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Research Article

On FTCS Approach for Box Model of Three-Dimension Advection-Diffusion Equation

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This paper describes a numerical solution for mathematical model of the transport equation in a simple rectangular box domain. The model of street tunnel pollution distribution using two-dimension advection and three-dimension diffusion is solved numerically. Because of the nature of the problem, the model is extended to become three-dimension advection and three-dimension diffusion to study the sea-sand mining pollution distribution. This model with various advection and diffusion parameters and the boundaries conditions is also solved numerically using a finite difference (FTCS) method.

1. Introduction

The solution of partial differential equation and their associated boundary and initial condition play an important role in modelling of phenomena in fields as diverse as physics, chemistry, geology, biology, engineering, and economics. The transport of pollutants occurs in a large variety of environmental, agricultural, and industrial processes. The phenomenon is usually modelled into partial differential equations with boundary and/or initial conditions. The models, however, in most cases have no analytical solution. Numerical solution becomes an alternative solution to models such as partial differential equation models in order to investigate, predict, and conclude the models.

Numerical solution for an advection-diffusion equation or transport equation had become an interesting subject for many authors recently. Several improvements in finite difference approach had been noted. A stability limit for a finite difference scheme such as the forward time and space-centered numerical scheme applied the convection-diffusion equation is discussed in [1]. Three-dimension solution of advection-diffusion base on the two-level fully explicit and fully implicit finite difference approximation is discussed in [2]. The comparison of the two standard finite difference schemes such as FTCS and Crank-Nicolson methods is carried out by [3]. The method of time splitting to divide complicated time dependent partial differential equation into sets of simpler equations which could then be solved separately by numerical means over fraction time step had been done as in [4]. Numerical solution of a three-dimension advection-diffusion for models in a street tunnel is discussed in [5]. More recently, an application of the generalized finite difference method to solve the advection-diffusion equation using the explicit method is discussed in [6]. Also, practical use of finite difference methods in order to study the pollution distribution on Unhas lake had been carried out as in [7].

The studying of a box model had been carried out numerically as in [5] using two-dimension advection and three-dimension diffusion. In this paper we extend the box model into three-dimension advection and three-dimension diffusion to determine the pollutants spread in the water or in the air. This is because of the nature of the pollutant particles. For certain dimension of particle, gravitational force due to the particle mass should be taken into account. We solve the three-dimension advection-diffusion equation by using the forward in time, center in space (FTCS) finite difference method. The domain of this study model is a rectangular box with length L, width W, and height H. Numerical results for several different pollutant source configurations are presented and discussed.

The paper is organized as follows. In Section 2, we introduce the basic equation and the problems. In Section 3,

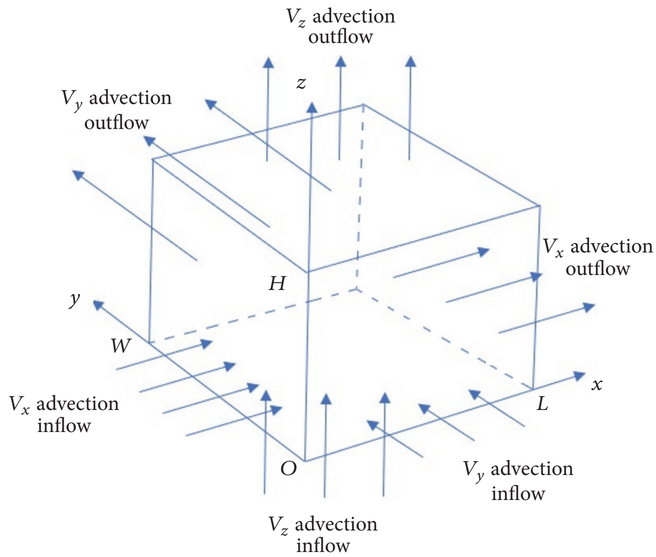

FIGURE 1: Domain of advection-diffusion.

the finite difference schemes for the computation approach are introduced. The stability condition for FTCS scheme is discussed in Section 4. In Section 5, some numerical models, results, and discussions are presented. Finally, conclusions are found in Section 6.

2. Basic Equation and Problems

In this paper we consider the three-dimension advection-diffusion equation [2, 5, 6]

$$\frac{\partial C}{\partial t} + V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y} + V_z \frac{\partial C}{\partial z} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2}, \quad 0 < t \le T, \tag{1}$$

in the domains $0 \le x \le L$, $0 \le y \le W$, and $0 \le z \le H$, with initial condition

$$C(x, y, z, 0) = f(x, y, z), \tag{2}$$

and the boundary conditions

$$C(0, y, z, t) = g_0(y, z, t), \quad 0 < t \le T, \tag{3}$$

$$C(L, y, z, t) = g_L(y, z, t), \quad 0 < t \le T, \tag{4}$$

$$C(x, 0, z, t) = h_0(x, z, t), \quad 0 < t \le T, \tag{5}$$

$$C(x, W, z, t) = h_W(x, z, t), \quad 0 < t \le T, \tag{6}$$

$$C(x, y, 0, t) = k_0(x, y, t), \quad 0 < t \le T, \tag{7}$$

$$C(x, y, H, t) = k_H(x, y, t), \quad 0 < t \le T, \tag{8}$$

where $f, g_0, g_L, h_0, h_W, k_0$, and k_H are known functions, while the function C is unknown. Here $C(x, y, z, t)$ denote materials concentration which is transported. The constants V_x, V_y, V_z represent speeds of advection with respect to $x-$axis, $y-$axis, and $z-$axis, respectively. Also, the constants D_x, D_y, D_z, represent the speeds of diffusivities with respect to $x-$axis, $y-$axis, and $z-$axis, respectively (see illustration in Figure 1).

3. Finite Difference Schemes

The main idea behind the finite difference schemes for obtaining the solution of a given partial differential equation is to approximate the derivatives appearing in the equation by a set of values of the function at selected number of points. The most usual way to generate these approximations is through the use of Taylor series.

The solution domain of the problem over a time $0 \le t \le T$ is covered by a mesh of uniformly spaced grid-lines parallel to the space and time coordinates axes, respectively.

$$x_i = i\Delta x, \quad i = 0, 1, 2, \ldots, M; \tag{9}$$

$$y_j = j\Delta y, \quad j = 0, 1, 2, \ldots, N; \tag{10}$$

$$z_k = k\Delta z, \quad k = 0, 1, 2, \ldots, O; \tag{11}$$

$$t_n = n\Delta t, \quad n = 0, 1, 2, \ldots, R; \tag{12}$$

Approximations $C_{i,j,k}^n$ to $C(i\Delta x, j\Delta y, k\Delta z, n\Delta t)$ are calculated at the point of intersection of these lines according to the (i, j, k, n) grid points. The uniform spatial and temporal grid spacings are $\Delta x = L/M$, $\Delta y = W/N$, $\Delta z = H/O$, and $\Delta t = T/R$, where L is the length, W is the width, and H is the height of the domain of the interested rectangular box.

Since the grid points are in three dimensions of the box form, several numerical approximations are needed for the grid points. These depend on the position of the grid points. The approximation for interior points is forward time center space (FTCS). The points obtained from intersection of two planes of the box are approximated using forward time center space forward or backward space while the edge points with intersection of three planes are approximated using forward time and combination of forward and backward approximation.

Forward time centered space (FTCS) approximation of (1) for the interior points as in [2, 5, 7] is

$$\begin{aligned}\frac{C_{i,j,k}^{n+1}-C_{i,j,k}^{n}}{\Delta t}&+V_x\left(\frac{C_{i+1,j,k}^{n}-C_{i-1,j,k}^{n}}{2\Delta x}\right)\\&+V_y\left(\frac{C_{i,j+1,k}^{n}-C_{i,j-1,k}^{n}}{2\Delta y}\right)\\&+V_z\left(\frac{C_{i,j,k+1}^{n}-C_{i,j,k-1}^{n}}{2\Delta z}\right)\\=D_x&\left(\frac{C_{i+1,j,k}^{n}-2C_{i,j,k}^{n}+C_{i-1,j,k}^{n}}{\Delta x^2}\right)\\&+D_y\left(\frac{C_{i,j+1,k}^{n}-2C_{i,j,k}^{n}+C_{i,j-1,k}^{n}}{\Delta y^2}\right)\\&+D_z\left(\frac{C_{i,j,k+1}^{n}-2C_{i,j,k}^{n}+C_{i,j,k-1}^{n}}{\Delta z^2}\right).\end{aligned}\tag{13}$$

Another finite difference approximation such as forward time backward space x center space y center space z is used for the boundary points on $C(L,y,z,t)=g_L(y,z,t)$

$$\begin{aligned}\frac{C_{i,j,k}^{n+1}-C_{i,j,k}^{n}}{\Delta t}&+V_x\left(\frac{3C_{i,j,k}^{n}-4C_{i-1,j,k}^{n}+3C_{i-2,j,k}^{n}}{2\Delta x}\right)\\&+V_y\left(\frac{C_{i,j+1,k}^{n}-C_{i,j-1,k}^{n}}{2\Delta y}\right)\\&+V_z\left(\frac{C_{i,j,k+1}^{n}-C_{i,j,k-1}^{n}}{2\Delta z}\right)\\=D_x&\left(\frac{C_{i,j,k}^{n}-2C_{i-1,j,k}^{n}+C_{i-2,j,k}^{n}}{\Delta x^2}\right)\\&+D_y\left(\frac{C_{i,j+1,k}^{n}-2C_{i,j,k}^{n}+C_{i,j-1,k}^{n}}{\Delta y^2}\right)\\&+D_z\left(\frac{C_{i,j,k+1}^{n}-2C_{i,j,k}^{n}+C_{i,j,k-1}^{n}}{\Delta z^2}\right).\end{aligned}\tag{14}$$

Forward time backward space x backward space y center space z is used for the boundary points on $C(L,W,z,t)$ which is

$$\begin{aligned}\frac{C_{i,j,k}^{n+1}-C_{i,j,k}^{n}}{\Delta t}&+V_x\left(\frac{3C_{i,j,k}^{n}-4C_{i-1,j,k}^{n}+3C_{i-2,j,k}^{n}}{2\Delta x}\right)\\&+V_y\left(\frac{3C_{i,j,k}^{n}-4C_{i,j-1,k}^{n}+3C_{i,j-2,k}^{n}}{2\Delta y}\right)\\&+V_z\left(\frac{C_{i,j,k+1}^{n}-C_{i,j,k-1}^{n}}{2\Delta z}\right)\\=D_x&\left(\frac{C_{i,j,k}^{n}-2C_{i-1,j,k}^{n}+C_{i-2,j,k}^{n}}{\Delta x^2}\right)\\&+D_y\left(\frac{C_{i,j,k}^{n}-2C_{i,j-1,k}^{n}+C_{i,j-2,k}^{n}}{\Delta y^2}\right)\\&+D_z\left(\frac{C_{i,j,k+1}^{n}-2C_{i,j,k}^{n}+C_{i,j,k-1}^{n}}{\Delta z^2}\right).\end{aligned}\tag{15}$$

Also forward time backward space x forward space y center space z is used for the boundary points on $C(L,0,z,t)$ which is

$$\begin{aligned}\frac{C_{i,j,k}^{n+1}-C_{i,j,k}^{n}}{\Delta t}&+V_x\left(\frac{3C_{i,j,k}^{n}-4C_{i-1,j,k}^{n}+3C_{i-2,j,k}^{n}}{2\Delta x}\right)\\&+V_y\left(\frac{C_{i,j,k}^{n}-4C_{i,j+1,k}^{n}+3C_{i,j+2,k}^{n}}{2\Delta y}\right)\\&+V_z\left(\frac{C_{i,j,k+1}^{n}-C_{i,j,k-1}^{n}}{2\Delta z}\right)\\=D_x&\left(\frac{C_{i,j,k}^{n}-2C_{i-1,j,k}^{n}+C_{i-2,j,k}^{n}}{\Delta x^2}\right)\\&+D_y\left(\frac{C_{i,j,k}^{n}-2C_{i,j+1,k}^{n}+C_{i,j+2,k}^{n}}{\Delta y^2}\right)\\&+D_z\left(\frac{C_{i,j,k+1}^{n}-2C_{i,j,k}^{n}+C_{i,j,k-1}^{n}}{\Delta z^2}\right).\end{aligned}\tag{16}$$

All approximations have error of first order in time interval and second order is spatial coordinate grid spacing.

4. Stability

To ensure obtained solutions have a nonpropagate error, the approximation or schemes should meet certain conditions. The approximation schemes on the boundary usually have a nonpropagate error since boundary exact conditions are supplied on the boundary. However, for the interior points the error might propagate. For the FTCS approximations, (13) might be rewritten as

$$\begin{aligned}C_{i,j,k}^{n+1}=&\left(\frac{D_x\Delta t}{\Delta x^2}+\frac{V_x\Delta t}{2\Delta x}\right)C_{i-1,j,k}^{n}\\&+\left(\frac{D_y\Delta t}{\Delta y^2}+\frac{V_y\Delta t}{2\Delta y}\right)C_{i,j-1,k}^{n}\\&+\left(\frac{D_z\Delta t}{\Delta z^2}+\frac{V_z\Delta t}{2\Delta z}\right)C_{i,j,k-1}^{n}\end{aligned}$$

$$+\left(\frac{D_x\Delta t}{\Delta x^2}-\frac{V_x\Delta t}{2\Delta x}\right)C^n_{i+1,j,k}$$
$$+\left(\frac{D_y\Delta t}{\Delta y^2}-\frac{V_y\Delta t}{2\Delta y}\right)C^n_{i,j+1,k}$$
$$+\left(\frac{D_z\Delta t}{\Delta z^2}-\frac{V_z\Delta t}{2\Delta z}\right)C^n_{i,j,k-1}$$
$$+\left(1-2\left[\frac{D_x\Delta t}{\Delta x^2}+\frac{D_y\Delta t}{\Delta y^2}+\frac{D_z\Delta t}{\Delta z^2}\right]\right)C^n_{i,j,k} \quad (17)$$

or simply

$$\begin{aligned}C^{n+1}_{i,j,k} &= \left(s_x+\frac{c_x}{2}\right)C^n_{i-1,j,k}+\left(s_y+\frac{c_y}{2}\right)C^n_{i,j-1,k}\\ &+\left(s_z+\frac{c_z}{2}\right)C^n_{i,j,k-1}+\left(s_x-\frac{c_x}{2}\right)C^n_{i+1,j,k}\\ &+\left(s_y-\frac{c_y}{2}\right)C^n_{i,j+1,k}+\left(s_z-\frac{c_z}{2}\right)C^n_{i,j,k+1}\\ &+\left(1-2\left[s_x+s_y+s_z\right]\right)C^n_{i,j,k}\end{aligned} \quad (18)$$

in which

$$s_x=\frac{D_x\Delta t}{\Delta x^2},\quad s_y=\frac{D_y\Delta t}{\Delta y^2},\quad s_z=\frac{D_z\Delta t}{\Delta z^2}, \quad (19)$$

$$c_x=\frac{V_x\Delta t}{\Delta x},\quad c_y=\frac{V_y\Delta t}{\Delta y},\quad c_z=\frac{V_z\Delta t}{\Delta z}. \quad (20)$$

The stability of this three-dimensional difference scheme may be investigated using the von Neumann method. As in [2, 5] application of this method of stability analysis shows that (18) will be stable if it satisfies both equations

$$s_x+s_y+s_z\le\frac{1}{2} \quad (21)$$

and

$$\frac{c_x^2}{s_x}+\frac{c_y^2}{s_y}+\frac{c_z^2}{s_z}\le 3. \quad (22)$$

For one-dimension version of the forward time centered space (FTCS) type formula for the case that $s_x = s_y = s_z = s$ and $c_x = c_x = c_x = c$, it should also satisfy

$$c^2\le s\le\frac{1}{6}. \quad (23)$$

This clearly is much more strict than one-dimension advection-diffusion equation of the forward time centered space (FTCS) approximation which is $s \le 1/2$.

5. Numerical Results and Discussions

Problem 1. In order to verify the accuracy of the procedure that has been built, we consider the three-dimension advection-diffusion equation (1), with initial condition (2) and boundary conditions (3)-(8). By taking $V_y = V_z = 0$, $L = 1, W = 1, H = 1$, this procedure simply reduced to three-dimension advection-diffusion equation for transport of pollutants in street tunnel as discussed in [5]

$$\frac{\partial C}{\partial t}+V_x\frac{\partial C}{\partial x}=D_x\frac{\partial^2 C}{\partial x^2}+D_y\frac{\partial^2 C}{\partial y^2}+D_z\frac{\partial^2 C}{\partial z^2}, \quad 0<t<T. \quad (24)$$

Furthermore, taking appropriate function for $f, g_0, g_L, h_0, h_W, k_0, k_H$, the initial condition and boundary conditions yield

$$\begin{aligned}&C(x,y,z,0)=0 \quad 0\le x\le 1;\ 0\le y\le 1;\ 0\le z\le 1\\ &C(0,y,z,t)=0 \quad 0\le y<0.5;\ 0\le z\le 1\\ &C(0,y,z,t)=1 \quad 0.5\le y\le 1;\ 0\le z\le 1\\ &C(1,y,z,t)=0 \quad 0\le y\le 1;\ 0\le z\le 1\\ &\frac{\partial C}{\partial y}(x,0,z,t)=0 \quad t>0\\ &\frac{\partial C}{\partial y}(x,1,z,t)=0 \quad t>0\\ &\frac{\partial C}{\partial z}(x,y,0,t)=0 \quad t>0\\ &\frac{\partial C}{\partial z}(x,y,1,t)=0 \quad t>0\end{aligned} \quad (25)$$

This partial differential equation and its initial and boundary conditions come from pollution distribution on a street tunnel, where the wind flows steadily only in the x direction. There is no disperse flux of the pollutant through the solid side-walls nor through the base and the roof. By using nondimensionalised parameters $\Delta x = \Delta y = \Delta z = 0.1$, $\Delta t = 0.01$, $D_x = D_y = D_z = 0.1$, $V_x = 0.02$, and for the time $T = 20$. These given values will certainly satisfy both conditions $s_x+s_y+s_z = 3/10 \le 1/2$ and $c_x^2/s_x+c_y^2/s_y+c_z^2/s_z = 3/25000 \le 3$. Thus numerical results are stable and can be found as in the Figure 2.

There is no significant difference from the results obtained from [5], as from Figure 2 left one can see the mesh plot on $z = 0$. This is reasonable, with the boundary conditions satisfied, the concentration decreases away from the source and is less than one-half of the source value over more than three-quarters of the tunnel. Also, the solutions are

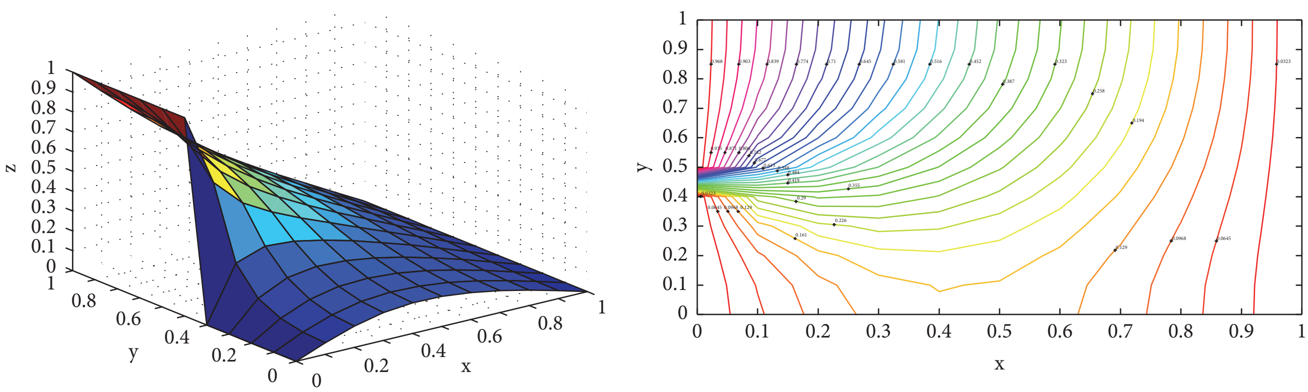

Figure 2: Pollutant distribution in a street tunnel with advection only in x direction.

independent of height since all contour plots on $z = 0, z = 0.2, \ldots, z = 1$ are the same as Figure 2 right. The difference may be noted from the previous work as in [5] lies only on $x = 0.1$.

Problem 2. Here, we consider the three-dimension advection-diffusion equation (1), with initial condition (2) and boundary conditions (3)-(8). By taking $V_z = 0$, $L = 1, W = 1, H = 1$, this simply reduces to two-dimension advection three-dimension diffusion equation for transport of pollutants in street tunnel problem as discussed in [5]

$$\frac{\partial C}{\partial t} + V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2}, \quad 0 < t < T. \tag{26}$$

Furthermore, taking appropriate function for $f, g_0, g_L, h_0, h_W, k_0, k_H$, the initial condition and boundary conditions yield

$$\begin{aligned}
&C(x, y, z, 0) = 0 \quad 0 \le x \le 1;\ 0 \le y \le 1;\ 0 \le z \le 1\\
&C(0, y, z, t) = 0 \quad 0 \le y < 0.5;\ 0 \le z \le 1\\
&C(0, y, z, t) = 1 \quad 0.5 \le y \le 1;\ 0 \le z \le 1\\
&C(x, 0, z, t) = 0 \quad 0 \le y < 0.3;\ 0 \le z \le 1\\
&C(x, 0, z, t) = 1 \quad 0.3 \le y \le 0.6;\ 0 \le z \le 1\\
&C(x, 0, z, t) = 0 \quad 0.6 < y \le 1;\ 0 \le z \le 1\\
&\frac{\partial C}{\partial y}(x, 0, z, t) = 0 \quad t > 0\\
&\frac{\partial C}{\partial y}(x, 1, z, t) = 0 \quad t > 0\\
&\frac{\partial C}{\partial z}(x, y, 0, t) = 0 \quad t > 0\\
&\frac{\partial C}{\partial z}(x, y, 1, t) = 0 \quad t > 0
\end{aligned} \tag{27}$$

This model of partial differential equation and its initial and boundary conditions comes from pollution distribution on a street tunnel, where the wind flows steadily only in the x and y directions. There is no disperse flux of the pollutant through the solid side-walls nor through the base and the roof. By using nondimensionalised parameters $\Delta x = \Delta y = \Delta z = 0.1$, $\Delta t = 0.005$, $D_x = D_y = D_z = 0.2$, $V_x = 0.6$, $V_y = 0.4$ and for the time $T = 20$. These values satisfy both conditions $s_x + s_y + s_z = 3/10 \le 1/2$ and $c_x^2/s_x + c_y^2/s_y + c_z^2/s_z = 13/1000 \le 3$. The numerical results are found as in Figure 3.

Figure 3 shows a mesh plot and a contour plot on $z = 0.2$. The other mesh plot and contour plot on the other z show no difference in $z = 0.2$. This is reasonable, with the boundary conditions satisfied, the concentration decreases away from the source faster in the x direction rather than in the y direction.

Problem 3. We consider a three-dimensional advection-diffusion equation for transport of pollutants in the water such as a sea-sand mining or in the air such as limestone rock mining to produce cement. Such activity usually spread pollutants to the environment nearby. By assuming that the domain of the physical problem had been nondimensionalized, and taking the box model, it will satisfy three-dimension advection and three-dimension diffusion equation. There is no disperse flux of the pollutant through the base and the roof. The boundary condition on top of the box will be sea surface and the bottom of the box will be sea floor and satisfy $(\partial C/\partial z)(x, y, H, t) = 0$ and $(\partial C/\partial z)(x, y, 0, t) = 0$. The advection constant $V_z \neq 0$ due to the gravitational force acting on the particle. The constants V_x and V_y are the sea current speed in x and y directions. Another initial condition and boundary conditions yield

$$\begin{aligned}
&C(x, y, z, 0) = 0 \quad 0 \le x \le 1;\ 0 \le y \le 1;\ 0 \le z \le 1\\
&C(0, y, z, t) = 0\\
&\qquad\qquad 0 \le y < 0.4;\ 0.6 < y \le 1;\ 0 \le z \le 1\\
&C(0, y, z, t) = 1 \quad 0.4 \le y \le 0.6;\ 0 \le z \le 1\\
&C(x, 0, z, t) = 0\\
&\qquad\qquad 0 \le x < 0.4;\ 0.6 < x \le 1;\ 0 \le z \le 1
\end{aligned}$$

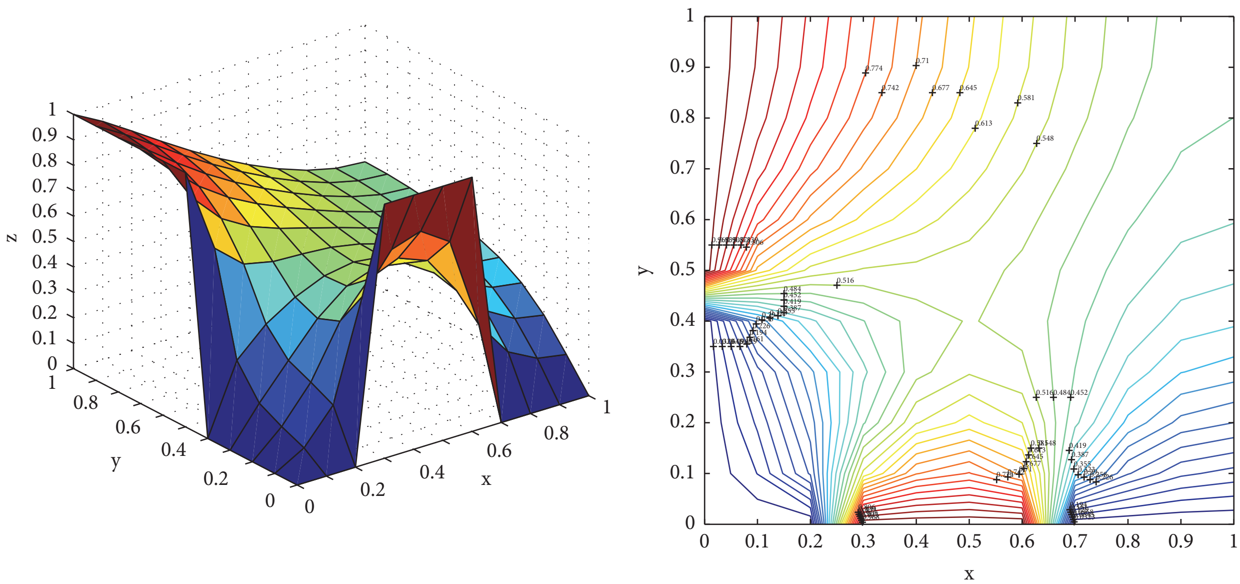

Figure 3: Pollutant distribution in a street tunnel with advection in x and y directions.

$$
\begin{aligned}
&C(x,0,z,t)=1 \quad 0.4\le x\le 0.6;\ 0\le z\le 1\\
&C(1,y,z,t)=0 \quad 0\le y\le 1;\ 0\le z\le 1\\
&\frac{\partial C}{\partial y}(x,0,z,t)=0 \quad t>0\\
&\frac{\partial C}{\partial y}(x,1,z,t)=0 \quad t>0\\
&\frac{\partial C}{\partial z}(x,y,0,t)=0 \quad t>0\\
&\frac{\partial C}{\partial z}(x,y,1,t)=0 \quad t>0
\end{aligned}
\tag{28}
$$

Using nondimensionalised parameters $\Delta x=\Delta y=\Delta z=0.1$, $\Delta t=0.005$; $D_x=D_y=D_z=0.1$, $V_x=1.0$, $V_y=0.3$, $V_z=-0.3$ and for the time $T=2$. Here, the values satisfy also both stability conditions $s_x+s_y+s_z=3/20\le 1/2$ and $c_x^2/s_x+c_y^2/s_y+c_z^2/s_z=59/1000\le 3$. The numerical results for surface plot and contour plot on $z=0.5$ and several values of time t are found as in Figure 4. Surface plot and contour plot for other values of z are similar as in Figure 4.

Problem 4. We consider a three-dimensional advection-diffusion equation for transport of pollutants in the water such as a sea-sand mining again. Such activity usually lasts for certain period of time and spreads pollutants to the environment nearby. Of our interest is to find out the pollutant distribution of this activity. By assuming that the domain of the physical problem had been nondimensionalized and taking the box model, it will satisfy three-dimension advection and three-dimension diffusion equation. There is no disperse flux of the pollutant through the base and the roof. The boundary condition on top of the box will be sea surface and the bottom of the box will be sea floor and satisfy $(\partial C/\partial z)(x,y,H,t)=0$ and $(\partial C/\partial z)(x,y,0,t)=0$. Other initial condition and boundary conditions yield

$$
\begin{aligned}
&C(x,y,z,0)=0 \quad 0\le x\le 1;\ 0\le y\le 1;\ 0\le z\le 1\\
&C(0,y,z,t)=0\\
&\qquad 0\le y<0.4;\ 0.6<y\le 1;\ 0\le z\le 1\\
&C(0,y,z,t)=1\\
&\qquad 0.4\le y\le 0.6;\ 0\le z\le 1;\ 0\le t\le 0.5\\
&C(x,0,z,t)=0\\
&\qquad 0\le x<0.4;\ 0.6<x\le 1;\ 0\le z\le 1\\
&C(x,0,z,t)=1\\
&\qquad 0.4\le x\le 0.6;\ 0\le z\le 1;\ 0\le t\le 0.5\\
&C(1,y,z,t)=0 \quad 0\le y\le 1;\ 0\le z\le 1\\
&C(0,y,z,t)=0\\
&\qquad 0.4\le y\le 0.6;\ 0\le z\le 1;\ t>0.5\\
&C(x,0,z,t)=0\\
&\qquad 0.4\le x\le 0.6;\ 0\le z\le 1;\ t>0.5\\
&\frac{\partial C}{\partial y}(x,0,z,t)=0 \quad t>0\\
&\frac{\partial C}{\partial y}(x,1,z,t)=0 \quad t>0
\end{aligned}
$$

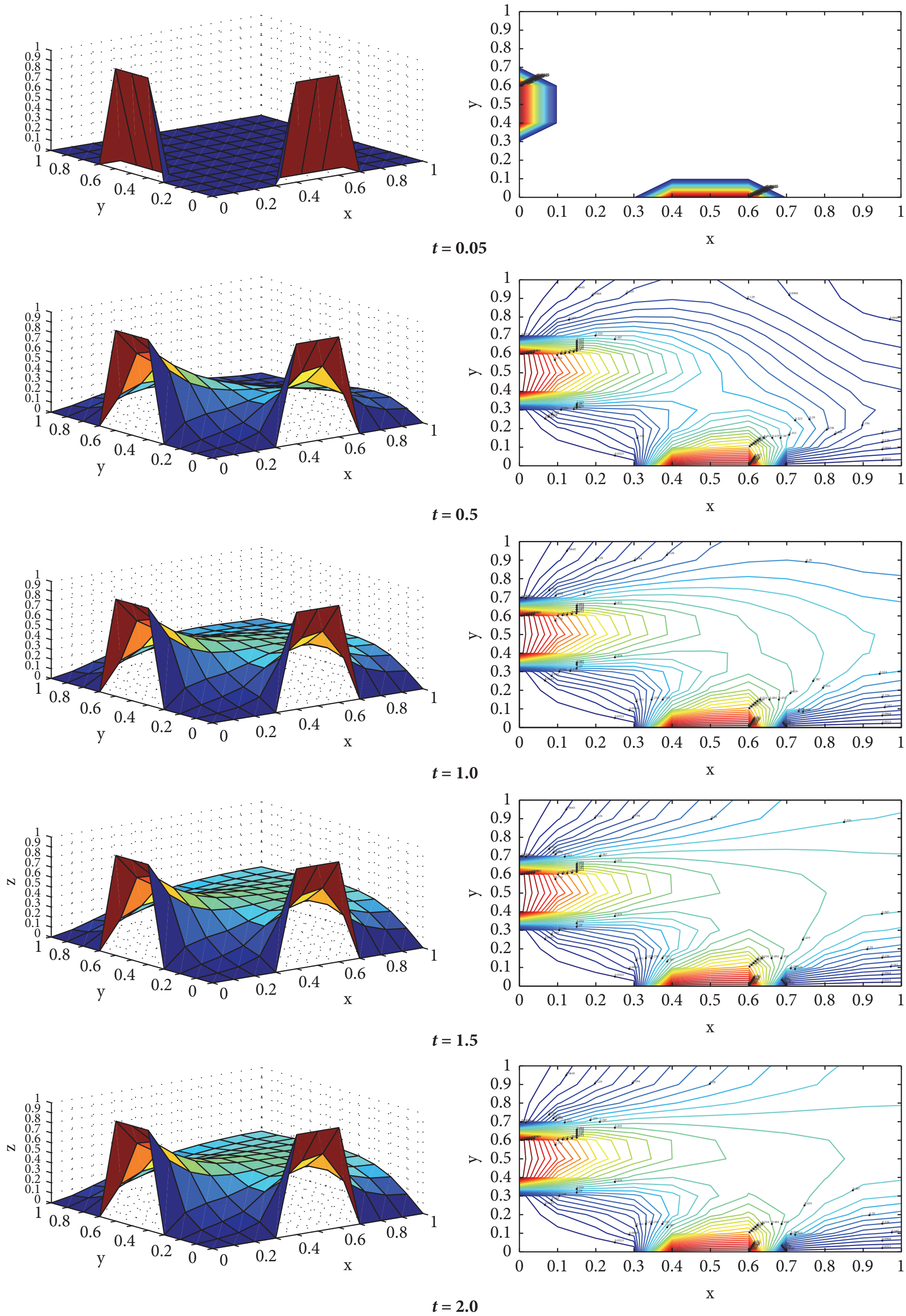

FIGURE 4: Pollutant distribution in a box model under the sea with advection in x, y, and z directions.

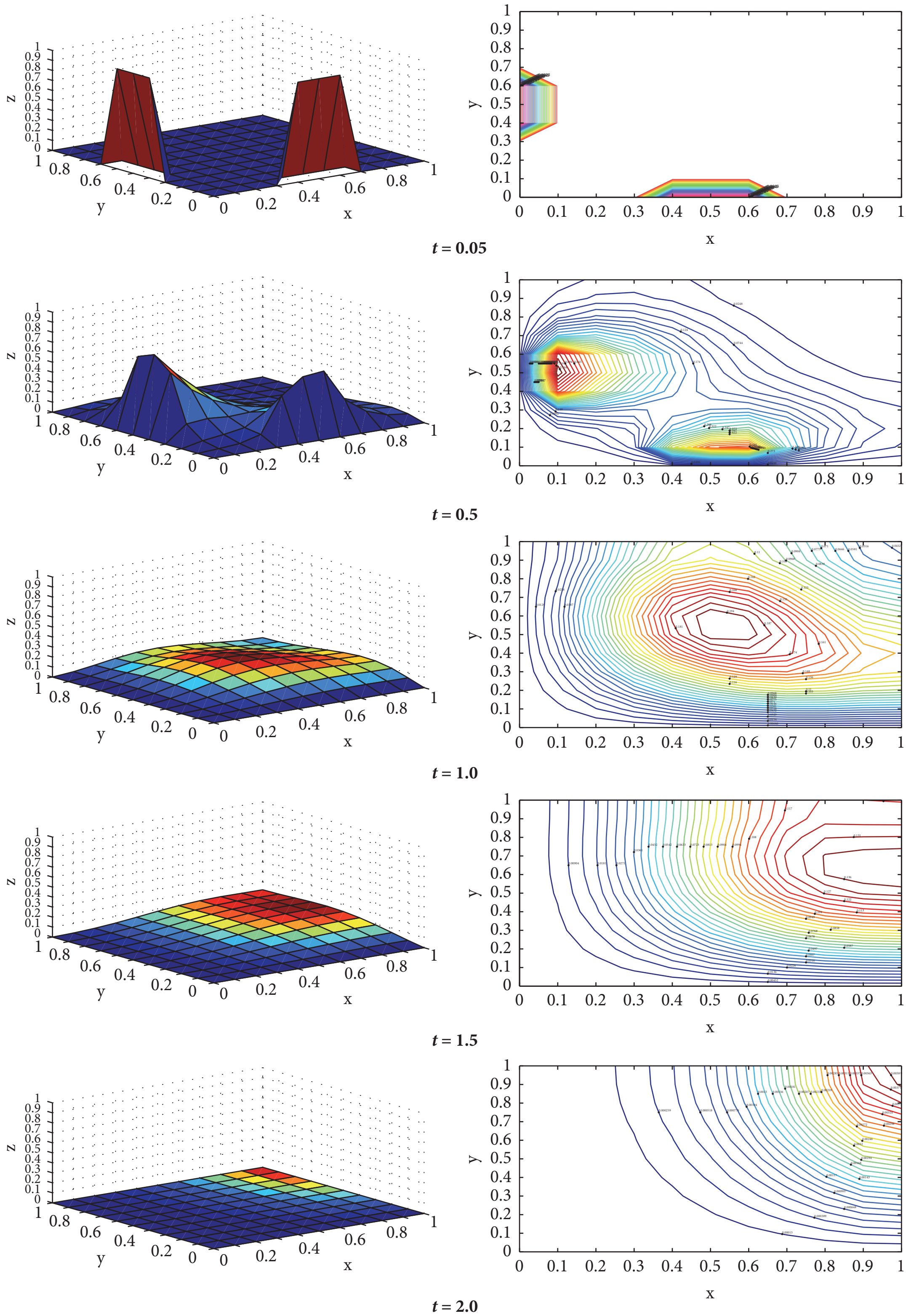

Figure 5: Pollutant distribution in a box model under the sea with advection in x, y, and z directions.

$$\frac{\partial C}{\partial z}(x, y, 0, t) = 0 \quad t > 0$$
$$\frac{\partial C}{\partial z}(x, y, 1, t) = 0 \quad t > 0 \tag{29}$$

Using nondimensionalised parameters $\Delta x = \Delta y = \Delta z = 0.1$, $\Delta t = 0.005$; $D_x = D_y = D_z = 0.2$, $V_x = 1.0$, $V_y = 0.3$, $V_z = -0.3$ and for the time $T = 2$. These values also satisfy both stability conditions $s_x + s_y + s_z = 3/10 \leq 1/2$ and $c_x^2/s_x + c_y^2/s_y + c_z^2/s_z = 295/10000 \leq 3$. There are two pollutant sources on the x-axis and y-axis and they last for certain period of time $0 \leq t \leq 0.5$. The numerical results for surface plot and contour plot on $z = 0.4$ for several different times are found as in Figure 5. Unlike the previous Problem 3, surface and contour plot for other values of z show that pollutant materials move to x, y, z directions as expected.

6. Conclusions

We have employed a standard finite difference scheme to study the pollution distribution for two-dimension advection and three-dimension diffusion equation and extend into three-dimension advection and three-dimension diffusion equation. The stability conditions ensure that the solution of all the interior points can be obtained. The scheme works well as one can see numerical results obtained in solutions of the problems above.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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