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LEVITIN-POLYAK WELL-POSEDNESS OF GENERALIZED VECTOR EQUILIBRIUM PROBLEMS

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Abstract. In this paper, four types of Levitin-Polyak well-posedness of generalized vector equilibrium problems with both abstract set constraints and functional constraints are investigated. Criteria and characterizations for these types of Levitin-Polyak well-posedness of generalized vector equilibrium problems are obtained.

1. INTRODUCTION

It is well known that the well-posedness is very important for both optimization theory and numerical methods of optimization problem, which guarantees that, for approximating solution sequences, there is a subsequence which converges to a solution. The study of well-posedness started from Tykhonov [1] and Levitin and Polyak [2]. Since then, various notions of well-posedness for scalar optimization optimizations have been defined and studied in [3-7] and the references therein. Recent studies on various notions of well-posedness for vector optimization problems can be found in [8-13]. The study of Levitin-Polyak well-posedness for convex scalar optimization problems with functional constraints originates from [4]. Recently, this research was extended to nonconvex optimization problems with both abstract set constraints and functional constraints [6], nonconvex vector optimization problems with abstract set constraints and functional constraints [13], variational inequalities with abstract set constraints and functional constraints [14], generalized variational inequalities with abstract set constraints and functional constraints [15], generalized vector variational inequalities with abstract set constraints

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and functional constraints [16], equilibrium problems with abstract set constraints and functional constraints [17], and equilibrium problems with abstract set constraints and functional constraints [18]. Well-posedness of variational inequalities, mixed variational inequalities, variational inclusions, mixed quasi-variational-like inequalities and generalized mixed variational inequalities, vector equilibrium problems and vector quasi-equilibrium problems without explicit constraints have been intensively investigated (see [19-32] and the references therein). However, there is no study for the Levitin-Polyak well-posedness for generalized vector equilibrium problems with abstract set constraints and functional constraints.

In this paper, we will introduce four types of Levitin-Polyak well-posedness for a generalized vector equilibrium problem with abstract set constraints and functional constraints. In section 2, by a gap function for a generalized vector equilibrium problem, we show equivalent relations between the Levitin-Polyak well-posedness of the optimization problem and the Levitin-Polyak well-posedness of a generalized vector equilibrium problem. In section 3, we derive some various criteria and characterizations for the (generalized) LP well-posedness of a generalized vector equilibrium problem. The results in this paper unify, generalize and extend some known results in [14-18, 25].

2. Preliminaries

Throughout this paper, unless otherwise specified, we use the following notations and assumptions:

Let $(X, \|\cdot\|)$ be a normed space and (Z, d_1) be a metric space. Let $X_1 \subset X$, $K \subset Z$ be nonempty and closed sets. Let Y be a locally convex space and $C : X \to 2^Y$ be a set-valued map such that for any $x \in X$, $C(x)$ is a pointed, closed and convex cone in Y with nonempty interior $intC(x)$. Let V be a topological space, and $T : X_1 \to 2^V$ be a strict set-valued map (i.e., $T(x) \neq \emptyset, \forall x \in X_1$). Let X^* and Y^*, respectively, be the dual spaces of X and Y, and X, Y, V be equipped with the norm topology. Let $e : X \to Y$ be a continuous vector-valued map and satisfy that for any $x \in X$, $e(x) \in intC(x)$, $g : X_1 \to Z$ be a continuous vector-valued map, and $f : X \times V \times X_1 \to Y$ be a vector-valued map. Let $X_0 = \{x \in X_1 : g(x) \in K\}$ be nonempty. We consider the following explicit constrained generalized vector equilibrium problem with variable domination structures: Find a point $\bar{x} \in X_0$ and some point $\bar{z} \in T(\bar{x})$, such that

$$\text{(GVEP)} \qquad f(\bar{x}, \bar{z}, y) \notin -intC(\bar{x}), \forall y \in X_0.$$

The solution set of (GVEP) is denoted by Ω_1.

Let (P, d) be a metric space, $P_1 \subseteq P$ and $x \in P$. We denote by $d(x, P_1) = \inf\{d(x, p) : p \in P_1\}$ the distance function from the point $x \in P$ to the set P_1.

Definition 2.1.

(i) A sequence $\{x_n\} \subset X_1$ is called a type I Levitin-Polyak (in short LP) approximating solution sequence for (GVEP) if there exist $\{\epsilon_n\} \subseteq \mathbf{R}_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that

$$d(x_n, X_0) \leq \epsilon_n, \tag{2.1}$$

and

$$f(x_n, z_n, y) + \epsilon_n e(x_n) \notin -intC(x_n), \forall y \in X_0. \tag{2.2}$$

(ii) $\{x_n\} \subset X_1$ is called type II LP approximating solution sequence for (GVEP) if there exist $\{\epsilon_n\} \subseteq \mathbf{R}_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that (2.1) and (2.2) hold and for any $z \in T(x_n)$ there exists $y_n \in X_0$ such that

$$f(x_n, z, y_n) - \epsilon_n e(x_n) \in -C(x_n). \tag{2.3}$$

(iii) $\{x_n\} \subset X_1$ is called a generalized type I LP approximating solution sequence for (GVEP) if there exist $\{\epsilon_n\} \subseteq \mathbf{R}_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ satisfying

$$d(g(x_n), K) \leq \epsilon_n, \tag{2.4}$$

and (2.2);

(iv) $\{x_n\} \subset X_1$ is called a generalized type II LP approximating solution sequence for (GVEP) if there exist $\{\epsilon_n\} \subseteq \mathbf{R}_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that (2.2) and (2.4) hold and for any $z \in T(x_n)$ there exists $y_n \in X_0$ satisfying (2.3).

Definition 2.2 (GVEP) is said to be type I (resp. type II, generalized type I, generalized type II) LP well-posed if $\Omega_1 \neq \emptyset$ and for any type I (resp. type II, generalized type I, generalized type II) LP approximating solution sequence $\{x_n\}$ of (GVEP), there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and $\bar{x} \in \Omega_1$ such that $x_{n_j} \to \bar{x}$.

Remark 2.1

(i) It is clear that any (generalized) type II LP approximating solution sequence of (GVEP) is a (generalized) type I LP approximating solution sequence of (GVEP). Thus the (generalized) type I LP well-posedness of (GVEP) implies the (generalized) type II LP well-posedness of (GVEP).

(ii) If there exists some $\delta_0 > 0$ such that g is uniformly continuous on the set

$$S(\delta_0) = \{x \in X_1 : d(X_0, x) \leq \delta_0\},$$

then it is not difficult to see that generalized type I (resp. generalized type II) LP well-posedness of (GVEP) implies type I (resp. type II) LP well-posedness of (GVEP).

(iii) Any one type of (generalized) LP well-posedness defined above implies that the solution set Ω_1 of (GVEP) is nonempty and compact.

(iv) If $T(x) = \bar{z}$ for all $x \in X_1$, $K = Z$ and define a function $\varphi : X \times X_1 \to Y$ as $\varphi(x, y) = f(x, \bar{z}, y)$, then the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of (GVEP) defined in Definition 2.2 reduces to the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of the vector equilibrium problem with abstract set constraints and functional constraints introduced by Peng, Wang and Zhao [18]. Moreover, if $Y = \mathbf{R}$, $C(x) = \mathbf{R}_+$ for all $x \in X$, then the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of (GVEP) defined in Definition 2.2 reduces to the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of the scalar equilibrium problem with abstract set constraints and functional constraints introduced by Long, Huang and Teo [17].

(v) Let $V = L(X, Y)$ be the space of all the linear continuous operators from X to Y, $C(x) = C$ and $e(x) = e$ for all $x \in X$, and let $\langle z, x\rangle$ denote the function value $z(x)$, where $z \in L(X, Y)$, $x \in X_1$. If $f(x, z, y) = \langle z, y - x\rangle$ for all $x \in X, z \in V, y \in X_1$, then the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of (GVEP) reduces to the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of the set-valued vector variational inequality problem with abstract set constraints and functional constraints introduced by Xu, Zhu and Huang [16]. Moreover, if $V = X^*$, and $C(x) = \mathbf{R}_+$ for all $x \in X$, then the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of (GVEP) reduces to the type I (resp. type II, generalized type I, generalized type II) LP well-posedness of the generalized variational inequality problem with abstract set constraints and functional constraints introduced by Huang and Yang [15], which contains as special cases for the type I (resp. type II, generalized type I, generalized type II) LP well-posedness for the variational inequality with abstract set constraints and functional constraints introduced by Huang, Yang and Zhu [14].

Definition 2.3. (GVEP) is said to be type I (resp. generalized type I, type II, generalized type II) well-set if $\Omega_1 \neq \emptyset$ and for any type I (resp. generalized type I, type II, generalized type II) LP approximating solution sequence $\{x_n\}$ for (GVEP), we have $\lim_{n\to\infty} d(x_n, \Omega_1) \to 0$.

From Definitions 2.2 and 2.3, we can easily obtain the following result about the relations between (generalized) type LP well-posedness and (generalized) well set of (GVEP):

Proposition 2.1. $($*GVEP*$)$ *is type I* $($*resp. type II, generalized type I, generalized type II*$)$ *LP well-posed if and only if* $($*GVEP*$)$ *is type I* $($*resp. type II, generalized type I, generalized type II*$)$ *well-set and* Ω_1 *is compact.*

To see the various LP well-posednesses of (GVEP) are adaptations of the corresponding LP well-posednesses in minimizing problems by using the Auslender gap function, we consider the following general constrained optimization problem introduced and researched by Huang and Yang [6]:

$$\text{(P)} \ \min \phi(x)$$
$$\text{s.t. } x \in X_1, \ g(x) \in K.$$

We use $\bar{\Omega}$ and $\bar{v}$ to denote the optimal set and value of (P), respectively.

Now, we recall the following definitions about well-posedness for (P) introduced by Huang and Yang [6].

Definition 2.4.

(i) A sequence $\{x_n\} \subset X_1$ is called a type I LP minimizing sequence for (P) if

$$\limsup_{n \to +\infty} \phi(x_n) \leq \bar{v}, \tag{2.5}$$

and

$$d(x_n, X_0) \to 0. \tag{2.6}$$

(ii) $\{x_n\} \subset X_1$ is called a type II LP minimizing sequence for (P) if

$$\lim_{n \to \infty} \phi(x_n) = \bar{v}, \tag{2.7}$$

and (2.6) hold.

(iii) $\{x_n\} \subset X_1$ is called a generalized type I LP minimizing sequence for (P) if

$$d(g(x_n), K) \to 0, \tag{2.8}$$

and (2.5) hold.

(iv) $\{x_n\} \subset X_1$ is called a generalized type II LP minimizing sequence for (P) if (2.8) and (2.7) hold.

Definition 2.5. (P) is said to be type I (resp: generalized type I, type II, generalized type II) LP well-posed if $\bar{\Omega} \neq \emptyset$, and for any type I (resp: generalized type I, type II, generalized type II) LP minimizing sequence $\{x_n\}$ for (P), there exist a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and $\bar{x} \in \bar{\Omega}$ such that $x_{n_j} \to \bar{x}$.

Chen, Yang and Yu [33] introduced a nonlinear scalarization function $\xi_e : X \times Y \to \mathbf{R}$ defined by:

$$\xi_e(x, y) = \inf\{\lambda \in \mathbf{R} : y \in \lambda e(x) - C(x)\}.$$

Definition 2.6. The function $h : X_1 \to \mathbf{R} \cup \{+\infty\}$ is said to be a gap function on X_0 for (GVEP) if $h(x) \geq 0, \forall x \in X_0$, and for any $x^* \in X_0$, $h(x^*) = 0$ iff $x^* \in \Omega_1$.

We define a function ϕ_1 on X_1 as follows:

$$\phi_1(x) = \inf_{z \in T(x)} \sup_{y \in X_0} \{-\xi_e(x, f(x, z, y))\}, \forall x \in X_1. \tag{2.9}$$

Now we present some properties of the function ϕ_1 which are generalizations and extensions of Lemmas 2.1 and 2.2 in [16], Propositions 4.1 and 4.2 in [25], and Lemma 1.1 in [15].

Proposition 2.2. *Assume that for any $x \in X_0$ and $z \in T(x)$, there holds $f(x, z, x) \in -\partial C(x)$, the set-valued map T is compact-valued on X_1, and for any $(x, y) \in X \times X_1$, the vector-valued function $z \mapsto f(x, z, y)$ is continuous. Then ϕ_1 defined by (2.9) is a gap function on X_0 for (GVEP).*

Proof. We now prove that $\phi_1(x) \geq 0$ for all $x \in X_0$. Suppose to the contrary that $\phi_1(x) < 0$ for some $x \in X_0$. Then, there exists a $\delta > 0$ such that $\phi_1(x) < -\delta$. By definition, for $\delta/2 > 0$, there exists a $z \in T(x)$, such that

$$\sup_{y \in X_0} \{-\xi_e(x, f(x, z, y))\} \leq \phi_1(x) + \frac{\delta}{2} < -\frac{\delta}{2} < 0$$

Thus, we have

$$\xi_e(x, f(x, z, y)) > 0, \forall y \in X_0.$$

It follows from Proposition 2.3 in [33] that

$$f(x, z, y) \notin -C(x), \forall y \in X_0,$$

which contradicts to the assumption when $y = x$.

Next we will show that for any $x \in X_0$, $\phi_1(x) = 0$ if and only if $x \in \Omega_1$. Indeed, we suppose that there exists $x \in X_0$ such that $\phi_1(x) = 0$. Then, there exist $z_n \in T(x)$ and $0 < \epsilon_n \to 0$ such that

$$\sup_{y \in X_0} \{-\xi_e(x, f(x, z_n, y))\} \leq \phi_1(x) + \epsilon_n = \epsilon_n.$$

Thus,

$$\xi_e(x, f(x, z_n, y)) \geq -\epsilon_n, \forall y \in X_0.$$

It follows from Proposition 2.3 in [33] that

$$f(x, z_n, y) + \varepsilon_n e(x) \notin -intC(x), \forall y \in X_0. \tag{2.10}$$

By the compactness of $T(x)$, there exist a sequence $\{z_{n_j}\}$ of $\{z_n\}$ and some $z \in T(x)$ such that

$$z_{n_j} \to z.$$

This fact, together with (2.10), implies that $f(x, z, y) \notin -intC(x), \forall y \in X_0$. Hence $x \in \Omega_1$.

Conversely, suppose $\tilde{x} \in \Omega_1$. Then $\tilde{x} \in X_0$, and there exists $z \in T(\tilde{x})$ such that $f(\tilde{x}, z, y) \notin -intC(x), \forall y \in X_0$. It follows from proposition 2.3 in [33] that

$$\xi_e(\tilde{x}, f(\tilde{x}, z, y)) \geq 0, \forall y \in X_0.$$

Hence

$$\phi_1(\tilde{x}) = \inf_{z \in T(\tilde{x})} \sup_{y \in X_0} \{-\xi_e(\tilde{x}, f(\tilde{x}, z, y))\} \leq 0.$$

We have proved that $\phi_1(x) \geq 0$ for all $x \in X_0$. It follows that $\phi_1(\tilde{x}) = 0$. Thus $\phi_1(x)$ is a gap function of (GVEP). This completes the proof. ∎

Proposition 2.3. *Assume that for any $y \in X_1$, the vector-valued function $(x, z) \mapsto f(x, z, y)$ is continuous, the set-valued map T is upper semi-continuous and compact-valued on X_1, and the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is upper semi-continuous. Then ϕ_1 defined by (2.9) is a lower semi-continuous function from X_1 to $\mathbf{R} \cup \{+\infty\}$. Further assume that the solution set Ω_1 of (GVEP) is nonempty, then $Dom(\phi_1) \neq \varnothing$.*

Proof. First, it is obvious that $\phi_1(x) > -\infty, \forall x \in X_1$. Otherwise, suppose that there exists $x_0 \in X_1$ satisfying $\phi_1(x_0) = -\infty$. Then, there exist $z_n \in T(x_0)$ and $\{M_n\} \subset \mathbf{R}^+$ with $M_n \to +\infty$ such that

$$\sup_{y \in X_0} \{-\xi_e(x_0, f(x_0, z_n, y))\} \leq -M_n.$$

Hence,

$$\xi_e(x_0, f(x_0, z_n, y)) \geq M_n, \forall y \in X_0.$$

By the compactness of $T(x_0)$, there exist a sequence $\{z_{n_j}\}$ of $\{z_n\}$ and some $z \in T(x_0)$ such that

$$z_{n_j} \to z.$$

It follows from Theorem 2.1 in [33] that ξ_e is upper semi-continuous, and so

$$\xi_e(x_0, f(x_0, z, y)) \geq \limsup_{j \to +\infty} \xi_e(x_0, f(x_0, z_{n_j}, y)) = +\infty, \forall y \in X_0,$$

which is impossible, since $\xi_e(x_0,\cdot)$ is a finite function on Y.

Second, we show that ϕ_1 is lower semi-continuous on X_1. Let $a \in R$, suppose that $\{x_n\} \subset X_1$ satisfies $\phi_1(x_n) \leq a, \forall n$ and $x_n \to x_0$. It follows that for each n there exist $z_n \in T(x_n)$ and $0 < \delta_n \to 0$ such that

$$\xi_e(x_n, f(x_n, z_n, y)) \geq -a - \delta_n, \forall y \in X_0. \tag{2.11}$$

By the upper semi-continuity of T at x_0 and the compactness of $T(x_0)$, we obtain a sequence $\{z_{n_j}\}$ of $\{z_n\}$ and some $z_0 \in T(x_0)$ such that $z_{n_j} \to z_0$. It follows from Theorem 2.1 in [33] and (2.11) that

$$\xi_e(x_0, f(x_0, z_0, y)) \geq \limsup_{j\to+\infty} \xi_e(x_0, f(x_0, z_{n_j}, y)) \geq -a, \forall y \in X_0,$$

which implies that $\phi_1(x_0) \leq a$. This completes the proof. ∎

The following result shows the equivalent relationship between the various types of LP well-posedness of (P) and the corresponding ones of LP well-posedness of (GVEP), which is a generalization of Lemma 2.3 in [16] and Theorem 3.13 in [18].

Theorem 2.1. *Assume that for any $x \in X_0$ and $z \in T(x)$, there holds $f(x,z,x) \in -\partial C(x)$, the set-valued map T is compact-valued on X_1, and for any $(x,y) \in X \times X_1$, the vector-valued map $z \mapsto f(x,z,y)$ is continuous, the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is upper semi-continuous and the function $\phi(x)$ is replaced by $\phi_1(x)$ defined by (2.9). Then (GVEP) is type I (resp. generalized type I, type II, generalized type II) LP well-posed if and only if (P) is type I (resp. generalized type I, type II, generalized type II) LP well-posed.*

Proof. We only need to prove that (GVEP) is type I LP well-posed if and only if (P) is type I LP well-posed. The others can be proved similarly and they are omitted here.

By Proposition 2.2, we know that ϕ_1 is a gap function of (GVEP) on X_0, $\bar{x} \in \Omega_1$ if and only if $\bar{x} \in X_0$ with $\bar{v} = \phi_1(\bar{x}) = 0$.

Assume that $\{x_n\}$ is any type I LP approximating solution sequence for (GVEP). Then there exist $\epsilon_n > 0$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that (2.1) and (2.2) hold. It follows from (2.1) that (2.6) holds. It follows from proposition 2.3 in [33] and (2.2) that

$$\xi_e(x_n, f(x_n, z_n, y)) \geq -\epsilon_n, \forall y \in X_0.$$

Hence, we obtain

$$\phi_1(x_n) = \inf_{z\in T(x_n)} \sup_{y\in X_0} \{-\xi_e(x_n, f(x_n, z, y))\} \leq \epsilon_n.$$

Thus,

$$\limsup_{n\to\infty} \phi_1(x_n) \leq 0 \text{ since } \epsilon_n \to 0,$$

which implies that $\{x_n\}$ is a type I LP approximating solution sequence for (P).

Conversely, assume that $\{x_n\}$ is any type I LP approximating solution sequence for (P). Then $d(x_n, X_0) \to 0$ and $\limsup_{n\to\infty} \phi_1(x_n) \leq 0$.

Thus, there exists $\epsilon_n > 0$ with $\epsilon_n \to 0$ satisfying (2.1) and

$$\phi_1(x_n) = \inf_{z\in T(x_n)} \sup_{y\in X_0} \{-\xi_e(x_n, f(x_n, z, y))\} \leq \epsilon_n.$$

It follows from the upper semi-continuity of ξ_e, we know that

$$\exists z_n \in T(x_n),\ s.t.\ \xi_e(x_n, f(x_n, z_n, y)) \geq -\epsilon_n, \forall y \in X_0.$$

Equivalently, (2.2) holds. Hence, $\{x_n\}$ is a type I LP approximating solution sequence for (GVEP). Hence, (GVEP) is type I LP well-posed if and only if (P) is type I LP well-posed. This completes the proof. ■

3. Criteria and Characterizations for LP Well-posednesses of (GVEP)

In this section, we present necessary and/or sufficient conditions for those types of (generalized) LP well-posedness of (GVEP) defined in section 2.

Now we introduce the Kuratowski measure of noncompactness for a nonempty subset A of X (see [34]) defined by

$$\alpha(A) = \inf\{\epsilon > 0 : A \subset \cup_{i=1}^{n} A_i,\ \text{for every } A_i, \mathrm{diam} A_i < \epsilon\},$$

where $diamA_i$ is the diameter of A_i defined by

$$\mathrm{diam} A_i = \sup\{d(x_1, x_2) : x_1, x_2 \in A_i\}.$$

Given two nonempty subsets A and B of X, the excess of set A to set B is defined by

$$e(A, B) = \sup\{d(a, B) : a \in A\}$$

and the Hausdorff distance between A and B is defined by

$$H(A, B) = \max\{e(A, B), e(B, A)\}.$$

For any $\epsilon > 0$, two types of approximating solution sets for (GVEP) are defined by

$$\Theta_1(\epsilon) := \{x \in X_1 : d(x, X_0) \leq \epsilon \text{and } \exists z \in T(x), s.t.\ f(x, z, y) + \epsilon e(x) \notin -intC(x), \forall y \in X_0\},$$

$$\Theta_2(\epsilon) := \{x \in X_1 : d(g(x), K) \leq \epsilon and\ \exists z \in T(x), s.t.\ f(x,z,y) + \epsilon e(x) \notin -intC(x), \forall y \in X_0\}.$$

Now we will present some metric characterizations of various types of LP well-posedness of (GVEP).

Theorem 3.1. *Assume that for any $y \in X_1$, the vector-valued function $(x, z) \mapsto f(x, z, y)$ is continuous, the set-valued map T is upper semi-continuous and compact-valued on X_1 and the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is closed. Then the following results hold:*

(a) $(GVEP)$ is type I LP well-posed if and only if the solution set Ω_1 is nonempty, compact and

$$e(\Theta_1(\epsilon), \Omega_1) \to 0 \text{ as } \epsilon \to 0. \tag{3.1}$$

(b) $(GVEP)$ is type I LP well-posed if and only if

$$\Theta_1(\epsilon) \neq \varnothing, \forall \epsilon > 0 \text{ and } \lim_{\epsilon \to 0} \alpha(\Theta_1(\epsilon)) = 0. \tag{3.2}$$

(c) $(GVEP)$ is generalized type I LP well-posed if and only if the solution set Ω_1 is nonempty, compact and

$$e(\Theta_2(\epsilon), \Omega_1) \to 0 \text{ as } \epsilon \to 0.$$

(d) $(GVEP)$ is generalized type I LP well-posed if and only if

$$T_2(\epsilon) \neq \varnothing, \forall \epsilon > 0 \text{ and } \lim_{\epsilon \to 0} \alpha(T_2(\epsilon)) = 0.$$

Proof. We only prove (a) and (b). The proofs of (c) and (d) are similar and they are omitted here.

(a) Let (GVEP) be type I LP well-posed. Then Ω_1 is nonempty and compact. Now we show that (3.1) holds. Suppose to the contrary that there exist $l > 0$, $\epsilon_n > 0$ with $\epsilon_n \to 0$ and $x_n \in \Theta_1(\epsilon_n)$ such that

$$d(x_n, \Omega_1) \geq l. \tag{3.3}$$

Since $\{x_n\} \subset \Theta_1(\epsilon_n)$ we know that $\{x_n\}$ is type I LP approximating solution for (GVEP). By the type I LP well-posedness of (GVEP), there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ converging to some element of Ω_1. This contradicts (3.3). Hence (3.1) holds.

Conversely, suppose that Ω_1 is nonempty, compact and (3.1) holds. Let $\{x_n\}$ be a type I LP approximating solution for (GVEP). Then there exist a sequence $\{\epsilon_n\}$

with $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ and $\epsilon_n \to 0$ such that (2.1) and (2.2) hold. Thus, $\{x_n\} \subset \Theta_1(\epsilon)$. It follows from (3.1) that there exists a sequence $\{\omega_n\} \subseteq \Omega_1$ such that

$$d(x_n, \omega_n) = d(x_n, \Omega_1) \leq e(T_1(\epsilon), \Omega) \to 0.$$

Since Ω_1 is compact, there exists a subsequence $\{\omega_{n_k}\}$ of $\{\omega_n\}$ converging to $x_0 \in \Omega_1$. And so the corresponding subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converges to x_0. Therefore (GVEP) is type I LP well-posed.

(b) First we show that for every $\epsilon > 0$, $\Theta_1(\epsilon)$ is closed. In fact, let $\{x_n\} \subset \Theta_1(\epsilon)$ and $x_n \to \bar{x}$. Then

$$d(x_n, X_0) \leq \epsilon, \tag{3.4}$$

and

$$\exists z_n \in T(x_n), s.t.\ f(x_n, z_n, y) + \epsilon e(x_n) \notin -intC(x_n), \forall y \in X_0. \tag{3.5}$$

From (3.4) and (3.5), we get

$$d(\bar{x}, X_0) \leq \epsilon,$$

and

$$f(x_n, z_n, y) + \epsilon e(x_n) \in W(x_n), \forall y \in X_0. \tag{3.6}$$

By the upper semi-continuity of T at $\bar{x}$ and the compactness of $T(\bar{x})$, there exist a subsequence $\{z_{n_j}\} \subset \{z_n\}$ and some $\bar{z} \in T(\bar{x})$ such that $z_{n_j} \to \bar{z}$. It follows from (3.6) that $f(\bar{x}, \bar{z}, y) + \epsilon e(\bar{x}) \notin -intC(\bar{x}), \forall y \in X_0$. Hence $\bar{x} \in \Theta_1(\epsilon)$.

Second, we show that

$$\Omega_1 = \cap_{\epsilon>0} \Theta_1(\epsilon). \tag{3.7}$$

It is obvious that $\Omega_1 \subset \cap_{\epsilon>0} \Theta_1(\epsilon)$.

Now suppose that $\epsilon_n > 0$ with $\epsilon_n \to 0$ and $x^* \in \cap_{n=1}^{\infty} \Theta_1(\epsilon_n)$. Then

$$d(x^*, X_0) \leq \epsilon_n, \forall n \in \mathbf{N}, \tag{3.8}$$

and

$$\exists z \in T(x^*), s.t.\ f(x^*, z, y) + \epsilon_n e(x^*) \notin -intC(x^*), \forall y \in X_0. \tag{3.9}$$

It is easy to see that X_0 is closed. By (3.8), we get $x^* \in X_0$. By (3.9) and closedness of $W(x^*)$, we know that

$$\exists z \in T(x^*), s.t.\ f(x^*, z, y) \in W(x^*), \forall y \in X_0.$$

That is, $x^* \in \Omega_1$. Hence (3.7) holds.

Now we assume that (3.1) holds. Clearly, $\Theta_1(.)$ is increasing with $\epsilon > 0$. By the Kuratowski theorem (see [34]), we have

$$H(\Theta_1(\epsilon), \Omega_1) \to 0, \text{ as } \epsilon \to 0.$$

where $\Omega_1 = \cap_{\epsilon>0}\Theta_1(\epsilon)$ is nonempty and compact. Since

$$H(\Theta_1(\epsilon), \Omega_1) = \max\{e(\Theta_1(\epsilon), \Omega_1), e(\Omega_1, \Theta_1(\epsilon))\} = e(\Theta_1(\epsilon), \Omega_1),$$

we get $\lim_{\epsilon\to 0} e(\Theta_1(\epsilon), \Omega_1) \to 0$. It follows from (a) that (GVEP) is type I LP well-posed.

Conversely, let (GVEP) be type I LP well-posed. Note that for every $\epsilon > 0$, we have

$$\alpha(\Theta_1(\epsilon)) \leq 2H(\Theta_1(\epsilon), \Omega_1) + \alpha(\Omega_1) = 2e(\Theta_1(\epsilon), \Omega_1),$$

where $\alpha(\Omega_1) = 0$ since Ω_1 is compact. It follows from (a) that $\lim_{\epsilon\to 0}\alpha(\Theta_1(\epsilon)) = \lim_{\epsilon\to 0} e(\Theta_1(\epsilon), \Omega_1) = 0$. This completes the proof. ∎

Theorem 3.2. *Let X be finite dimensional. Assume that for any $y \in X_1$, the vector-valued map $(x, z) \mapsto f(x, z, y)$ is continuous, the set-valued map T is upper semi-continuous and compact-valued on X_1, the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is closed, and Ω_1 is nonempty.*

(i) *If there exists $\epsilon_0 > 0$ such that $\Theta_1(\epsilon_0)$ is bounded, then (GVEP) is type I LP well-posed.*

(ii) *If there exists $\epsilon_0 > 0$ such that $\Theta_2(\epsilon_0)$ is bounded, then (GVEP) is generalized type I LP well-posed.*

Proof. We only prove (i). The proof of (ii) is similar and they are omitted here. Let $\{x_n\}$ be a type I LP approximating solution sequence for (GVEP). Then there exist a sequence $\{\epsilon_n\}$ with $\{\epsilon_n\} \subseteq \mathbf{R}_+$ and $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that (2.1) and (2.2) hold. From (2.1) and (2.2), without loss of generality, we can assume that $\{x_n\} \subset \Theta_1(\epsilon_0)$. Hence, $\{x_n\}$ is bounded. Since X is finite dimensional, let $\{x_{n_j}\}$ be any subsequence of $\{x_n\}$ such that $x_{n_j} \to \bar{x} \in X_1$. From (2.1) and (2.2), we get

$$d(x_{n_j}, X_0) \leq \epsilon_{n_j}, \tag{3.10}$$

and

$$\exists z_{n_j} \in T(x_{n_j}), \text{ s.t. } f(x_{n_j}, z_{n_j}, y) + \epsilon_{n_j} e(x_{n_j}) \notin -intC(x_{n_j}), \forall y \in X_0. \tag{3.11}$$

Since X_0 is closed and by (3.10), we get $\bar{x} \in X_0$. By the upper semi-continuity of T at $\bar{x}$ and the compactness of $T(\bar{x})$, there exist a subsequence $\{z_{n_{j_k}}\} \subset \{z_{n_j}\}$

and some $\bar{z} \in T(\bar{x})$ such that $z_{n_{j_k}} \to \bar{z}$. It follows from (3.12) that $f(\bar{x}, \bar{z}, y) \notin -intC(\bar{x}), \forall y \in X_0$. Hence, $\bar{x} \in \Omega_1$ and (GVEP) is type I LP well-posed. This completes the proof. ∎

Corollary 3.1. *Assume that for any $y \in X_1$, the vector-valued function $(x, z) \mapsto f(x, z, y)$ is continuous, the set-valued map T is upper semi-continuous and compact-valued on X_1, the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is closed, and there exists $\epsilon_0 > 0$ such that $\Theta_1(\epsilon_0)$ (resp. $\Theta_2(\epsilon_0)$) is compact. If Ω_1 is nonempty, then (GVEP) is type I (resp. generalized type I) LP well-posed.*

Proof. The proof is similar to that of Theorem 3.3 and is omitted. This completes the proof. ∎

Remark 3.1 Theorems 3.1 is an extension and generalization of Theorem 2.3 in [14], Theorem 2.3 in [15], Lemma 2.6 in [16], Theorems 3.1 and 3.4-3.5 in [17], Theorems 3.1 and 3.2 in [18], and Theorem 3.1 in [25]. Theorem 3.2 and Corollary 3.1, respectively, extend and generalize Theorem 3.3 and Corollary 3.1 in [25], Theorem 3.6 and Corollary 3.7 in [18].

The following results show the equivalent relations between the (generalized) type II LP well-posedness of (GVEP) and the (generalized) type II LP well-posedness of (P).

Now we consider a real-valued function $c = c(t, s)$ defined for $t, s \geq 0$ sufficiently small, such that

$$c(t, s) \geq 0, \forall t, s,\ c(0, 0) = 0, \tag{3.12}$$

$$s_n \to 0,\ t_n \geq 0,\ c(t_n, s_n) \to 0 \text{ imply } t_n \to 0. \tag{3.13}$$

The following theorem follows immediately from Theorem 2.1 in [6] and Theorem 2.1 with $\bar{v} = 0$.

Theorem 3.3. *Assume that for any $x \in X_0$ and $z \in T(x)$, there holds $f(x, z, x) \in -\partial C(x)$, the set-valued map T is compact-valued on X_1, and for any $(x, y) \in X \times X_1$, the vector-valued map $z \mapsto f(x, z, y)$ is continuous, the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is upper semi-continuous and the function $\phi(x)$ is replaced by $\phi_1(x)$ defined by (2.9).*

(i) If (GVEP) is type II LP well-posed, then there exists a function c satisfying (3.13) and (3.14) such that

$$|\phi_1(x)| \geq c(d(x, \Omega), d(x, X_0)), \forall x \in X_1. \tag{3.14}$$

(ii) *If* Ω_1 *is nonempty compact, and* (3.15) *holds for some* c *satisfying* (3.13) *and* (3.14)*, then* $(GVEP)$ *is type II LP well-posed.*

The following theorem follows immediately from Theorem 2.2 in [6] and Theorem 2.1 with $\bar{v} = 0$.

Theorem 3.4. *Assume that for any* $x \in X_0$ *and* $z \in T(x)$*, there holds* $f(x, z, x) \in -\partial C(x)$*, the set-valued map* T *is compact-valued on* X_1*, and for any* $(x, y) \in X \times X_1$*, the vector-valued map* $z \mapsto f(x, z, y)$ *is continuous, the set-valued map* $W : X \to 2^Y$ *defined by* $W(x) = Y \setminus -intC(x)$ *is upper semi-continuous and the function* $\phi(x)$ *is replaced by* $\phi_1(x)$ *defined by* (2.9)*.*

(i) *If* $(GVEP)$ *is generalized type II LP well-posed, then there exists a function* c *satisfying* (3.13) *and* (3.14) *such that*

$$(3.15) \qquad |\phi_1(x)| \geq c(d(x, \Omega_1), d(g(x), K)), \forall x \in X_1;$$

(ii) *If* Ω_1 *is nonempty compact, and* (3.16) *holds for some* c *satisfying* (3.13) *and* (3.14)*, then* $(GVEP)$ *is generalized type II LP well-posed.*

It is easy to see that Theorems 3.4 and 3.5 generalize and extend the corresponding results in [14-18] and [25].

Definition 3.5.

(i) Let Z be a topological space and let $Z_1 \subset Z$ be a nonempty subset. Suppose that $G : Z \to \mathbf{R} \cup \{+\infty\}$ is an extend real-valued function. Then function G is said to be level-compact on Z_1 if for any $s \in \mathbf{R}$ the subset $\{z \in Z_1 : G(z) \leq s\}$ is compact.

(ii) Let Z be a finite dimensional normed space and $Z_1 \subset Z$ be nonempty. A function $h : Z \to \mathbf{R} \cup \{+\infty\}$ is said to be level-bounded on Z_1 if Z_1 is bounded or

$$\lim_{z \in Z_1, ||z|| \to +\infty} h(z) = +\infty.$$

Now we give some sufficient conditions for the (generalized) type I LP well-posedness of (GVEP) as follows:

Proposition 3.1. *Assume that for any* $y \in X_1$*, the vector-valued map* $(x, z) \mapsto f(x, z, y)$ *is continuous, the set-valued map* T *is upper semi-continuous and compact-valued on* X_1*, the set-valued map* $W : X \to 2^Y$ *defined by* $W(x) = Y \setminus -intC(x)$ *is upper semi-continuous, and* Ω_1 *is nonempty. Then,* $(GVEP)$ *is type I LP well-posed if one of the following conditions holds:*

(i) *there exists* $\delta_1 > 0$ *such that* $S(\delta_1)$ *is compact, where*

$$S(\delta_1) = \{x \in X_1 : d(x, X_0) \le \delta_1\}; \tag{3.16}$$

(ii) *the function* ϕ_1 *defined by* (*2.9*) *is level-compact on* X_1*;*

(iii) X *is a finite-dimensional normed space and*

$$\lim_{x \in X_1, ||x|| \to +\infty} \max\{\phi_1(x), d(x, X_0)\} = +\infty; \tag{3.17}$$

(iv) *there exists* $\delta_1 > 0$ *such that* ϕ_1 *is level-compact on* $S(\delta_1)$ *defined by* (*3.17*)*;*

Proof. It is easy to see that condition (i) and (ii) imply condition (iv). Now we show that condition (iii) implies condition (iv). It follows from Proposition 2.3 that the function ϕ_1 defined by (2.9) is lower semi-continuous, and thus for any $t \in \mathbf{R}$, the set $\{x \in S(\delta_1) : \phi(x) \le t\}$ is closed. Since X is a finite dimensional space, we only need to show that for any $t \in \mathbf{R}$, the set $\{x \in S(\delta_1) : \phi(x) \le t\}$ is bounded. Suppose to the contrary, there exist $t \in \mathbf{R}$ and $\{x'_n\} \subset S(\delta_1)$ and $\phi(x'_n) \le t$ such that $||x'_n|| \to +\infty$. It follows from $\{x'_n\} \subset S(\delta_1)$ that $d(x'_n, X_0) \le \delta_1$ and so

$$\max\{\phi(x'_n), d(x'_n, X_0)\} \le \max\{t, \delta_1\},$$

which contradicts with (3.18).

Therefore, we only need to prove that if condition (iv) holds, then (GVEP) is type I LP well-posed. Suppose that condition (iv) holds and $\{x_n\}$ is a type I LP approximating solution sequence for (GVEP). Then there exist $\{\epsilon_n\} \subset \mathbf{R}_+$ with $\epsilon_n > 0$ and $z_n \in T(x_n)$ such that (2.1) and (2.2) hold. By (2.1), we can assume without loss of generality that $\{x_n\} \subset S(\delta_1)$. It follows from (2.2) that $\xi_e(x_n, f(x_n, z_n, y)) \ge -\epsilon_n, \forall y \in X_0$. Thus

$$\phi(x_n) \le \epsilon_n, \forall n. \tag{3.18}$$

From (3.19), without loss of generality that $\{x_n\} \subseteq \{x \in S(\delta_1) : \phi(x) \le b\}$ for some $b > 0$. Since ϕ is level-compact on $S(\delta_1)$, the subset $\{x \in S(\delta_1) : \phi(x) \le b\}$ is compact. It follows that there exist a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and $\bar{x} \in S(\delta_1)$ such that $x_{n_j} \to \bar{x}$. The rest of the proof is similar with that of Theorem 3.3. This completes the proof. ∎

Similarly, we can prove the following results:

Proposition 3.2. *Assume that for any* $y \in X_1$*, the vector-valued map* $(x, z) \mapsto f(x, z, y)$ *is continuous, the set-valued map* T *is upper semi-continuous and compact-valued on* X_1*, the set-valued map* $W : X \to 2^Y$ *defined by* $W(x) = Y \setminus -intC(x)$

is upper semi-continuous, and Ω_1 *is nonempty. Then,* $($*GVEP*$)$ *is generalized type I LP well-posed if one of the following conditions holds:*

(i) *there exists* $\delta_1 > 0$ *such that* $S_1(\delta_1)$ *is compact where*

$$S_1(\delta_1) = \{x \in X_1 : d(g(x), K) \le \delta_1\}; \tag{3.20}$$

(ii) *the function* ϕ_1 *defined by* $($*2.9*$)$ *is level-compact on* X_1*;*

(iii) X *is a finite-dimensional normed space and*

$$\lim_{x \in X_1, ||x|| \to +\infty} max\{\phi_1(x), d(g(x), K)\} = +\infty;$$

(iv) *there exists* $\delta_1 > 0$ *such that* ϕ_1 *is level-compact on* $S_1(\delta_1)$ *defined by* $($*3.19*$)$*.*

Definition 3.5. Let $\emptyset \ne D \subset X_1$. A vector-valued map $t(x)$ from D to Z (resp. $L(X,Y)$) is called a selection of the set-valued map T (resp. Q) if $t(x) \in T(x)(\text{resp.}, \ t(x) \in Q(x)) \ \forall \in D$.

Proposition 3.3. *Let* X *be finite dimensional. Assume that for any* $y \in X_1$*, the vector-valued map* $(x,z) \mapsto f(x,z,y)$ *is continuous, the set-valued map* T *is upper semi-continuous and compact-valued on* X_1*, the set-valued map* $W : X \to 2^Y$ *defined by* $W(x) = Y \setminus -intC(x)$ *is closed, and* Ω_1 *is nonempty. If there exist* $\delta_1 > 0$ *and* $x_0 \in X_0$ *such that*

$$\lim_{x \in S(\delta_1), \|x\| \to +\infty} \xi_e(x, f(x, z(x), x_0)) = -\infty, \tag{3.20}$$

for any selection $z(x)$ *of* T*, where* $S(\delta_1)$ *is defined by* $($*3.17*$)$*, then,* $($*GVEP*$)$ *is type I LP well-posed.*

Proof. Let $\{x_n\}$ be a type I LP approximating solution sequence for (GVEP). Then there exists $\{\epsilon_n\} \subset \mathbf{R}_+$ with $\epsilon_n > 0$ and $z_n \in T(x_n)$ such that (2.1) and (2.2) hold. By (2.1), we can assume without loss of generality that $\{x_n\} \subset S(\delta_1)$. It follows from (2.2) that

$$\xi_e(x_n, f(x_n, z_n, y)) \ge -\epsilon_n, \forall y \in X_0. \tag{3.21}$$

Next we show that $\{x_n\}$ is bounded. Otherwise, we assume without loss of generality that $\| x_n \| \to +\infty$. By (3.21), We have

$$\lim_{n \to +\infty} \xi_e(x_n, f(x_n, z_n, x_0)) = -\infty,$$

contradicting (3.22) (with y is replaced by x_0) when n is sufficiently large. Consequently, we can assume without loss of generality that $x_n \to \bar{x} \in X_1$. This fact, together with (2.1), yields $\bar{x} \in X_0$. Furthermore, from the upper semi-continuity of T at $\bar{x}$, the compactness of $T(\bar{x})$, we deduce that there exist $\{z_{n_j}\} \subset \{z_n\}$ and some $z \in T(\bar{x})$ such that $z_{n_j} \to \bar{t}$. Taking the limit in (2.2) with z_n replaced by z_{n_j} as $j \to +\infty$, by the continuity of f and the closedness of W, we have $\bar{x} \in \Omega_1$.

Similarly, we can prove the next proposition.

Proposition 3.4. *Let X be finite dimensional. Assume that for any $y \in X_1$, the vector-valued map $(x, z) \mapsto f(x, z, y)$ is continuous, the set-valued map T is upper semi-continuous and compact-valued on X_1, the set-valued map $W : X \to 2^Y$ defined by $W(x) = Y \setminus -intC(x)$ is closed, and Ω_1 is nonempty. If there exist $\delta_1 > 0$ and $x_0 \in X_0$ such that*

$$\lim_{x \in S_1(\delta_1), \|x\| \to +\infty} \xi_e(x, f(x, z(x), x_0)) = -\infty,$$

for any selection $z(x)$ of T, where $S_1(\delta_1)$ is defined by (3.20), then, (GVEP) is generalized type I LP well-posed.

Remark 3.2.

(i) If X is a finite dimensional space, then the ”level-compactness” condition in Propositions 3.1 - 3.2 can be replaced by the ”level-boundedness” condition.

(ii) Propositions 3.1-3.4 are generalizations of Propositions 2.2-2.5 in [14] and [16], Propositions 2.1, 2.2 and 2.5 in [15], and Propositions 4.2-4.4 and 4.6 in [17]. Propositions 3.1 and 3.2, respectively, generalize and extend Propositions 3.18 and 3.17 in [18].

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