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Research Article

A Weak Solution of a Stochastic Nonlinear Problem

M. L. Hadji

Laboratory of Probability and Statistics (LAPS), Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria

Correspondence should be addressed to M. L. Hadji; mohamedlakhdarhadji@gmail.com

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We consider a problem modeling a porous medium with a random perturbation. This model occurs in many applications such as biology, medical sciences, oil exploitation, and chemical engineering. Many authors focused their study mostly on the deterministic case. The more classical one was due to Biot in the 50s, where he suggested to ignore everything that happens at the microscopic level, to apply the principles of the continuum mechanics at the macroscopic level. Here we consider a stochastic problem, that is, a problem with a random perturbation. First we prove a result on the existence and uniqueness of the solution, by making use of the weak formulation. Furthermore, we use a numerical scheme based on finite differences to present numerical results.

1. Introduction

The theory of linear poroelasticity has been introduced and rigorously improved by many authors, among others, Biot [1–4], Bear and Bachmat [5], Bémer et al. [6], Barucq et al. [7], and Zenisek [8]. Nowadays, this theory leads to many applications in different disciplines, such as oil exploration, biological phenomena, medical sciences [9], and military applications [10].

In our work, we are interested in the study of fluid-saturated porous media, subject to a random disturbance when the phenomenon of consolidation is realized (see [11, 12]). The study of poroelastic properties or the problem of acoustic wave propagation in saturated porous media, for example, in the oil exploration, has been based on two approaches (see, e.g., [9, 10, 13]). The first focuses on microscopic laws; that is, the pore becomes an entire field of study and then derives the macroscopic laws that are involved across the porous media as a whole. This is where the homogenization techniques are used, by considering the fact that the microscopic structure is repeated periodically which leads to the periodicity of the solutions [14]. The second approach, the more classical, was due to Biot in the 50s, where he suggested to ignore everything that happens at the microscopic level, to apply the continuum mechanics principles at the macroscopic level.

2. The Model

Let Ω be a porous medium and $\rho(x)$ the density; we consider the porous matrix (porous or skeleton) to be filled with a relatively incompressible viscous fluid which diffuses through. The small movements of both solid and liquid phases are verified, neglecting the speed of filtration. We write the coupled system of hyperbolic-parabolic type as

$$\begin{aligned}\rho(x)\frac{\partial^2 u}{\partial t^2} &- \nabla\left(\lambda^*(x)\frac{\partial}{\partial t}\operatorname{div} u\right) \\ &- \nabla\left(\left(\lambda(x)+\mu(x)\right)\operatorname{div} u\right) - \operatorname{div}\left(\mu(x)\nabla u\right) \\ &+ \alpha\nabla p = f(t,x), \\ c_0(x)\frac{\partial p}{\partial t} &+ \alpha\operatorname{div}\frac{\partial u}{\partial t} - \operatorname{div}\left(k(x)\nabla p\right) = h(t,x),\end{aligned} \quad (1)$$

where $u(t,x)$, $p(t,x)$, and $f(t,x)$ are the velocity of the solid matrix, the fluid pressure, and the external forces acting on the macroscopic element, for all x in Ω and all $t > 0$, respectively, with $h(t,x)$ a source term. The parameters $\lambda(x)$ and $\mu(x)$ denote the expansion and the shear modulus, respectively, whereas the coefficients $\lambda^*(x)$, $c_0(x)$, and $k(x)$ are positive such that $\lambda^*(x)$ is associated with the consolidation side effects and may cancel, $c_0(x) \geq 0$ is the coefficient that combines the porosity of the medium and the compressibility of the fluid-solid structure and $k(x)$ is the one which takes into account the permeability of the medium and the viscosity of the fluid, since it is a measure of flow obeying the Darcy law for a given pressure gradient. The constant α is a positive real number, representing the Biot-Willis constant which takes into account the effects of coupling the deformation and the pressure: it is actually a measure of the amount of fluid that can be placed in the porous matrix by increasing the pressure at constant volume.

3. One-Dimensional Biot Model

We are interested in the one-dimensional nonlinear model of the following form:

$$\begin{aligned} &\rho\partial_t^2 u - \lambda^*\partial_t\partial_x^2 u - (\lambda+2\mu)\,\partial_x^2 u - \mu^*\partial_x\left(\left|\partial_x u\right|^{q-2}\partial_x u\right) \\ &\quad + \alpha\partial_x p = f, \\ &c_0\partial_t p + \alpha\partial_t\partial_x u - k\partial_x^2 p = h. \end{aligned} \tag{2}$$

The nonlinear term in the first equation of (2) is due to the local geometry of the medium, such as sudden changes in contact areas or occlusions of cracks, with $\mu^*(x) \geq 0$. Here $q \geq 0$ and λ^* is a positive constant if $q \neq 2$ and vanishes when $q = 2$. Besides the system physical parameters are assumed to be constant and independent of the space variable.

This system appears when we consider the particular case of a wave propagating in a single direction. In this case the displacement depends only on one variable denoted by x and the scalar u represents the component along the x-direction and the same for the pressure p.

Let us consider $\Omega = [a,b]$, $a, b \in \mathbb{R}$, $a < b$, to be the saturated porous medium which occurs in the propagation of the wave and let Q be the cylinder $Q = [0,T] \times \Omega$.

When $\rho = 0$, the system is transformed into a quasistatic system as follows:

$$\begin{aligned} &-\lambda^*\partial_t\partial_x^2 u - (\lambda+2\mu)\,\partial_x^2 u - \mu^*\partial_x\left(\left|\partial_x u\right|^{q-2}\partial_x u\right) \\ &\quad + \alpha\partial_x p = f, \\ &c_0\partial_t p + \alpha\partial_t\partial_x u - k\partial_x^2 p = h \end{aligned} \tag{3}$$

with the initial conditions

$$(u(0,x), p(0,x)) = (u_0(x), p_0(x)). \tag{4}$$

In addition, when $\rho > 0$, we can write

$$\partial u(0,x) = u_1(x) \tag{5}$$

and the homogeneous Dirichlet boundary conditions

$$u(t,a) = u(t,b) = p(t,a) = p(t,b) = 0. \tag{6}$$

Note that λ, μ, α, and c_0 are strictly positive constants.

3.1. Galerkin Method. The Galerkin method is used to prove the existence of the discrete solution. We consider a family of vector spaces $(V_m)_{m\in\mathbb{N}^*}$ that approaches an infinite dimensional Hilbert space V satisfying the following:

(i) $(V_m)_{m\in\mathbb{N}^*} \subset V$;

(ii) $V_m \to V$ when $m \to +\infty$ in the sense that there exists a dense subspace v of V, such that, for all $v \in V$, we can find a sequence $(v_m)_{m\in\mathbb{N}^*}$ satisfying: for all m, $v_m \in V_m$, and $v_m \to v$ in V when $m \to +\infty$.

These approximation spaces are generated using a family $(w_j)_{j\in\mathbb{N}^*}$ for V such that, for $m \in \mathbb{N}^*$, $V_m = \mathrm{Vect}\{w_1, w_2, \dots, w_m\}$. According to the choice of this family, we can construct solutions to problems that can be more or less regular.

In order to have sufficient regularity for the nonlinear term treatment, we make use of the eigenfunctions of the Laplace operator in $L^2(\Omega)$. First let us set the following results.

3.2. Known Results for Laplace's Operator

Proposition 1. *There exists a sequence $(w_j)_{j\in\mathbb{N}^*}$ such that*

$$\begin{aligned} \forall j \in \mathbb{N}^*, \quad &w_j \in H_0^1(\Omega) \cap H^2(\Omega) \\ &-\Delta w_j = \lambda_j w_j \end{aligned} \tag{7}$$

with $w_j \neq 0$. The set $(w_j)_{j\in\mathbb{N}^*}$ is a Hilbertian basis for $L^2(\Omega)$ and the space of the finite linear combinations of w_j is dense in $H_0^1(\Omega)$ and in $H_0^1(\Omega) \cap H^2(\Omega)$.

Proposition 2. *Let $V_m = \mathrm{Vect}\{w_1, w_2, \dots, w_m\}$ where $(w_j)_{j\in\mathbb{N}^*}$ is defined by (7) and let P_m be the projection onto V_m defined on $(H_0^1(\Omega) \cap H^2(\Omega))'$ by*

$$P_m(v) = \sum_{i=1}^{m} \langle v, w_i\rangle\, w_i, \tag{8}$$

where $\langle\cdot,\cdot\rangle$ denotes the duality product between $H_0^1(\Omega) \cap H^2(\Omega)$ and its dual. Then the restriction of P_m to $L_2(\Omega)$ on V_m is $P_m \in \mathscr{L}(L_2(\Omega))$ with $\|P_m\|_{\mathscr{L}(L_2(\Omega))} = 1$. In addition, the following properties

$$\begin{aligned} &P_m \in \mathscr{L}\left(H_0^1(\Omega) \cap H^2(\Omega)\right), \\ &P_m \in \mathscr{L}\left(\left(H_0^1(\Omega) \cap H^2(\Omega)\right)'\right) \end{aligned} \tag{9}$$

are satisfied and the norms $\|P_m\|_{\mathscr{L}((H_0^1(\Omega)\cap H^2(\Omega))')}$ and $\|P_m\|_{\mathscr{L}(H_0^1(\Omega)\cap H^2(\Omega))}$ are independent of m.

4. One-Dimensional Nonlinear Stochastic Biot Model

Here we are interested in studying the Biot model in the presence of a stochastic perturbation. The idea is to develop a mathematical analysis of the above equations with a stochastic perturbation. This is based on analyzing the problem in a weak form by making use of appropriate functional spaces.

4.1. Weak Formulation. We study the model of consolidation in the case where $\rho \geq 0$ and $q \geq 2$ by using the weak formulation.

Find $(u, p) \in L^\infty([0,T]; W_0^{1,q}(\Omega)) \times L^\infty([0,T]; H_0^1(\Omega))$ such that $u \in L^2([0,T]; W^2(\Omega))$, $\partial_t u \in L^2([0,T]; W_0^1(\Omega))$, $\partial_t p \in L^2([0,T]; L^2(\Omega))$, and $\rho\partial_t^2 u \in L^2([0,T]; W^{-1,q^*}(\Omega))$, satisfying

$$\begin{gathered}\rho \left\langle \partial_t^2 u, v\right\rangle_{W^{-1,q^*}(\Omega), W_0^{1,q}(\Omega)} + \lambda^* \int_\Omega \partial_t \partial_x u \partial_x v\, dx \\ - \alpha \int_\Omega p \partial_x v\, dx + (\lambda + 2\mu) \int_\Omega \partial_x u \partial_x v\, dx \\ + \mu^* \int_\Omega \left|\partial_x u\right|^{q-2} \partial_x u \partial_x v\, dx = \int_\Omega f v\, dx + \int_\Omega v\, dG \\ c_0 \int_\Omega \partial_t p r\, dx + \alpha \int_\Omega \partial_t \partial_x u r\, dx \\ + k \int_\Omega \partial_x p \partial_x r\, dx = \int_\Omega h r\, dx \\ (u(0,x), p(0,x)) = (u_0(x), p_0(x)) \\ \partial u(0,x) = u_1(x)\end{gathered} \tag{10}$$

for almost every t in $]0,T[$ and $\forall (v, r) \in W_0^{1,q}(\Omega) \times H_0^1(\Omega)$. Here q^* is real and is defined by the relation $1/q + 1/q^* = 1$. It is assumed that the initial conditions (u_0, p_0) and u_1 (when $\rho > 0$) are in $H_0^1(\Omega) \cap H^2(\Omega) \times H_0^1(\Omega)$ and $L^2(\Omega)$, respectively, and the source terms belong to $L^2([0,T]; L^2(\Omega))$. We are also supposed to have a major regularity for the disturbance G, so that the resolution of the stochastic equation is reduced for each element ω of the probability space Ω, to a deterministic equation. Hence G is a continuous stochastic process with values in Ω (i.e., continuous trajectory of the disturbance) and defined on a probability space (Ω, F, P). Any solution of the variational formulation (10) is called the solution of the nonlinear stochastic consolidation Biot model. This solution is obtained by solving the equations in (10) for each $\omega \in \Omega$.

Theorem 3. *Let q be real such that $q \geq 2$, $u_0 \in H_0^1(\Omega) \cap H^2(\Omega)$, $p_0 \in H_0^1(\Omega)$, $u_1 \in L^2(\Omega)$, and f and $h \in L^2([0,T]; L^2(\Omega))$. Let G be a stochastic process defined in Ω. There is a single pair of random variables (u, p) such that $u \in L^\infty([0,T]; W^{1,\infty}(\Omega))$ and $p \in L^\infty([0,T]; H_0^1(\Omega))$, satisfying the system of (10).*

4.2. Existence of the Solution. In this section, we propose to formulate the equations whose solutions constitute the Faedo-Galerkin approximation type of our problem, in $H_0^1(\Omega) \cap H^2(\Omega)$ (for more details on the Faedo-Galerkin approximations the reader is referred to [15, 17] and references therein). A solution is constructed as the limit of a sequence of approximate solutions denoted by $(u_m, p_m)_{m\in\mathbb{N}^*}$. This sequence $(u_m, p_m)_{m\in\mathbb{N}^*}$ is defined from $]0,T[$ in $V_m \times V_m$ by

$$\begin{aligned} u_m(t) &= \sum_{j=1}^{m} u_{jm}(t)\, w_j, \\ p_m(t) &= \sum_{j=1}^{m} p_{jm}(t)\, w_j, \end{aligned} \tag{11}$$

where $(w_j)_{j\in\mathbb{N}^*}$ is the sequence defined in Proposition 1 of Section 3.2. To each integer $m \in \mathbb{N}^*$, we associate a new discrete unknown by the use of the sequence $(u_{jm(t)}, p_{jm(t)})_{1<j<m}$ that is defined by solving the differential system

$$\begin{aligned} &\rho \int_\Omega \partial_t^2 u_m w_j\, dx + \lambda^* \int_\Omega \partial_t \partial_x u_m \partial_x w_j\, dx \\ &\quad + (\lambda + 2\mu) \int_\Omega \partial_x u_m \partial_x w_j\, dx \\ &\quad + \mu^* \int_\Omega \left|\partial_x\right|^{q-2} u_m \partial_x u_m \partial_x w_j\, dx \\ &\quad - \alpha \int_\Omega p_m \partial_x w_j\, dx = \int_\Omega f w_j\, dx + \int_\Omega w_j\, dG_m \quad p.s \\ &c_0 \int_\Omega \partial_t p_m w_j\, dx + \alpha \int_\Omega \partial_t \partial_x u_m w_j\, dx \\ &\quad + k \int_\Omega \partial_x p_m \partial_x w_j\, dx = \int_\Omega h w_j\, dx, \end{aligned} \tag{12}$$

for all $1 \leq j \leq m$, with initial conditions

$$\begin{gathered} (u_m(0), p_m(0)) = (u_{0m}, p_{0m}) \in V_m \times V_m \text{ such that} \\ u_{0m} \longrightarrow u_0 \quad \text{in } H_0^1(\Omega) \cap H^2(\Omega), \\ p_{0m} \longrightarrow p_0 \quad \text{in } H_0^1(\Omega) \\ \rho \partial_t u_m(0) = \rho u_{1m} \in V_m \text{ such that} \\ p_{1m} \longrightarrow p_1 \quad \text{in } L^2(\Omega), \end{gathered} \tag{13}$$

where $u(\omega, t) \in V_m$, $\forall t \in [0,T]$ $p.s.$

G_m is a continuous stochastic process with values in V_m and defined on a probability space (Ω, F, P). The solution of (12) is obtained by solving the equations for each fixed $\omega \in \Omega$. Therefore we consider the following deterministic equations:

$$\begin{aligned} &\rho \int_\Omega \partial_t^2 u_m w_j\, dx + \lambda^* \int_\Omega \partial_t \partial_x u_m \partial_x w_j\, dx \\ &\quad + (\lambda + 2\mu) \int_\Omega \partial_x u_m \partial_x w_j\, dx \\ &\quad + \mu^* \int_\Omega \left|\partial_x\right|^{q-2} \partial_x u_m \partial_x w_j\, dx \\ &\quad - \alpha \int_\Omega p_m \partial_x w_j\, dx \\ &\quad = \int_\Omega f w_j\, dx + \int_\Omega G_m w_j\, dx \\ &c_0 \int_\Omega \partial_t p_m w_j\, dx + \alpha \int_\Omega \partial_t \partial_x u_m w_j\, dx \\ &\quad + k \int_\Omega \partial_x p_m \partial_x w_j\, dx = \int_\Omega h w_j\, dx, \end{aligned} \tag{12 bis}$$

for all $1 \le j \le m$, with initial conditions

$$
\begin{gathered}
(u_m(0), p_m(0)) \\
= (u_{0m}, p_{0m}) \in V_m \times V_m \text{ such that} \\
u_{0m} \longrightarrow u_0 \quad \text{in } H_0^1(\Omega) \cap H^2(\Omega), \\
p_{0m} \longrightarrow p_0 \quad \text{in } H_0^1(\Omega) \\
\rho \partial_t u_m(0) = \rho u_{1m} \in V_m \text{ such that} \\
u_{1m} \longrightarrow u_1 \quad \text{in } L^2(\Omega)
\end{gathered}
\tag{13 bis}
$$

with $u(t) \in V_m$, $\forall t \in [0, T]$, and G_m is a continuous function with values in V_m.

Theorem 4. *The existence of sequences $(u_{0m})_{m\in\mathbb{N}^*}$, $(u_{1m})_{m\in\mathbb{N}^*}$, and $(p_{0m})_{m\in\mathbb{N}^*}$ satisfying the properties* (13 bis) *is a consequence of Propositions 1 and 2 in Section 3.2. Problem* ((12 bis)-(13 bis)) *satisfies the Cauchy-Lipschitz conditions and, from the nonlinear differential equations theory,* ((12 bis) (13 bis)) *admits a unique maximal solution $(u_m, p_m)_{m\in\mathbb{N}^*}$ in $H^1([0,T_m]; H_0^1(\Omega) \cap H^2(\Omega)) \times H^1([0,T_m]; H_0^1(\Omega))$, $T_m > 0$, such that $\rho u_m \in H^2([0,T_m]; H_0^1(\Omega) \cap H^2(\Omega))$.*

The idea is to show that there exists $(u_m, p_m)_{m\in\mathbb{N}^*}$ that converges to problem ((12 bis)-(13 bis)) solution (u, p). It is sufficient to extract a converging sequence $(u_m, p_m)_{m\in\mathbb{N}^*}$, where its existence is satisfied by a priori estimates which prove that the sequence is bounded in suitable functional spaces from the bounded differential equation solution's principle. The time T_m for which solutions exist is equal to the initially given time T.

Lemma 5. *The sequence $(u_m, p_m)_{m\in\mathbb{N}^*}$ of the solutions of problem* ((12 bis)-(13 bis)) *satisfies the following properties:*

(i) $(\rho \partial_t^2 u_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^2([0,T]; (H_0^1(\Omega) \cap H^2(\Omega)'))$,

(ii) $(\partial_t u_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^2([0,T]; H_0^1(\Omega))$,

(iii) $(\sqrt{\rho}\partial_t u_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^\infty([0,T]; L^2(\Omega))$,

(iv) $(u_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^\infty([0,T]; W^{1,q}(\Omega))$,

(v) $(\partial_t p_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^2([0,T]; L^2(\Omega))$,

(vi) $(p_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^\infty([0,T]; H_0^1(\Omega))$.

Each estimate is independent of the physical parameter ρ but depends only on (u_0, p_0) and u_1 from $\|u_0\|_{W^{1,q}(\Omega)}$, $\|p_0\|_{H^1(\Omega)}$, and $\|u_1\|_{L^2(\Omega)}$.

Lemma 6. *Let $(u_m, p_m)_{m\in\mathbb{N}^*}$ be the sequence of the solutions of* ((12 bis)-(13 bis)). *Then*

(i) $(\partial_x^2 u_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^\infty(0,T; L^2(\Omega))$,

(ii) $(u_m)_{m\in\mathbb{N}^*}$ *is bounded in* $L^\infty(0,T; W^{1,q}(\Omega))$.

For the proof of Lemma 6, we use the same as arguments as [7].

For the proof of Theorem 3, we proceed as follows.

From Theorem 4 we have, for almost every ω, problem ((12 bis), (13 bis)) with $G_m = G_m(\omega)$ having one and only one solution: $(u, p) = (u(\omega), p(\omega)) \in C(0,T; V_m) \times C^1(0,T; V_m)$, satisfying ((12), (13)). It is sufficient to prove that $G_m \in C(0,T; V_m) \rightarrow (u, p)$ defined by the unique solution in Theorem 4 is measurable. Let G_m and $\overline{G_m}$ be two elements of $C(0,T; V_m)$ and let (u, p) be the solution of ((12 bis), (13 bis)) corresponding to G_m and $\overline{G_m}$, respectively. Using the uniqueness of the solution we have

$$
\begin{gathered}
v = u - \overline{u} = 0 \\
\psi = p - \overline{p} = 0.
\end{gathered}
\tag{14}
$$

That is to say $(u - \overline{u}) \rightarrow 0$ in $L^\infty(0,T; W_0^{1,q}(\Omega))$ and $(p - \overline{p}) \rightarrow 0$ in $L^\infty(0,T; H_0^1(\Omega))$.

This proves that the mapping: $G_m \rightarrow (u, p)$ is continuous and, from the compacity, G_m is measurable in the considered topology.

5. Numerical Results

5.1. Discretisation and Stability of the Scheme. We consider (2), taking $q = 2$ and $f = f + dW$:

$$
\begin{gathered}
-\lambda^* \partial_t \partial_x^2 u - (\lambda + 2\mu + \mu^*) \partial_x^2 u + \alpha \partial_x p = f + dW \\
c_0 \partial_t p + \alpha \partial_t \partial_x u - k \partial_x^2 p = h.
\end{gathered}
\tag{15}
$$

We use the Euler scheme for the time discretisation, the central finite differences for the space variable, and

$$
dW = W(n+1) - W(n) = \sqrt{n}\ \text{rand}\, n \tag{16}
$$

to obtain

$$
\begin{aligned}
U_{j+1}^{n+1} &= a_1 U_j^{n+1} + U_{j-1}^{n+1} + a_2 U_{j+1}^n + a_3 U_j^n \\
&\quad + a_4 U_{j-1}^n a_5 U_j^{n-1} a_6 \left(P_{j+1}^n - P_j^n\right) + a_7 \\
P_j^{n+1} &= b_1 \left(P_{j+1}^n + P_{j-1}^n\right) + b_2 P_j^n \\
&\quad + b_3 \left(U_{j-1}^{n+1} - U_j^{n+1} - U_{j+1}^n + U_j^n\right) + h^* dt
\end{aligned}
\tag{17}
$$

with $a_1 = (1/\rho^* dt + 2/(dx)^2)$; $a_2 = (dtA^*/\rho^*(dx)^2 - 1/(dx)^2)$; $a_3 = (2/\rho^* dt + 2/(dx)^2 - 2dtA^*/\rho^*(dx)^2)$; $a_4 = (dtA^*/\rho^*(dx)^2 - 1/(dx)^2)$; $a_5 = (-1/\rho^* dt)$; $a_6 = (\alpha^* dt/\rho^* dx)$, $a_7 = (B^* dt/\rho^*)$, $b_1 = dtk^*/(dx)^2$; $b_2 = (1 - 2dtk^*/(dx)^2)$, $b_3 = (-c^*/dx)$, $A = \lambda + 2\mu + \mu^*$; $B = f + dW$; $c^* = \alpha/c_0$; $k^* = k/c_0$; $h^* = h/c_0$; $\rho^* = \lambda^*/\rho$; $A^* = A/\rho$; $\alpha^* = \alpha/\rho$; $B^* = B/\rho$.

Using the Fourier stability analysis, we write

$$
\frac{2}{(dx)^2} + \frac{1}{\rho^* dt} < 1. \tag{18}
$$

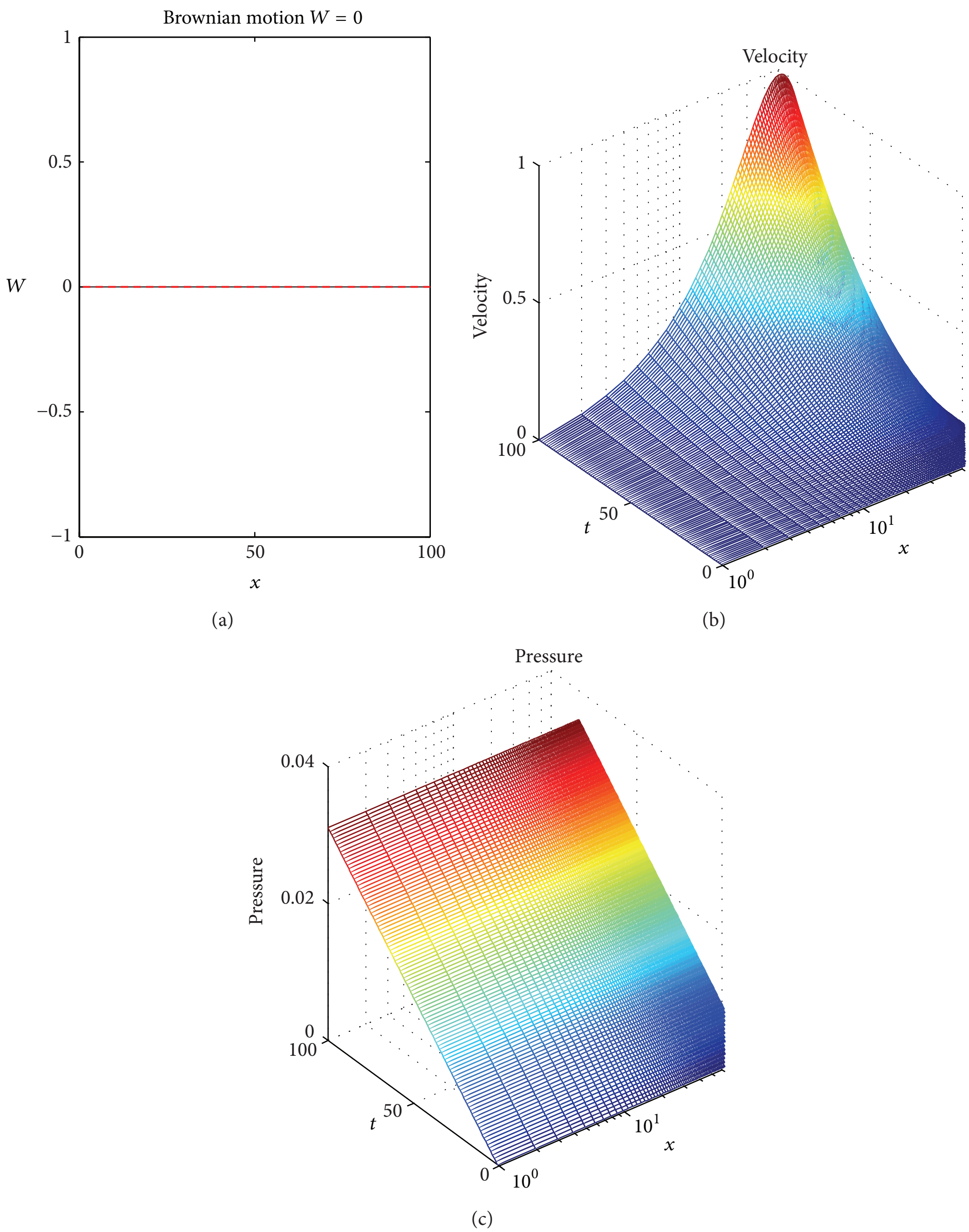

Figure 1: A null Brownian motion and the associated velocity and pressure.

This is the so-called CFL condition, which is the stability condition leading us to the right choice of the discretisation time and space steps.

5.2. Numerical Results and Comments. In Figures 1–3, we present the Brownian motion, the associated velocities, and pressures for different space and time steps as solutions of (15) using the finite difference scheme considered in Section 5.1. The numerical simulations were carried out using MATLAB with the parameters set as $\rho = 0.04$, $\lambda = 0.02$, $\lambda^* = 0.03$, $\mu = 0.03$, $\mu^* = 0.07$, $c_0 = 0.05$, $\alpha = 0.01$, and $k = 0.03$. The data are set as $f = 10$ and $h = 2.0$.

5.2.1. Comments on the Numerical Results. According to the numerical experimentations, in Figure 1, we present results for the deterministic case where the Brownian motion W is equal to 0. As shown in Figures 2 and 3, it is clear that the velocity and the pressure of the fluid behave randomly because of the stochastic part in (10). We also noticed that we experimented no unstability behavior as long as we respect the CFL condition (the stability condition) that is deducted from the stability analysis of the finite difference scheme considered (see Figure 3). To show this numerical instability behavior, we present numerical results for a condition equal to twice the CFL condition (see Figure 2).

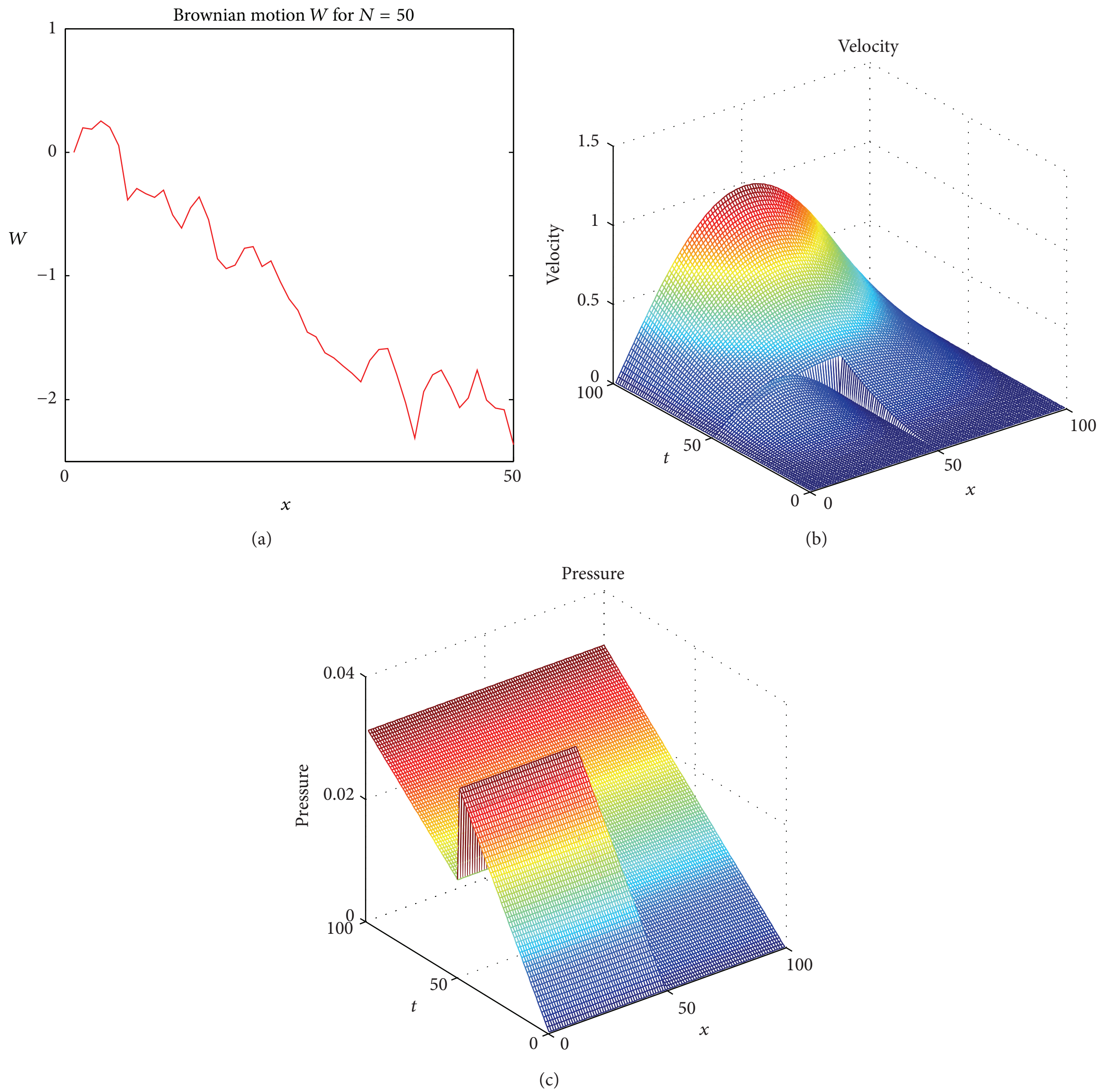

FIGURE 2: The Brownian, the associated velocity and pressure, and instable case.

5.3. Conclusion. We solved a system of two partial differential equations (PDEs) in a bounded domain, modeling the velocity and pressure; the first equation has a nonlinear part and another random part. A weak solution has been found in the space $H_0^1(\Omega) \cap H^2(\Omega)$ by the principle of the resolution of a stochastic differential equation (SDE) trajectory path. The velocity and the pressure were approached numerically by the Euler method for the time and the central finite difference for the space variable, where the time and space discretisation steps were chosen according to the numerical stability condition (CFL). The numerical results are resumed in Figures 1–3 and commented on above. It has to be pointed out that this model is quite important and can be used in many applications, for example, in image processing, and for this purpose a study on a comparison of this model with the one in [16] is under consideration.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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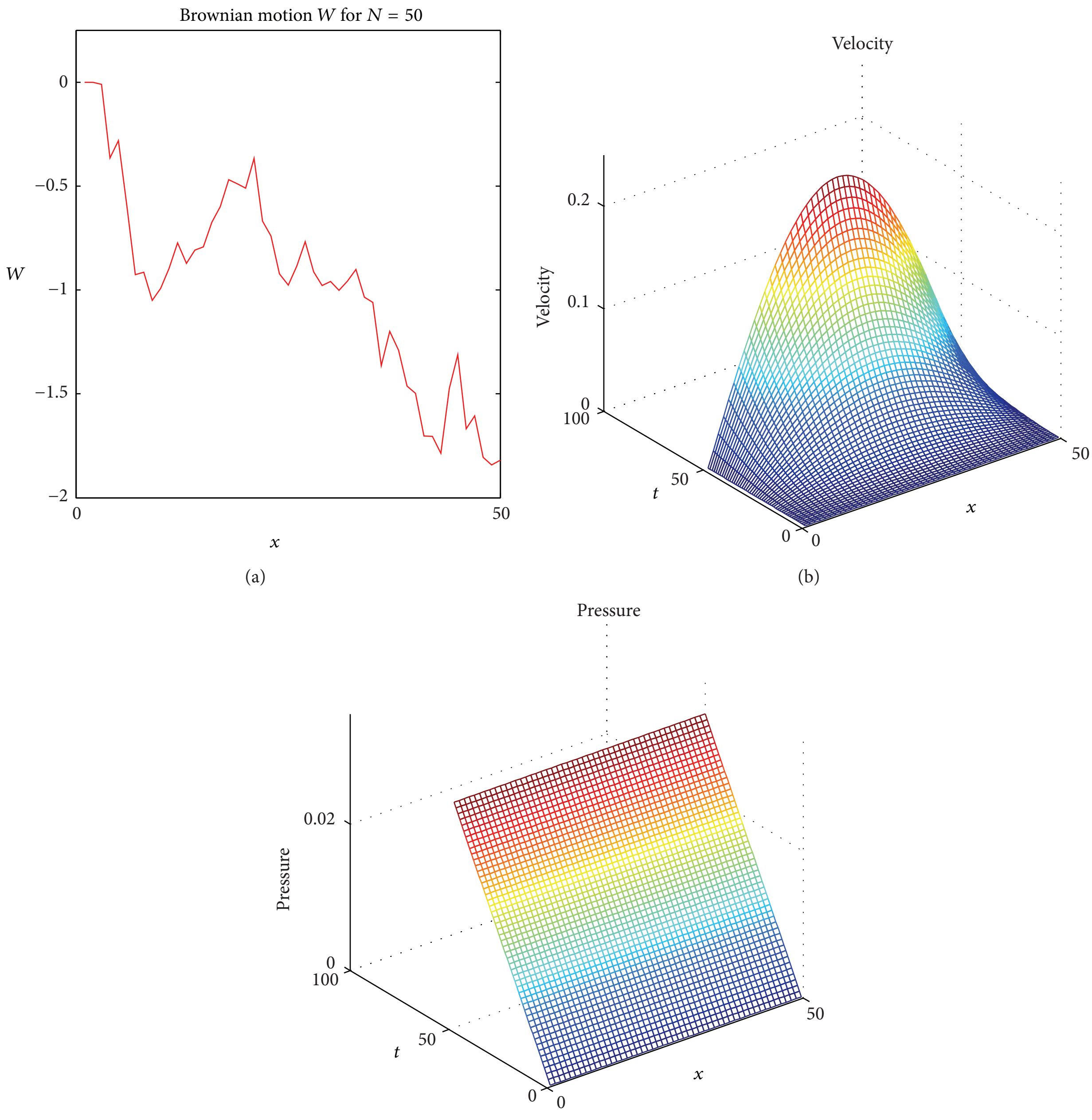

FIGURE 3: The Brownian, the associated velocity and pressure, and stable case.

recommendations, and suggestions that made the accomplishment of this work possible.

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