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ON THE INTEGRAL CONVOLUTION OF CERTAIN CLASSES OF ANALYTIC FUNCTIONS

Urszula Bednarz and Janusz Sokół

Abstract. In this paper we consider the convolution properties of the classes of *k*-uniformly convex and *k*-starlike functions which have been introduced in [4], [6]. We investigate the problem of stability of integral convolution on certain pairs of such classes and we give the upper and lower bounds of their radius of stability. In the present paper we improve some of the results obtained in [1].

1. Introduction

Let $\mathcal{A}$ denote the class of functions f of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

which are analytic in the open unit disk $\mathcal{U} = \{z : z \in \mathbf{C} \text{ and } |z| < 1\}$. As usual, we denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions which are also univalent in $\mathcal{U}$. Furthermore, we denote by k-$\mathcal{UCV}$ and k-$\mathcal{ST}$ two interesting subclasses of $\mathcal{S}$ consisting, respectively, of functions which are *k*-uniformly convex and *k*-starlike in $\mathcal{U}$. Thus we have

$$k\text{-}\mathcal{UCV} := \left\{ f \in \mathcal{S} : \operatorname{Re}\left[1 + \frac{zf''(z)}{f'(z)}\right] > k\left|\frac{zf''(z)}{f'(z)}\right|, \quad (z \in \mathcal{U}; 0 \le k < \infty) \right\},$$

$$k\text{-}\mathcal{ST} := \left\{ f \in \mathcal{S} : \quad \operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] > k\left|\frac{zf'(z)}{f(z)} - 1\right|, \quad (z \in \mathcal{U}; 0 \le k < \infty) \right\}.$$

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The class k-$\mathcal{UCV}$ was introduced by Kanas and Wiśniowska [4], where its geometric definition and connections with the conic domains were considered. The class k-$\mathcal{UCV}$ was defined pure geometrically as a subclass of univalent functions, that map each circular arc contained in the unit disk $\mathcal{U}$ with a center $\xi, |\xi| \le k \ (0 \le k < \infty)$, onto a convex arc. It can be seen that k-uniformly convex functions are functions with positive curvature on each circular arc contained in $\mathcal{U}$ and centered at any point ξ, $|\xi| \le k$ (see [4]). The notion of k-uniformly convex function is a natural extension of the classical convexity. Observe that, if $k = 0$ then the center ξ is the origin and the class k-$\mathcal{UCV}$ reduces to the class of convex univalent functions $\mathcal{CV}$. Moreover for $k = 1$ corresponds to the class of uniformly convex functions $\mathcal{UCV}$ introduced by Goodman [2] and studied extensively by Rønning [10] and independently by Ma and Minda [8]. The class k-$\mathcal{ST}$ was investigated in [6]. Note that the case $k = 0$ coincides with the usual case of starlike functions $\mathcal{ST}$ and if we take $k = 1$ we recover the class called $\mathcal{S}_p$ by Rønning [10]. The properties of these classes have been studied in [3], [5], [7]. The function $g(z) = z/(1 - Az)^2$ is in the class k-$\mathcal{ST}$ if and only if $|A| \le 1/(2k + 1)$, thus the Koebe function $k(z) = z/(1 - e^{i\theta})^2$ is not k-starlike for any $k > 0$. Similarly, the function $h(z) = z/(1 - Bz)$ is in $\mathcal{UCV}$ if and only if $|B| \le 1/3$ (see [2]). The class k-$\mathcal{ST}$ is related to the class k-$\mathcal{UCV}$ by means of the well-known Alexander equivalence between the usual classes of convex $\mathcal{CV}$ and starlike $\mathcal{ST}$ functions (see also the work of Kanas and Srivastava [7] for further developments involving each of the classes k-$\mathcal{UCV}$ and k-$\mathcal{ST}$). In [1] the authors studied the properties of the integral convolution of the neighborhoods of these classes.

Recall that the Hadamard product or convolution of two power series

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad \text{and} \quad g(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

is defined as

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n \tag{1.1}$$

and the integral convolution is defined by

$$(f \otimes g)(z) = z + \sum_{n=2}^{\infty} \frac{a_n b_n}{n} z^n. \tag{1.2}$$

Note that by (1.1) and (1.2) we have

$$(f \otimes g)(z) = \int_0^z \frac{(f * g)(t)}{t} dt,$$

therefore

$$(f \otimes g)(z) \in k\text{-}\mathcal{UCV} \Longleftrightarrow (f * g)(z) \in k\text{-}\mathcal{ST}. \tag{1.3}$$

In accordance to Ruscheweyh [12], let $\mathcal{V}^*$ denote the dual set of $\mathcal{V} \subset \mathcal{A}$. Then

$$\mathcal{V}^* = \left\{ g \in \mathcal{A} : \frac{(f * g)(z)}{z} \neq 0,\ \forall f \in \mathcal{V},\ \forall z \in \mathcal{U} \right\}.$$

Dual sets for the classes k-$\mathcal{ST}$ and k-$\mathcal{UCV}$ were found in [3], [4]. Let us denote the dual set for k-$\mathcal{ST}$ by $\mathcal{B}$ and for k-$\mathcal{UCV}$ by $\mathcal{G}$. Then for $f \in \mathcal{A}$

$$f \in k\text{-}\mathcal{ST} \Longleftrightarrow \frac{(f * h)(z)}{z} \neq 0,\ \forall h \in \mathcal{B},\ \forall z \in \mathcal{U}$$

and

$$f \in k\text{-}\mathcal{UCV} \Longleftrightarrow \frac{(f * h)(z)}{z} \neq 0,\ \forall h \in \mathcal{G},\ \forall z \in \mathcal{U},$$

respectively. For $h(z) = z + c_2 z^2 + \ldots \in \mathcal{B}$ we have the following estimates [3]

$$|c_n| \leq n + (n-1)k, \quad n \geq 2 \tag{1.4}$$

and if $h \in \mathcal{G}$, then

$$|c_n| \leq n[n + (n-1)k], \quad n \geq 2. \tag{1.5}$$

For $\delta \geq 0$ Ruscheweyh [11] defined N_δ-neighborhood of a function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ by

$$N_\delta(f) = \left\{ g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A} : \sum_{n=2}^{\infty} n|a_n - b_n| \leq \delta \right\}.$$

By $N_\delta(A), A \subset \mathcal{A}$, we denote the union of all neighborhoods $N_\delta(f)$ with f ranging over the class A. Ruscheweyh proved certain inclusions for the neighborhood mentioned above, in particular that $N_{1/4}(f) \subset \mathcal{ST}$ holds for all $f \in \mathcal{CV}$.

Assume that A, B are subclasses of the class $\mathcal{A}$. Then the set of all function $f * g$ and $f \otimes g$, where $f \in A$ and $g \in B$, will be denoted by $A * B$ and $A \otimes B$, respectively. Let $A * B \subset C$, the Hadamard product is called stable on the pair of classes (A, B) if there exists $\delta > 0$ such that $N_\delta(A) * N_\delta(B) \subset C$ and unstable otherwise (see [9]). Stability of integral convolution is defined in a similar way. The constant δ which characterizes the stability of Hadamard or integral convolution is called the radius of stability and it is defined as follows.

Definition 1. Let A, B, C be the subclasses of the class $\mathcal{A}$ and $A * B \subset C$. Then a constant $\delta(A * B, C)$, such that

$$\delta(A * B, C) = \sup\{\delta : N_\delta(A) * N_\delta(B) \subset C\} \tag{1.6}$$

is called the radius of stability of the convolution on the pair (A, B). And a constant $\delta(A \otimes B, C)$, such that

$$\delta(A \otimes B, C) = \sup\{\delta : N_\delta(A) \otimes N_\delta(B) \subset C\} \tag{1.7}$$

is called the radius of stability of the integral convolution on the pair (A, B).

If the related value (1.6) or (1.7) is positive, then there exists a neighbourhood of the class A and B mapped by the convolution or the integral convolution into C. Problem of stability of the Hadamard product and the integral convolution in the classes of univalent, starlike and convex functions was considered by Nezhmetdinov [9]. Numerous results concerning the stability of the Hadamard product and the integral convolution were obtained by Bednarz and Kanas (see [1], [3]).

In the present paper we investigate the problem of stability of integral convolution on the pairs $((k-\mathcal{ST}), \mathcal{CV})$, $((k-\mathcal{UCV}), \mathcal{CV})$ and we give the upper and lower bounds of their radius of stability. We improve some of the results obtained in [1].

2. Main Results

In order to establish our main theorems, we shall require the following lemmas.

Lemma 1. (see [3]). *Let* $0 \le k < \infty$. *If* $f \in k-\mathcal{UCV}$, *then* $F(z) = \frac{f(z)+\varepsilon z}{1+\varepsilon} \in k-\mathcal{ST}$ *for* $|\varepsilon| < \frac{1}{4}$.

Lemma 2. (see [5]).

(*i*) $(k-\mathcal{UCV}) * \mathcal{CV} = k-\mathcal{UCV}$,

(*ii*) $(k-\mathcal{ST}) * \mathcal{CV} = k-\mathcal{ST}$,

(*iii*) $(k-\mathcal{UCV}) * \mathcal{ST} = k-\mathcal{ST}$.

Let us denote (see [5])

$$P_1(k) = \begin{cases} \dfrac{8(\arccos k)^2}{\pi^2(1-k^2)} & \text{for} \quad 0 \le k < 1 \\ \dfrac{8}{\pi^2} & \text{for} \quad k = 1 \\ \dfrac{\pi^2}{4\sqrt{t}(1+t)(k^2-1)\mathcal{K}^2(t)} & \text{for} \quad k > 1 \end{cases},$$

where $t \in (0,1)$ is determined by $k = \cosh(\pi\mathcal{K}'(t)/[4\mathcal{K}(t)])$, $\mathcal{K}$ is the Legendre's complete Elliptic integral of the first kind

$$\mathcal{K}(t) = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-t^2x^2)}}$$

and $\mathcal{K}'(t) = \mathcal{K}(\sqrt{1-t^2})$ is the complementary integral of $\mathcal{K}(t)$. Let Ω_k be a domain such that $1 \in \Omega_k$ and

$$\partial\Omega_k = \left\{w = u + iv : \quad u^2 = k^2(u-1)^2 + k^2v^2\right\}, \quad 0 \le k < \infty.$$

The domain Ω_k is elliptic for $k > 1$, hyperbolic when $0 < k < 1$, parabolic when $k = 1$, and a right half-plane when $k = 0$. If p is an analytic function with $p(0) = 1$ which maps the unit disc $\mathcal{U}$ conformally onto the region Ω_k, then $P_1(k) = p'(0)$. $P_1(k)$ is strictly decreasing function of the variable k and it values are included in the interval $(0, 2]$.

Lemma 3. (see [5]). *Let $0 \le k < \infty$ and let $f \in k - \mathcal{ST}$ be of the form*

$$f(z) = z + \sum_{n=2}^{\infty} a_n(k)z^n \quad (|z| < 1),$$

then

$$|a_n(k)| \le \frac{(P_1(k))_{(n-1)}}{(n-1)!}, \quad n = 2, 3, \dots ,$$

where $(\lambda)_n$ is the Pochhammer symbol defined by

$$(\lambda)_n = \begin{cases} 1 & (n = 0) \\ \lambda(\lambda+1)\cdot \ldots \cdot(\lambda+n-1) & (n \in \mathbf{N}). \end{cases}$$

For $k = 0$ the estimates are sharp; otherwise only the bound on $|a_2(k)|$ is sharp.

Lemma 4. (see [5]). *Let $0 \le k < \infty$ and let $f \in k - \mathcal{UCV}$ be of the form*

$$f(z) = z + \sum_{n=2}^{\infty} a_n(k)z^n \quad (|z| < 1),$$

then

$$|a_n(k)| \le \frac{(P_1(k))_{(n-1)}}{n!}, \quad n = 2, 3, \dots ,$$

where $P_1(k)$ is given above. For $k = 0$ the estimates are sharp; otherwise only the bound on $|a_2(k)|$ is sharp.

Corollary 1. *If $f \in k - \mathcal{ST}$, then*

$$|a_n(k)| \leq \frac{nP_1(k)}{2}, \quad n = 2, 3, \ldots. \tag{2.1}$$

For $k = 0$ the estimates are sharp; otherwise only the bound on $|a_2(k)|$ is sharp.

Proof. We have $0 < P_1(k) \leq 2$, thus

$$\frac{(P_1(k))_{(n-1)}}{(n-1)!} = \frac{P_1(k)[P_1(k)+1] \cdot \ldots \cdot [P_1(k)+n-2]}{1 \cdot 2 \ldots \cdot (n-1)} \leq \frac{nP_1(k)}{2}. \qquad \blacksquare$$

In the same way we can prove the following Corollary 2.

Corollary 2. *If $f \in k - \mathcal{UCV}$, then*

$$|a_n(k)| \leq \frac{P_1(k)}{2}, \quad n = 2, 3, \ldots . \tag{2.2}$$

Theorem 1. *If $f \in k - \mathcal{UCV}$ and $h \in \mathcal{B}$, then*

$$\left|\frac{(f * h)(z)}{z}\right| \geq \frac{1}{4}. \tag{2.3}$$

Proof. Let $f \in k - \mathcal{UCV}, h \in \mathcal{B}, z \in \mathcal{U}$. Then, in view of Lemma 1

$$\frac{1}{z}\left[h(z) * \frac{f(z) + \varepsilon z}{1 + \varepsilon}\right] \neq 0 \quad (|\varepsilon| < 1/4).$$

Therefore

$$\frac{1}{z}\left[h(z) * f(z)\right] \neq -\varepsilon \quad (|\varepsilon| < 1/4),$$

that is

$$\frac{1}{z}\left[h(z) * f(z)\right] \geq 1/4. \qquad \blacksquare$$

Theorem 2. *Let $k \in [0; +\infty)$. If*

$$0 \leq \delta < \sqrt{[P_1(k)+1]^2 + \frac{1}{k+1}} - [P_1(k)+1],$$

then

$$N_\delta(k - \mathcal{ST}) \otimes N_\delta(\mathcal{CV}) \subseteq k - \mathcal{ST}.$$

Proof. Let $f_0(z) = z+\sum_{n=2}^{\infty} a_{0n}z^n \in k-\mathcal{ST}$, $g_0(z) = z+\sum_{n=2}^{\infty} b_{0n}z^n \in \mathcal{CV}$ and $f \in N_\delta(f_0)$, $g \in N_\delta(g_0)$ and $h \in \mathcal{B}, z \in \mathcal{U}$. We want to show that

$$\frac{1}{z}\left(f \otimes g * h\right)(z) \neq 0, \quad z \in \mathcal{U}.$$

By the identity

$$f\otimes g*h = f_0\otimes g_0*h+f_0\otimes(g-g_0)*h+(f-f_0)\otimes g_0*h+(f-f_0)\otimes(g-g_0)*h$$

we obtain

$$\begin{aligned}\left|\frac{(f\otimes g*h)(z)}{z}\right| &\geq \left|\frac{(f_0\otimes g_0*h)(z)}{z}\right| - \left|\frac{(f_0\otimes(g-g_0)*h)(z)}{z}\right| \\ &- \left|\frac{((f-f_0)\otimes g_0*h)(z)}{z}\right| - \left|\frac{((f-f_0)\otimes(g-g_0)*h)(z)}{z}\right|.\end{aligned} \tag{2.4}$$

From Lemma 2 (ii) we have $f_0 * g_0 \in k-\mathcal{ST}$ hence from (1.3) $f_0\otimes g_0 \in k-\mathcal{UCV}$. Therefore by using Theorem 1 we obtain

$$\left|\frac{(f_0\otimes g_0*h)(z)}{z}\right| \geq 1/4 \quad (|z|<1). \tag{2.5}$$

Moreover, the coefficients of $g_0 \in \mathcal{CV}$ satisfy inequality $|b_{0n}| \leq 1$, then making use of the bounds of the coefficients of functions (1.4), (2.1) and from (2.4), (2.5) we get

$$\left|\frac{(f\otimes g*h)(z)}{z}\right| \geq \frac{1}{4} - (k+1)\frac{\delta P_1(k)}{2} - (k+1)\frac{\delta}{2} - (k+1)\frac{\delta^2}{4}. \tag{2.6}$$

The right side of (2.6) is positive whenever

$$0 \leq \delta < \sqrt{[P_1(k)+1]^2 + \frac{1}{k+1}} - [P_1(k)+1]\,. \qquad \blacksquare$$

Corollary 3. *The radius of stability of the integral convolution* (1.7) *on the pair* $(k\text{-}\mathcal{ST},\mathcal{CV})$ *satisfy*

$$\delta\left((k-\mathcal{ST})\otimes\mathcal{CV}; k-\mathcal{ST}\right) \geq \sqrt{[P_1(k)+1]^2 + \frac{1}{k+1}} - [P_1(k)+1]\,.$$

For $k>0$ Corollary 3 improves an earlier result from [1] of the form

$$\delta\left((k-\mathcal{ST})\otimes\mathcal{CV}; k-\mathcal{ST}\right) \geq \sqrt{9+\frac{1}{(k+1)^2}} - 3.$$

For $k = 0$ the bounds are equal each other.

Theorem 3. *Let* $k \in [0; +\infty)$. *If*

$$0 \le \delta < \sqrt{\frac{[P_1(k)+2]^2}{4} + \frac{1}{2(k+1)}} - \frac{P_1(k)+2}{2},$$

then

$$N_\delta(k-\mathcal{UCV}) \otimes N_\delta(\mathcal{CV}) \subseteq k-\mathcal{UCV}.$$

Proof. Observe that from (1.3) it is sufficient to prove that

$$N_\delta(k-\mathcal{UCV}) * N_\delta(\mathcal{CV}) \subseteq k-\mathcal{ST}.$$

Let $f_0 \in k-\mathcal{UCV}$, $f \in N_\delta(f_0)$ and $g_0 \in \mathcal{CV}$ $g \in N_\delta(g_0)$ and $h \in \mathcal{B}, z \in \mathcal{U}$. We wish to show that

$$\frac{1}{z}\left(f * g * h\right)(z) \ne 0, \quad z \in \mathcal{U}.$$

By identity

$$f*g*h = f_0*g_0*h + f_0*(g-g_0)*h + (f-f_0)*g_0*h + (f-f_0)*(g-g_0)*h$$

we obtain in the same way as in the proof of Theorem 2

$$\left|\frac{(f*g*h)(z)}{z}\right| \ge \frac{1}{4} - \frac{\delta(k+1)P_1(k)}{2} - (k+1)\delta - (k+1)\frac{\delta^2}{2}. \tag{2.7}$$

The right side of (2.7) is positive if

$$0 \le \delta < \sqrt{\frac{[P_1(k)+2]^2}{4} + \frac{1}{2(k+1)}} - \frac{P_1(k)+2}{2}.$$ ■

Corollary 4. *The radius of stability of the integral convolution* (1.7) *on the pair* $(k\text{-}\mathcal{UCV}, \mathcal{CV})$ *satisfy*

$$\delta\left((k-\mathcal{UCV}) \otimes \mathcal{CV}; k-\mathcal{UCV}\right) \ge \sqrt{\frac{[P_1(k)+2]^2}{4} + \frac{1}{2(k+1)}} - \frac{P_1(k)+2}{2}.$$

For $k > 0$ Corollary 4 improves an earlier result from [1] of the form

$$\delta\left((k-\mathcal{UCV}) \otimes \mathcal{CV}; k-\mathcal{UCV}\right) \ge \sqrt{4 + \frac{1}{2(k+1)^2}} - 2.$$

For $k=0$ the bounds are equal each other.

Corollary 5. *The higher estimations for the radius of stability of the integral convolution* δ (1.7) *satisfy*

$$\delta\left((k-\mathcal{ST})\otimes\mathcal{CV};k-\mathcal{ST}\right)\le\delta_1=\frac{4}{\sqrt{k^2+10k+17}+3+k}<0,43845 \tag{2.8}$$

$$\delta\left((k-\mathcal{UCV})\otimes\mathcal{CV};k-\mathcal{UCV}\right)\le\delta_2=\frac{4}{\sqrt{4k^2+28k+41}+5+2k}<0,3508 \tag{2.9}$$

Proof. Let $g_0(z)=z+z^2+z^3+\cdots\in\mathcal{CV}$, $f_0(z)=z+\frac{1}{2(2+k)}z^2$, $h_0(z)=z+\frac{1}{2+k}z^2$. From [3] we have

$$z+Az^n\in k\text{-}\mathcal{UCV}\Longleftrightarrow|A|\le\frac{1}{n[n+k(n-1)]}$$

and

$$z+Az^n\in k\text{-}\mathcal{ST}\Longleftrightarrow|A|\le\frac{1}{n+k(n-1)}. \tag{2.10}$$

Thus $f_0\in k\text{-}\mathcal{UCV}$ and $h_0\in k\text{-}\mathcal{ST}$. Let

$$g(z)=z+\left(1+\frac{\delta}{2}\right)z^2+z^3+\cdots\in N_\delta(g_0)\subset N_\delta(\mathcal{CV}),$$

$$f(z)=z+\left(\frac{1}{2(2+k)}+\frac{\delta}{2}\right)z^2\in N_\delta(f_0)\subset N_\delta(k\text{-}\mathcal{UCV}),$$

$$h(z)=z+\left(\frac{1}{2+k}+\frac{\delta}{2}\right)z^2\in N_\delta(h_0)\subset N_\delta(k\text{-}\mathcal{ST}).$$

To show (2.8) and (2.9) it is sufficient to prove that

$$g\otimes h\notin k\text{-}\mathcal{ST}\quad\text{when}\quad\delta>\delta_1$$

and

$$g*f\notin k\text{-}\mathcal{ST}\quad\text{when}\quad\delta>\delta_2.$$

We have

$$(g\otimes h)(z)=z+\frac{1}{2}\left(1+\frac{\delta}{2}\right)\left(\frac{1}{2+k}+\frac{\delta}{2}\right)z^2.$$

Let $\phi(\delta)=\frac{1}{2}\left(1+\frac{\delta}{2}\right)\left(\frac{1}{2+k}+\frac{\delta}{2}\right)$. Then we have $\phi(\delta_1)=\frac{1}{2+k}$ and $\phi(\delta)>\frac{1}{2+k}$ for $\delta>\delta_1$, thus by 2.10 we have $(g\otimes h)(z)\notin k\text{-}\mathcal{ST}$ when $\delta>\delta_1$. Moreover

$$(g * f)(z) = z + \left(1 + \frac{\delta}{2}\right)\left(\frac{1}{2(2+k)} + \frac{\delta}{2}\right)z^2$$

and $\left(1 + \frac{\delta_2}{2}\right)\left(\frac{1}{2(2+k)} + \frac{\delta_2}{2}\right) = \frac{1}{2+k}$ therefore $g * f \notin k\text{-}\mathcal{ST}$ for $\delta > \delta_2$. ■

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