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AN ANSWER TO A CONJECTURE ON MULTIPLICATIVE MAPS ON $C(X, I)$

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Abstract. An answer to the conjecture in [1] is given.

The closed interval $[0, 1]$ (under the usual topology) is denoted by I. For a given topological space X, $C(X, I)$ denotes the set of continuous functions from X into I. The multiplication operation on $C(X, I)$ is defined pointwise, that is, $fg(x) := f(x)g(x)$. For each $c \in I$, $\mathbf{c} \in C(X, I)$ is defined by $\mathbf{c}(x) = c$. A map π from $C(X, I)$ into $C(X, I)$ is called *multiplicative* if

$$\pi(fg) = \pi(f)\pi(g)$$

for each $f, g \in C(X, I)$. The following conjecture is given in [1]. The answer of this conjecture is positive whenever X is first countable, which is the main result of [1].

The Conjecture. Let X be compact Hausdorff space and $\pi : C(X, I) \to C(X, I)$ be a bijective multiplicative map. Then there exists a homeomorphism $\sigma : X \to X$ and a continuous map $k : X \to (0, \infty)$ such that for $x \in X$

$$\pi(f)(x) = (f(\sigma(x)))^{k(x)}$$

for each $f \in C(X, I)$.

As usual the $Stone - \check{C}ech$ compactification of a completely regular Hausdorff space X is denoted by βX. The following example shows that the answer to the question is negative.

Example. For each $f \in C(\beta(0, 1), I)$, let $\alpha_f : (0, 1) \to I$ be defined by

$$\alpha_f(x) = (f(x))^{\frac{1}{x}}$$

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and let $\alpha_f^e \in C(\beta(0,1), I)$ be the extension of α_f. Then

$$T : C(\beta(0,1), I) \to C(\beta(0,1), I), \quad T(f) = \alpha_f^e$$

is bijective and multiplicative. We claim that there is no homeomorphism $\sigma : \beta(0,1) \to \beta(0,1)$ and a continuous function $k : \beta(0,1) \to (0,\infty)$ such that $T(f)(x) = (f(\sigma(x)))^{k(x)}$ for each $f \in C(\beta(0,1), I)$ and $x \in \beta(0,1)$. Indeed if such σ and k exist then for each $x, c \in (0,1)$ we have

$$c^{\frac{1}{x}} = T(\mathbf{c})(x) = c^{k(x)},$$

so $k(x) = \frac{1}{x}$. This is a contradiction to $k(\beta(0,1))$ being bounded.

References

[1] J. Marovt, *Multiplicative bijections of* $C(X, I)$, *Proc. Amer. Math. Soc.*, **134** (2006), 1065-1075.

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