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Rejoinder

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I thank Michael Larsen and Nathaniel Schenker for their thoughtful contributions, which usefully reinforce and expand my arguments.

Any differences between my perspective and that described by Schenker are minor. He defends a pragmatic approach to the Bayesian/frequentist divide, but in my applied work I confess I am pragmatic too. I provide confidence intervals and even *P* values to my biomedical collaborators, rather than posterior credibility intervals. I fear these would meet with head-scratching (not to mention article rejection) given current conventions of statistical reporting in medical journals. Like Schenker, I rely on similarities of frequentist and Bayesian interval estimates in many standard models. I am more focused on developing good scientific models than on elicitation of prior distributions.

Despite concessions to current-day realities, thinking about and articulating the underlying principles that should guide our methodology is worthwhile. For example, in the survey sampling setting, I do not believe design-based inference is appropriate for some problems, and model-based inference is appropriate for others.

Larsen calls for more examples of how to achieve Bayesian calibration in practice. A very reasonable request, but complete textbooks are needed to do any kind of justice to that aim; my examples barely scratch the surface. Concerning his specific comments, I think Bayesian hierarchical models are hugely valuable; model checks seeking a good fit to the observed data are important, but unfortunately not sufficient to guarantee good predictions for missing data and target unknowns. A calibrated Bayes perspective would I hope push the field toward more research and training on how to develop good models in practice.

Schenker's theoretical and applied work on multiple imputation is influential, and the examples cited in his discussion add real substance to my musings. Larsen's multiple imputation example allows me to illustrate Schenker's comment in his Section 4.4 on the danger of

Roderick Little is Richard D. Remington Collegiate Professor, Department of Biostatistics, University of Michigan, 1415 Washington Heights, Ann Arbor, Michigan 48109, USA. (e-mail: rlittle@umich.edu). omitting important variables in the imputation model. The relationship between parental longevity and off-spring's diabetes was under study, and age of death was missing for parents currently alive. Larsen's imputation model conditioned on diabetes status of the offspring. This is important, since a multiple imputation model for this variable that failed to condition on offspring's diabetes status would lead to attenuation of the estimated relationship between these variables—indeed, it might well be worse than discarding the incomplete cases, which would distort the distribution of parental age at death, but not necessarily the relationship under study.

Both discussants consider the Bayes/frequentist divide in the context of inference from survey samples. As Schenker states, the debate is particularly lively in that area, given that the prevailing philosophy is to base inference on the randomization distribution that governs sample selection. My perspective is described in Little (2004), but let me respond to some of Larsen's comments. The goal of design-based survey inference is "frequentist in nature," but for me (well-calibrated), Bayesian inference is just as useful and appropriate for inference about finite population quantities as it is for model parameters. The bald statement that "no model at all is involved in design-based inference" is oversimplified, since (as Larsen points out) design-based inference without any consideration of the implicit model underlying the choice of estimator leads to absurdities like Basu's (1971) famous elephant example.

Terminology can be confusing, and Larsen's comment allows me to draw distinctions between my use of the term "calibration" and other uses. Deville and Särndal (1992) discuss calibration of estimates to aggregate statistics. This form of data calibration is (in principle) automatically achieved by a Bayesian model for prediction that incorporates this information, without the need for Deville and Särndal's ad-hoc distance measure, though achieving it exactly may be challenging. Larsen also mentions "broader modeling options" in Särndal, Swensson and Wretman (1992). These modelassisted methods "calibrate" model predictions with design-weighted adjustments based on the model residuals. They lead to compromises between model predictions and direct design-based estimates, similar to the doubly robust competitors to the PPSP method in my 186 R. LITTLE

Example 4. In my view, these approaches, which are ultimately design-based, are inferior to judicious application of the calibrated Bayes approach. For simulation evidence in the context of my Example 4, see Kang and Schafer (2007) and Zhang and Little (2011).

In my discussion of Hansen, Madow and Tepping (1983), I opined that the way to mitigate the effects of model misspecification is not to modify the estimator, as in these model assisted methods, but to modify the model. I have not changed my opinion.

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