu_0 = 1.257$. Thus, for an acceptance number $c = 2$ and confidence level $P^* = 0.95$, the required n is found from Table 3 to be 4, that is, in this case, the acceptance sampling plan for the truncated life test from the alpha distribution is $(n = 4, c = 2, t/\mu_0 = 1.257)$. We considered the ordered sample size $n = 4$ with observations 519, 968, 1430, and 1893. Based on these data, we have to decide whether to accept or reject the lot. We will accept the lot only if the number of failures before 1257 hours is at most 2. The confidence level of the decision process is assured by the sampling plan only if the lifetimes follow the alpha distribution. We have verified this for the above sample data by the *Q–Q* plot. The Kolmogorov–Smirnov test is also applied to test the goodness of fit for the alpha distribution. Since for the given sample of $n = 4$ observations, there are only $c = 2$ failures at 519 and 968 hours before $t = 1257$, we shall accept the lot, assuring a specified mean life as 1000 hours with a confidence level $P^* = 0.95$.

5 Conclusions

In this paper, reliability sampling plans for truncated life tests are developed when the lifetimes of the items follow an alpha distribution. We have shown in general that under similar conditions, in order to ensure a specified mean life with a given confidence level, the alpha distribution model results in smaller sample sizes than some other models used in acceptance sampling. We have also demonstrated for a real data set that the alpha distribution fits the data better than the other models used in life testing. Some tables are provided so that the proposed method can be used conveniently for the practitioners in practical situations.

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