

Editorial

Qualitative Theory of Functional Differential and Integral Equations 2016

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The study of qualitative theory of various kinds of differential equations began with the birth of calculus, which dates to the 1660s. Part of Newton's motivation in developing calculus was to solve problems that could be attacked with differential equations. Now, with over 300 years of history, the subject of qualitative theory of differential equations, integral equations, and so on represents a huge body of knowledge including many subfields and a vast array of applications in many disciplines. It is beyond exposition as a whole. Qualitative theory refers to the study of the behavior of solutions without determining explicit formulas for the solutions. In addition, it should be noted that if solutions of an equation describing a dynamical system or of any kind of differential equations under consideration are known in closed form, one can determine the qualitative properties of the system or the solutions of that equations, by applying directly the definitions of relative mathematical concepts. As is also well-known, in general, it is not possible to find the solution of all linear and nonlinear differential equations, except numerically. Moreover, finding of solutions becomes very difficult for functional differential equations, integral equations, partially differential equations, and fractional differential equations rather than for ordinary differential equations. Thus, indirect methods are needed. Therefore, it is very important to obtain information on the qualitative

behavior of solutions of differential equations when there is no analytical expression for the solutions. So far, in the relevant literature, some methods have been improved to obtain information about qualitative behaviors of solutions of differential equations without solving them. Here, we would not like to give the details of methods.

It is worth mentioning that, in the last century, theory of ordinary differential equations, functional differential equations, partially differential equations, integral equations, and integrodifferential equations has developed quickly and played many important roles in qualitative theory and applications of that equations. Some problems of considerable interest in qualitative theory of ordinary differential equations, functional differential and integral equations, integrodifferential equations, fractional differential equations, partially differential equations, and so forth include many topics such as stability and instability of solutions, boundedness of solutions, convergence of solutions, existence of periodic solutions, almost periodic solutions, pseudo almost period solutions, existence and uniqueness of solutions, global existence of solutions, global stability, bifurcation analysis, control of chaos, boundary value problems, oscillation and nonoscillation of solutions, and global existence of solutions. Functional differential equations, which include ordinary and delay differential equations, partially differential equations,

and integral equations, have very important roles in many scientific areas such as mechanics, engineering, economy, control theory, physics, chemistry, biology, medicine, atomic energy, and information theory.

Over the years many scientific works have been dedicated to the mentioned problems for various differential equations, fractional differential equations, partially differential equations, and so forth. In particular, we can find many interesting results related to qualitative behaviors of solutions in the books or papers in [1–15] and in their references.

In response to the call for papers, 22 papers were received. After a rigorous refereeing process, 5 papers were accepted for publication in this special issue. The articles included in the issue cover novel contributions to qualitative theory of functional differential and integral equations, magnetohydrodynamics equations, partial differential equations.

The paper by G. Degla investigates the existence of a curve (with respect to the scalar delay) of periodic positive solutions for a smooth model of Cooke-Kaplan's integral equation by using the implicit function theorem under suitable conditions. The author also shows a situation in which any bounded solution with a sufficiently small delay is isolated, clearing an asymptotic stability result of Cooke and Kaplan.

In the paper by A. M. Kholkin, a resolvent for the Sturm-Liouville operator with a block triangular operator potential increasing at infinite is constructed. The structure of the spectrum of such an operator is obtained.

The paper by I. Ellahiani et al. deals with global existence of weak solutions to a one-dimensional mathematical model describing magnetoelastic interactions. The model is described by a fractional Landau-Lifshitz-Gilbert equation for the magnetization field coupled to an evolution equation for the displacement. They prove global existence by using Faedo-Galerkin/Penalty method. Some commutator estimates are used to prove the convergence of nonlinear terms.

In the paper by Y. Li et al., based on classical Lie Group method, a class of explicit solutions of two-dimensional ideal incompressible magnetohydrodynamics (MHD) equation by its infinitesimal generator is constructed. Via these explicit solutions, the authors study the uniqueness and stability of initial-boundary problem on MHD.

In the paper by D. P. D. Santos, the existence of solutions for certain nonlinear boundary value problems is investigated. All the contemplated boundary value problems are reduced to find a fixed point for one operator defined on a space of functions, and Schauder fixed point theorem or Leray-Schauder degree are used.

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