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Research Article

Modeling and Analysis of New Products Diffusion on Heterogeneous Networks

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We present a heterogeneous networks model with the awareness stage and the decision-making stage to explain the process of new products diffusion. If mass media is neglected in the decision-making stage, there is a threshold whether the innovation diffusion is successful or not, or else it is proved that the network model has at least one positive equilibrium. For networks with the power-law degree distribution, numerical simulations confirm analytical results, and also at the same time, by numerical analysis of the influence of the network structure and persuasive advertisements on the density of adopters, we give two different products propagation strategies for two classes of nodes in scale-free networks.

1. Introduction

The Bass model has become an important exemplar in marketing science. For over three decades, this model has been the main impetus underlying diffusion research and has been widely used to understand the diffusion of new products [1–3]. The following equation illustrates the classical Bass model [4]:

$$\frac{dN(t)}{dt} = \frac{q}{m} N(t)(m - N(t)) + p(m - N(t)), \quad (1)$$

where $N(t)$ is the number of cumulative adopters, p is the coefficient of external influence, q is the coefficient of internal influence, and m is the market potential or potential number of ultimate adopters. On the basis of the Bass model, assuming that populations on the market are homogeneous and well mixed, many dynamics models are used to study new products diffusion; Yu et al. proposed mathematical models to describe the dynamics of competitive products in the market and analyzed the stability of equilibria of the model [5–7]. Wang et al. proposed describing the dynamics of users of one product in two different patches; periodic advertisements are also incorporated and the existence and uniqueness of positive periodic solutions are investigated [8]. Considering that an observation process cannot be neglected for the consumers of many products, especially for the products with high values, Wang et al. established a new production diffusion model with two stages (the awareness stage and the decision-making stage) and obtained a threshold above which innovation diffusion is successful [9, 10]. All these models incorporate the imitation among the population but without the explicit network structure. In fact, the consumer's decision-making process on the new product adoption involves a complex interaction of various external and internal factors like mass media, advertising, word of mouth, personal preferences, and experience. Without a doubt, a friend's opinion or advice often can be a decisive argument for a purchase. So, network models considering social contacts and the individual heterogeneity about new products diffusion are relatively reasonable.

The products information spread in the diffusion process is similar to the typical virus spread model such as SIS model and SIR model. The spread mechanism of diseases on networks has been widely studied by many researchers. Many researchers have given different interpretations for the propagation of diseases on uncorrelated complex networks

[11–17]; Boguna et al. discuss the epidemic spreading on complex networks in which there are explicit correlations among the degrees of connected vertices [18]. With the development of the disease model on networks, researches about new products diffusion on complex networks have raised a growing interest [19, 20]; however, they still do not have specific network dynamics models or dynamics analysis of the model. So, we intend to model and analyze the process of new products diffusion on heterogeneous networks in this paper. Considering two stages, the awareness stage and the decision-making stage in the product diffusion process, and assuming that the product information is spread due to mouth-to-mouth method among people's direct contacts and mass media channels (in the awareness stage, mass media is informative, and in the decision-making stage, mass media is persuasive), we propose the heterogeneous networks model of new products diffusion with two stages.

The organization of this paper is as follows. In the next section, we set up a heterogeneous network model about new products diffusion and discuss its invariant set. In Section 3, when mass media is neglected in the decision-making stage, we discuss equilibria and their stabilities and obtain conditions that the diffusion is successful; when mass media is considered, we obtain that the system at least has a positive equilibrium. The theoretical results are also confirmed by numerical simulations in Section 4; furthermore, we also obtain two different propagation strategies by simulations analysis. Finally, we give a brief conclusion and discussion in Section 5.

2. Dynamics Model of New Products Diffusion

By incorporating the impact of social neighborhood in the new products diffusion process, we establish networks model describing the new products diffusion procedure. Here, we consider the whole population and their contacts in networks. Each individual in the region under consideration can be regarded as a vertex in the network, and each contact between two individuals is represented as an edge connecting the vertices. The number of edges emanating from a vertex, that is, the number of contacts a person has, is called the degree of the vertex. Therefore, we divide the population into n distinct groups of sizes N_k $(k = 1, 2, \ldots, n)$ such that each individual in group k has exactly k contacts per day. If the whole population size is N $(N = N_1 + N_2 + \cdots + N_n)$, then the probability that a uniformly chosen individual has k contacts is $p(k) = N_k/N$, which is called the degree distributions of the network. Let $U_k(t)$ denote the number of those individuals who have not been aware of the product within group k at time t, let $I_k(t)$ denote the number of those individuals who have been aware of the information about the product but have not yet adopted it within group k at time t, and let $A_k(t)$ denote the number of those individuals who have adopted the product at time t. We consider two stage: in the first stage, enterprises transfer new production information to consumers through informative advertisements, or individuals obtain new product information from their neighbors, so individuals become aware of information; in the decision-making stage, enterprises change consumers' preference through persuasive advertisements, or a friend's opinion or advice also influences individuals' preferences, so individuals try or adopt the product. We give a system of $3n$ ordinary differential equations representing the product diffusion dynamics on the scale-free network:

Table 1: Parameters of model 1.

Parameters	Description
p	The fraction of individuals who have not been aware of the product obtained the new product information from informative advertisements in the awareness stage
β_1	The effective transmission coefficient between communities U_k and I_k in the awareness stage
β_2	The effective transmission coefficient between communities U_k and A_k in the awareness stage
β_3	The effective transmission coefficient between communities I_k and A_k in the decision-making stage
γ	The rate at which individuals in awareness class forget the information of the product
μ	The coefficient of discontinuance rate of adopters
α	The fraction of individuals who have been aware of the product turned to be adopters owing to persuasive advertisements in the decision-making stage

$$\begin{aligned}\frac{dU_k(t)}{dt} &= -pU_k(t) - \beta_1 kU_k(t)\Theta_1(t) - \beta_2 kU_k(t)\Theta_2(t) \\ &\quad + \gamma I_k(t) + \mu A_k(t), \\ \frac{dI_k(t)}{dt} &= pU_k(t) + \beta_1 kU_k(t)\Theta_1(t) + \beta_2 kU_k(t)\Theta_2(t) \\ &\quad - \alpha I_k(t) - \gamma I_k(t) - \beta_3 kI_k(t)\Theta_2(t), \\ \frac{dA_k(t)}{dt} &= \alpha I_k(t) + \beta_3 kI_k(t)\Theta_2(t) - \mu A_k(t),\end{aligned} \tag{2}$$

where

$$\begin{aligned}\Theta_1(t) &= \frac{\sum_{k=1}^{n} kI_k}{\sum_{k=1}^{n} kN_k}, \\ \Theta_2(t) &= \frac{\sum_{k=1}^{n} kA_k}{\sum_{k=1}^{n} kN_k}.\end{aligned} \tag{3}$$

All parameters are positive constants and the meaning of parameters is summarized in Table 1. When $\beta_1 = 0$ and population contact is assumed to be homogeneous, system (2) becomes the model in [9].

Denote the relative densities of $U_k(t)$, $I_k(t)$, and $A_k(t)$ at time t by $u_k(t)$, $i_k(t)$, and $a_k(t)$, respectively; then, system (2) can be rewritten as

$$\begin{aligned}
\frac{du_k(t)}{dt} &= -pu_k(t) - \beta_1 k u_k(t)\Theta_1(t) - \beta_2 k u_k(t)\Theta_2(t) \\
&\quad + \gamma i_k(t) + \mu a_k(t), \\
\frac{di_k(t)}{dt} &= pu_k(t) + \beta_1 k u_k(t)\Theta_1(t) + \beta_2 k u_k(t)\Theta_2(t) \\
&\quad - \alpha i_k(t) - \gamma i_k(t) - \beta_3 k i_k(t)\Theta_2(t), \\
\frac{da_k(t)}{dt} &= \alpha i_k(t) + \beta_3 k i_k(t)\Theta_2(t) - \mu a_k(t),
\end{aligned} \tag{4}$$

with the normalization condition $u_k(t)+i_k(t)+a_k(t) = 1$, and $\Theta_1(t) = (1/\langle k\rangle)\sum_{k=1}^{n} kp(k)i_k$, $\Theta_2(t) = (1/\langle k\rangle)\sum_{k=1}^{n} kp(k)a_k$. Using this condition, system (4) is reduced to

$$\begin{aligned}
\frac{di_k(t)}{dt} &= \left(p + \beta_1 k\Theta_1(t) + \beta_2 k\Theta_2(t)\right)\left(1 - i_k(t) - a_k(t)\right) \\
&\quad - \gamma i_k(t) - \left(\alpha + \beta_3 k\Theta_2(t)\right) i_k(t), \\
\frac{da_k(t)}{dt} &= \left(\alpha + \beta_3 k\Theta_2(t)\right) i_k(t) - \mu a_k(t).
\end{aligned} \tag{5}$$

Prior to discussing the stability of system (5), we first study its variant set.

Let $i_k = y_k$, $k = 1,2,\ldots,n$, $a_k = y_{n+k}$, $k = 1,2,\ldots,n$, $y = (y_1, y_2, \ldots, y_{2n}) = (i_1, i_2, \ldots, i_n, a_1, a_2, \ldots, a_n)$. Denoting that $\Delta_{2n} = \sum_{l=1}^{2n}[0,1]$, we study system (5) for $y(t) \in \Delta_{2n}$.

Lemma 1. *The set Δ_{2n} is positively invariant set of system* (5).

Proof. We will show that all the solutions starting from any initial value $y(0) \in \Delta_{2n}$ of system (5) satisfy $y(t) \in \Delta_{2n}$. Denote that

$$\partial\Delta_{2n}^1 = \{y \in \Delta_{2n} \mid y_i = 0 \text{ for some } i\},$$

$$\partial\Delta_{2n}^2 = \{y \in \Delta_{2n} \mid y_i = 1 \text{ for some } i\},$$

where $i = 1,2,\ldots,2n$. Let the "outer normals" be denoted by $\eta_i^1 = (0,\ldots,-1,\ldots,0)$ and $\eta_i^2 = (0,\ldots,1,\ldots,0)$.

For arbitrary compact set Γ, Nagumo had proved that Γ is invariant for $dx/dt = f(x)$, if, at each point y in $\partial\Gamma$ (the boundary of Γ), the vector $f(x)$ is tangent or pointing into the set [21, 22]. We can easily apply the result here, since Γ is an $2n$-dimensional rectangle. Through Nagumo's result, it is not difficult to obtain that

$$\begin{aligned}
&\left(\left.\frac{dy}{dt}\right|_{y_i=0} \cdot \eta_i^1\right) \\
&\quad = -\left(p + \frac{\beta_1 i}{\langle k\rangle}\sum_{k\neq i} kp(k)\, y_k + i\beta_2\Theta_2\right)\left(1 - y_{i+n}\right) \\
&\quad \le 0, \quad i = 1,2,\ldots,n, \\
&\left(\left.\frac{dy}{dt}\right|_{y_{n+i}=0} \cdot \eta_i^1\right) = -\left(\alpha + \frac{\beta_3 i}{\langle k\rangle}\sum_{k\neq i} kp(k)\, y_{n+k}\right) \le 0, \\
&\qquad\qquad i = 1,2,\ldots,n, \\
&\left(\left.\frac{dy}{dt}\right|_{y_i=1} \cdot \eta_i^2\right) \le 0, \quad i = 1,2,\ldots,2n.
\end{aligned} \tag{6}$$

Hence, Lemma 1 is proved. □

3. Model Analysis

Next, for investigating the effect of mass media in the decision-making stage, we will analyze the dynamics of system (5) under two kinds of assumptions as followed:

(1) persuasive advertisements are neglected in the decision-making stage, which means $\alpha = 0$,

(2) persuasive advertisements are considered in the decision-making stage, which means $\alpha \neq 0$.

Case I ($\alpha = 0$). When $\alpha = 0$, equilibria for system (5) can be found by setting the right sides of two differential equations of system (5) equal to zero, giving the algebraic system

$$\begin{gathered}
\left(p + \beta_1 k\Theta_1 + \beta_2 k\Theta_2\right)\left(1 - i_k - a_k\right) - \gamma i_k - \beta_3 k i_k\Theta_2 = 0, \\
\beta_3 k i_k\Theta_2 - \mu a_k = 0.
\end{gathered} \tag{7}$$

We can get the equivalent system of system (7) as follows:

$$\begin{aligned}
i_k &= \frac{\mu\left(p + \beta_1 k\Theta_1 + \beta_2 k\Theta_2\right)}{\left(\mu + \beta_3 k\Theta_2\right)\left(p + \beta_1 k\Theta_1 + \beta_2 k\Theta_2\right) + \mu\left(\gamma + \beta_3 k\Theta_2\right)}, \\
a_k &= \frac{\beta_3 k\Theta_2\left(p + \beta_1 k\Theta_1 + \beta_2 k\Theta_2\right)}{\left(\mu + \beta_3 k\Theta_2\right)\left(p + \beta_1 k\Theta_1 + \beta_2 k\Theta_2\right) + \mu\left(\gamma + \beta_3 k\Theta_2\right)},
\end{aligned} \tag{8}$$

and it is obvious that $a_k = 0$ satisfies the second formula in (8); substituting $a_k = 0$ into the first formula in (8), we obtain

$$i_k = \frac{p + \beta_1 k\Theta_1}{p + \gamma + \beta_1 k\Theta_1} \tag{9}$$

and a self-consistency equation about Θ_1 as follows:

$$\Theta_1 = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\,\frac{p + \beta_1 k\Theta_1}{p + \gamma + \beta_1 k\Theta_1} \triangleq f\left(\Theta_1\right). \tag{10}$$

Through some calculation, we have that

$$0 < f(0) = \frac{p}{p+\gamma} < 1,$$
$$0 < f(1) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\frac{p+\beta_1 k}{p+\gamma+\beta_1 k} < 1, \tag{11}$$

$$f'(\Theta_1) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\frac{\beta_1\gamma k}{(p+\gamma+\beta_1 k\Theta_1)^2} > 0, \tag{12}$$

$$f''(\Theta_1) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\frac{-2\beta_1^2 k^2\gamma}{(p+\gamma+\beta_1 k\Theta_1)^3} < 0. \tag{13}$$

According to formula (11), we can know that (10) has at least a positive solution in $(0,1)$; formula (12) shows that the function $f(\Theta_1)$ is a rigorous monotone increasing function, and formula (13) means that the function $f(\Theta_1)$ is convex, so (10) must has a unique positive solution Θ_1^0 in $(0,1)$, and system (5) has a trivial solution $E^0 = (i_1^0, i_2^0, \ldots, i_n^0, 0, 0, \ldots, 0)$, where

$$i_k^0 = \frac{p+\beta_1 k\Theta_1^0}{p+\gamma+\beta_1 k\Theta_1^0}, \quad k = 1,2,\ldots,n, \tag{14}$$

$$\Theta_1^0 = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\, i_k^0. \tag{15}$$

The above results can be summarized in the following theorem.

Theorem 2. *When $\alpha = 0$, system (5) has a unique trivial equilibrium E^0.*

Furthermore, one will prove the local stability and global stability of the equilibrium E^0. The Jacobin matrix of system (5) at E^0 is

$$J|_{E^0} = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix}, \tag{16}$$

where

$$B = \begin{pmatrix} b_{11} & \beta_1\frac{2p(2)}{\langle k\rangle}u_1^0 & \cdots & \beta_1\frac{np(n)}{\langle k\rangle}u_1^0 \\ 2\beta_1\frac{p(1)}{\langle k\rangle}u_2^0 & b_{22} & \cdots & 2\beta_1\frac{np(n)}{\langle k\rangle}u_2^0 \\ \vdots & \vdots & \ddots & \vdots \\ n\beta_1\frac{p(1)}{\langle k\rangle}u_n^0 & n\beta_1\frac{2p(2)}{\langle k\rangle}u_n^0 & \cdots & b_{nn} \end{pmatrix},$$

$$C = \begin{pmatrix} c_{11} & \frac{2p(2)}{\langle k\rangle}(\beta_2 u_1^0 - \beta_3 i_1^0) & \cdots & \frac{np(n)}{\langle k\rangle}(\beta_2 u_1^0 - \beta_3 i_1^0) \\ 2\frac{p(1)}{\langle k\rangle}(\beta_2 u_2^0 - \beta_3 i_2^0) & c_{22} & \cdots & 2\frac{np(n)}{\langle k\rangle}(\beta_2 u_2^0 - \beta_3 i_2^0) \\ \vdots & \vdots & \ddots & \vdots \\ n\frac{p(1)}{\langle k\rangle}(\beta_2 u_n^0 - \beta_3 i_n^0) & n\frac{2p(2)}{\langle k\rangle}(\beta_2 u_n^0 - \beta_3 i_n^0) & \cdots & c_{nn} \end{pmatrix}, \tag{17}$$

$$D = \begin{pmatrix} \beta_3\frac{p(1)}{\langle k\rangle}i_1^0 - \mu & \beta_3\frac{2p(2)}{\langle k\rangle}i_1^0 & \cdots & \beta_3\frac{np(n)}{\langle k\rangle}i_1^0 \\ 2\beta_3\frac{p(1)}{\langle k\rangle}i_2^0 & 2\beta_3\frac{2p(2)}{\langle k\rangle}i_2^0 - \mu & \cdots & 2\beta_3\frac{np(n)}{\langle k\rangle}i_2^0 \\ \vdots & \vdots & \ddots & \vdots \\ n\beta_3\frac{p(1)}{\langle k\rangle}i_n^0 & n\beta_3\frac{2p(2)}{\langle k\rangle}i_n^0 & \cdots & n\beta_3\frac{np(n)}{\langle k\rangle}i_n^0 - \mu \end{pmatrix};$$

here

$$b_{ll} = -\left(p + l\beta_1\Theta_1^0 + \gamma\right) + l^2\beta_1\frac{p(l)}{\langle k\rangle}u_l^0, \quad l = 1,\dots,n,$$
$$c_{ll} = l^2\frac{p(l)}{\langle k\rangle}\left(\beta_2 u_l^0 - \beta_3 i_l^0\right) - \left(p + l\beta_1\Theta_1^0\right), \quad l = 1,\dots,n,$$
$$u_l^0 = 1 - i_l^0, \quad l = 1,\dots,n. \tag{18}$$

Next, for studying the local stability of the equilibrium E^0, we first estimate eigenvalues by carrying out the similarity transformation about the matrix B. Letting $T = \operatorname{diag}(\delta_1, \delta_2, \dots, \delta_n)$, $\delta_i = iu_i^0$, $i = 1, 2\dots, n$, then

$$T^{-1}BT = \begin{pmatrix} b_{11} & \beta_1\frac{2^2p(2)}{\langle k\rangle}u_2^0 & \cdots & \beta_1\frac{n^2p(n)}{\langle k\rangle}u_n^0 \\ \beta_1\frac{p(1)}{\langle k\rangle}u_1^0 & b_{22} & \cdots & \beta_1\frac{n^2p(n)}{\langle k\rangle}u_n^0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_1\frac{p(1)}{\langle k\rangle}u_1^0 & \beta_1\frac{2^2p(2)}{\langle k\rangle}u_2^0 & \cdots & b_{nn} \end{pmatrix} \triangleq B^*. \tag{19}$$

Obviously, B and B^* have same eigenvalues. We can obtain $\beta_1(\langle k^2u_k^0\rangle/\langle k\rangle) = p + \gamma - p/\Theta_1^0$ using formula (14), so $b_{ll} = -(p + l\beta_1\Theta_1^0 + \gamma) + l^2\beta_1(p(l)/\langle k\rangle)u_l^0 < -(p + l\beta_1\Theta_1^0 + \gamma) + \beta_1(\langle k^2u_k^0\rangle/\langle k\rangle) = -l\beta_1\Theta_1^0 - p/\theta_1^0 < 0$, $l = 1, 2, \dots, n$; furthermore, $|b_{ll}| - \sum_{j\neq l, j=1}^{n}|B_{lj}^*| = (p + l\beta_1\Theta_1^0 + \gamma) - \beta_1(\langle k^2u_k^0\rangle/\langle k\rangle) > 0$, so B^* is strictly diagonally dominant and the principle diagonal elements are negative, according to the results of estimating distribution of eigenvalues for generalized diagonally dominant matrices [23], and then every eigenvalue of the matrix B^* has negative real part; that is to say, every eigenvalue of the matrix B has negative real part.

To find eigenvalues of the matrix D, we carry out similarity transformation to the matrix D; namely, the j column multiplied by $-p(j-1)/jp(j)$ is added to the $j-1$ column, and the $j-1$ row multiplied by $p(j-1)/jp(j)$ is added to the j row, $j = 2, 3, \dots, n$. Then we obtain the similarity matrix D^* as follows:

$$D^* = \begin{pmatrix} -\mu & 0 & \cdots & \beta_3\frac{np(n)}{\langle k\rangle}i_1^0 \\ 0 & -\mu & \cdots & \beta_3\frac{np(n)}{\langle k\rangle}\left(\frac{p(1)}{2p(2)}i_1^0 + 2i_2^0\right) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_3\frac{\langle k^2 i_k^0\rangle}{\langle k\rangle} - \mu \end{pmatrix}. \tag{20}$$

It is straightforward that $J|_{E^0}$ has $2n-1$ negative characteristic roots. When $\beta_3(\langle k^2 i_k^0\rangle/\langle k\rangle) - \mu < 0$, E_0 is locally asymptotically stable; otherwise, it is not stable. Letting $R \triangleq (\beta_3/\mu)(\langle k^2 i_k^0\rangle/\langle k\rangle)$, we have the following theorem.

Theorem 3. *When $\alpha = 0$, if $R < 1$, E_0 is locally asymptotically stable, and if $R > 1$, E_0 is unstable.*

Actually, one can further obtain the global stability of E_0 under stronger parameter conditions.

Theorem 4. *When $\alpha = 0$, if $R < (\beta_3\langle k^2\rangle/\mu\langle k\rangle) < 1$, E_0 is globally asymptotically stable.*

Proof. Considering system (5) with $\alpha = 0$, by the second formula in system (5), we have

$$\frac{da_k(t)}{dt} \le \beta_3 k\Theta_2(t) - \mu a_k(t), \tag{21}$$

and we consider the following auxiliary system:

$$\frac{da_k(t)}{dt} = \beta_3 k\Theta_2(t) - \mu a_k(t). \tag{22}$$

Multiplying formula (22) by $kp(k)/\langle k\rangle$ and summing over k, we obtain

$$\frac{d\Theta_2(t)}{dt} = \mu\Theta_2\left(\frac{\beta_3}{\mu}\frac{\langle k^2\rangle}{\langle k\rangle} - 1\right), \tag{23}$$

and the solution of (23) tends to 0 when $\beta_3\langle k^2\rangle/\mu\langle k\rangle < 1$, since system (23) is a quasi-monotone system; by the comparison theorem, $\lim_{t\to\infty}a_k = 0$ $(k = 1, 2, \dots, n)$ in system (5), we consider the limit equation of the first equation of system (5):

$$\frac{di_k(t)}{dt} = \left(p + \beta_1 k\Theta_1(t)\right)\left(1 - i_k(t)\right) - \gamma i_k(t); \tag{24}$$

the above formula has a unique equilibrium $(i_1^0, i_2^0, \dots, i_n^0)$ and it is locally asymptotically stable. So $\lim_{t\to\infty}i_k = i_k^0$, $k = 1, 2, \dots, n$. E_0 is globally attractive when $\beta_3\langle k^2\rangle/\mu\langle k\rangle < 1$; this, combined with Theorem 3, implies that E_0 is globally asymptotically stable when $R < \beta_3\langle k^2\rangle/\mu\langle k\rangle < 1$. □

For proving the existence of the positive equilibrium of system (5) with $\alpha = 0$, we discuss the existence of the positive solution of (8); by (8), we obtain the following equations about Θ_1, Θ_2:

$$\begin{aligned}\Theta_1 &= \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\ &\quad\times \left(\mu\left(p+\beta_1 k\Theta_1+\beta_2 k\Theta_2\right)\right) \\ &\qquad\times \left(\left(\mu+\beta_3 k\Theta_2\right)\left(p+\beta_1 k\Theta_1+\beta_2 k\Theta_2\right)\right. \\ &\qquad\quad \left.+\,\mu\left(\gamma+\beta_3 k\Theta_2\right)\right)^{-1}, \\ \Theta_2 &= \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\ &\quad\times \left(\beta_3 k\Theta_2\left(p+\beta_1 k\Theta_1+\beta_2 k\Theta_2\right)\right) \\ &\quad\times \left(\left(\mu+\beta_3 k\Theta_2\right)\left(p+\beta_1 k\Theta_1+\beta_2 k\Theta_2\right)\right. \\ &\qquad \left.+\,\mu\left(\gamma+\beta_3 k\Theta_2\right)\right)^{-1}.\end{aligned} \tag{25}$$

In consideration of $a_k \neq 0$, $\Theta_2 \neq 0$, from formula (25), we can know that $(\Theta_1, \Theta_2) \in \Omega_1 \triangleq \{0 < \Theta_1 < 1, 0 < \Theta_2 < 1\}$. By the identical transformation of the second equation in formula (25), we have

$$\tilde{f}(\Theta_1, \Theta_2) = 0, \tag{26}$$

where

$$\begin{aligned}\tilde{f}(\Theta_1, \Theta_2) &\triangleq \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\ &\quad\times \left(\left(p+\beta_1 k\Theta_1+\beta_2 k\Theta_2\right)\left(\beta_3 k\left(1-\Theta_2\right)-\mu\right)\right. \\ &\qquad \left.-\,\mu\left(\gamma+\beta_3 k\Theta_2\right)\right) \\ &\quad\times \left(\left(\mu+\beta_3 k\Theta_2\right)\left(p+\beta_1 k\Theta_1+\beta_2 k\Theta_2\right)\right. \\ &\qquad \left.+\,\mu\left(\gamma+\beta_3 k\Theta_2\right)\right)^{-1},\end{aligned} \tag{27}$$

and $\tilde{f}(\Theta_1, \Theta_2)$ is continuous in $\Omega \triangleq [0,1]\times[0,1]$. It is easy to testify that $\tilde{f}(\Theta_1^0, 0) = R - 1 > 0$ when $R > 1$, $\tilde{f}(1,1) < 0$, and $\tilde{f}(\Theta_1, \Theta_2)$ is continuous about Θ_2 in $[0,1]$, so there must exist some sufficient small numbers ε_1 $(0 < \varepsilon_1 < 1)$ and ε_2 $(0 < \varepsilon_2 < 1)$ fully close to 1 that satisfy $\tilde{f}(\Theta_1^0, \varepsilon_1) > 0$ and $\tilde{f}(1, \varepsilon_2) < 0$, and $\tilde{f}(\Theta_1, \Theta_2)$ is continuous about Θ_1 in $[0,1]$; similarly, $\tilde{f}(\varepsilon_3, \varepsilon_2) < 0$, where ε_3 $(\Theta_1^0 < \varepsilon_3 < 1)$ is fully close to 1; thus there is at least one point $P(\widetilde{\Theta}_1, \widetilde{\Theta}_2)$ satisfying $\tilde{f}(\widetilde{\Theta}_1, \widetilde{\Theta}_2) = 0$, where $P \in [\Theta_1^0, \varepsilon_3]\times[\varepsilon_1, \varepsilon_2]$. According to the implicit function theorem, equation $\tilde{f}(\Theta_1, \Theta_2) = 0$ can establish only a continuous function in the neighbor domain $\Omega_2 (\subset \Omega_1)$ of P:

$$\Theta_2 = \tilde{g}(\Theta_1), \quad (\Theta_1, \Theta_2) \in \Omega_2. \tag{28}$$

Substituting (28) into the first equation in formula (25), we can get the following equation:

$$\begin{aligned}\Theta_1 &= \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\ &\quad\times \left(\mu\left(p+\beta_1 k\Theta_1+\beta_2 k\tilde{g}(\Theta_1)\right)\right) \\ &\quad\times \left(\left(\mu+\beta_3 k\tilde{g}(\Theta_1)\right)\left(p+\beta_1 k\Theta_1+\beta_2 k\tilde{g}(\Theta_1)\right)\right. \\ &\qquad \left.+\,\mu\left(\gamma+\beta_3 k\tilde{g}(\Theta_1)\right)\right)^{-1} \triangleq g(\Theta_1).\end{aligned} \tag{29}$$

Noting that $g(0) - 0 > 0$, $g(1) - 1 < 0$, by the continuity of function $g(\Theta_1)$, there at least exists $\Theta_1^* \in (0,1)$ satisfying formula (29), so formula (25) at least has a positive solution (Θ_1^*, Θ_2^*) $(\Theta_2^* = \tilde{g}(\Theta_1^*))$, which means that system (5) at least has a positive equilibrium $E_1(i_1^*, i_2^*, \ldots, i_n^*, a_1^*, a_2^*, \ldots, a_n^*)$, where

$$\begin{aligned}i_k^* &= \frac{\mu\left(p+\beta_1 k\Theta_1^*+\beta_2 k\Theta_2^*\right)}{\left(\mu+\beta_3 k\Theta_2^*\right)\left(p+\beta_1 k\Theta_1^*+\beta_2 k\Theta_2^*\right)+\mu\left(\gamma+\beta_3 k\Theta_2^*\right)}, \\ &\qquad k = 1, \ldots, n, \\ a_k^* &= \frac{\beta_3 k\Theta_2^*\left(p+\beta_1 k\Theta_1^*+\beta_2 k\Theta_2^*\right)}{\left(\mu+\beta_3 k\Theta_2^*\right)\left(p+\beta_1 k\Theta_1^*+\beta_2 k\Theta_2^*\right)+\mu\left(\gamma+\beta_3 k\Theta_2^*\right)}, \\ &\qquad k = 1, \ldots, n.\end{aligned} \tag{30}$$

We will subsequently show that system (5) has a unique positive equilibrium. We denote that $i_k^* = y_k^*$, $a_k^* = y_{n+k}^*$, $k = 1, \ldots, n$ and assume that $y = y^* > 0$ and $y = z^* > 0$ are two constant solutions of system (5). If $y^* \neq z^*$, then there exists at least one i_0, $i_0 = 1, 2, \ldots, 2n$, such that $y_{i_0}^* \neq z_{i_0}^*$, where $y_{i_0}^*$ is the i_0^{th} component of the vector y^*. Without loss of generality, we assume that $y_{i_0}^* > z_{i_0}^*$ and moreover that $y_{i_0}^*/z_{i_0}^* \geq y_i^*/z_i^*$ for all $i = 1, \ldots, 2n$. Since y^* and z^* are constant solutions of system (5), and if $1 \leq i_0 \leq n$, we obtain that

$$\begin{aligned}&\left(p+\beta_1 i_0\Theta_1(y^*)+\beta_2 i_0\Theta_2(y^*)\right)\left(1-y_{i_0}^*-y_{n+i_0}^*\right) \\ &\quad -\gamma y_{i_0}^* - \beta_3 k\Theta_2(y^*)\, y_{i_0}^* \\ &\quad = \left(p+\beta_1 i_0\Theta_1(z^*)+\beta_2 i_0\Theta_2(z^*)\right)\left(1-z_{i_0}^*-z_{n+i_0}^*\right) \\ &\quad -\gamma z_{i_0}^* - \beta_3 k\Theta_2(z^*)\, z_{i_0}^* = 0,\end{aligned} \tag{31}$$

or if $n < i_0 \leq 2n$, we have

$$\beta_3 i_0\Theta_2(y^*)\, y_{i_0-n}^* - \mu y_{i_0}^* = \beta_3 i_0\Theta_2(z^*)\, z_{i_0-n}^* - \mu z_{i_0}^* = 0, \tag{32}$$

where

$$\Theta_1(y^*) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\, y_k^*,$$
$$\Theta_2(y^*) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\, y_{n+k}^*,$$
$$\Theta_1(z^*) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\, z_k^*, \tag{33}$$
$$\Theta_2(z^*) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k)\, z_{n+k}^*.$$

After equivalent deformation, it follows that

$$\begin{aligned}
&\frac{z_{i_0}^*}{y_{i_0}^*}\left(p + \beta_1 i_0 \Theta_1(y^*) + \beta_2 i_0 \Theta_2(y^*)\right)\left(1 - y_{i_0}^* - z_{n+i_0}^*\right) \\
&\quad - \gamma z_{i_0}^* - \beta_3 k \Theta_2(z^*) z_{i_0}^* \\
&= \left(p + \beta_1 i_0 \Theta_1(z^*) + \beta_2 i_0 \Theta_2(z^*)\right)\left(1 - z_{i_0}^* - z_{n+i_0}^*\right) \\
&\quad - \gamma z_{i_0}^* - \beta_3 k \Theta_2(z^*) z_{i_0}^* = 0,
\end{aligned} \tag{34}$$

or

$$\frac{z_{i_0}^*}{y_{i_0}^*}\beta_3 i_0 \Theta_2(y^*) z_{i_0-n}^* - \mu z_{i_0}^* = \beta_3 i_0 \Theta_2(z^*) z_{i_0-n}^* - \mu z_{i_0}^* = 0. \tag{35}$$

But $y_{i_0}^* > z_{i_0}^*$, $(z_{i_0}^*/y_{i_0}^*) y_i^* \le z_i^*$ for all i, and $1 - y_{i_0}^* < 1 - z_{i_0}^*$; thus from the above equalities we get

$$\begin{aligned}
&\frac{z_{i_0}^*}{y_{i_0}^*}\left(p + \beta_1 i_0 \Theta_1(y^*) + \beta_2 i_0 \Theta_2(y^*)\right)\left(1 - y_{i_0}^* - z_{n+i_0}^*\right) \\
&\quad - \gamma z_{i_0}^* - \beta_3 k \Theta_2(z^*) z_{i_0}^* \\
&< \left(p + \beta_1 i_0 \Theta_1(z^*) + \beta_2 i_0 \Theta_2(z^*)\right)\left(1 - z_{i_0}^* - z_{n+i_0}^*\right) \\
&\quad - \gamma z_{i_0}^* - \beta_3 k \Theta_2(z^*) z_{i_0}^*, \\
&\frac{z_{i_0}^*}{y_{i_0}^*}\beta_3 i_0 \Theta_2(y^*) z_{i_0-n}^* - \mu z_{i_0}^* < \beta_3 i_0 \Theta_2(z^*) z_{i_0-n}^* - \mu z_{i_0}^*.
\end{aligned} \tag{36}$$

This is a contradiction. Therefore, system (5) has a unique positive equilibrium. $E_1(i_1^*, i_2^*, \ldots, i_n^*, a_1^*, a_2^*, \ldots, a_n^*)$ that is $y^* = (y_1^*, y_2^*, \ldots, y_{n+1}^*) \in \Delta_{2n}$.

Hence, the conclusion follows.

Theorem 5. *When $\alpha = 0$, if $R > 1$, system (5) has a unique positive equilibrium E_1.*

Case II ($\alpha \neq 0$). When $\alpha \neq 0$, we can get equations that the steady state of system (5) satisfies

$$\begin{aligned}
i_k &= \left(\mu\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right) \\
&\quad \times \left(\left(\alpha + \mu + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right. \\
&\qquad \left. + \mu\left(\alpha + \gamma + \beta_3 k \Theta_2\right)\right)^{-1}, \\
a_k &= \left(\left(\alpha + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right) \\
&\quad \times \left(\left(\alpha + \mu + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right. \\
&\qquad \left. + \mu\left(\alpha + \gamma + \beta_3 k \Theta_2\right)\right)^{-1},
\end{aligned} \tag{37}$$

and self-consistency equations about Θ_1, Θ_2 are obtained such that

$$\begin{aligned}
\Theta_1 &= \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\
&\quad \times \left(\mu\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right) \\
&\quad \times \left(\left(\alpha + \mu + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right. \\
&\qquad \left. + \mu\left(\alpha + \gamma + \beta_3 k \Theta_2\right)\right)^{-1}, \\
\Theta_2 &= \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\
&\quad \times \left(\left(\alpha + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right) \\
&\quad \times \left(\left(\alpha + \mu + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right. \\
&\qquad \left. + \mu\left(\alpha + \gamma + \beta_3 k \Theta_2\right)\right)^{-1}.
\end{aligned} \tag{38}$$

From formula (38), we can obtain $0 < \Theta_1 < 1$, $0 < \Theta_2 < 1$. By the identical transformation of the second equation in formula (38), we can obtain

$$f(\Theta_1, \Theta_2) = 0, \tag{39}$$

where

$$\begin{aligned}
&f(\Theta_1, \Theta_2) \\
&\quad \triangleq \frac{1}{\langle k\rangle}\sum_{k=1}^{n} kp(k) \\
&\qquad \times \left(\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\left(\alpha + \beta_3 k \Theta_2\right)\left(1 - \Theta_2\right)\right. \\
&\qquad\quad \left. - \mu \Theta_2\left(p + \alpha + \gamma + \beta_1 k \Theta_1 + \beta_2 k \Theta_2 + \beta_3 k \Theta_2\right)\right) \\
&\qquad \times \left(\left(\alpha + \mu + \beta_3 k \Theta_2\right)\left(p + \beta_1 k \Theta_1 + \beta_2 k \Theta_2\right)\right. \\
&\qquad\quad \left. + \mu\left(\alpha + \gamma + \beta_3 k \Theta_2\right)\right)^{-1},
\end{aligned} \tag{40}$$

and since

$$f(\Theta_1, 0) = \frac{1}{\langle k \rangle} \sum_{k=1}^{n} kp(k) \frac{(p + \beta_1 k \Theta_1)\alpha}{(\alpha + \mu)(p + \beta_1 k \Theta_1) + \mu(\alpha + \gamma)} > 0,$$

$$\begin{aligned} f(\Theta_1, 1) = {} & \frac{1}{\langle k \rangle} \sum_{k=1}^{n} kp(k) \\ & \times (-\mu(p + \alpha + \gamma + \beta_1 k \Theta_1 + \beta_2 k + \beta_3 k)) \\ & \times ((\alpha + \mu + \beta_3 k)(p + \beta_1 k \Theta_1 + \beta_2 k) \\ & + \mu(\alpha + \gamma + \beta_3 k))^{-1} < 0, \end{aligned} \quad (41)$$

$f(\widetilde{\Theta}_1, 0) > 0$ and $f(\widetilde{\Theta}_1, 1) < 0$ for any $\widetilde{\Theta}_1 \in (0,1)$, and $f(\widetilde{\Theta}_1, \Theta_2)$ is continuous about Θ_2, then there is at least $\widetilde{\Theta}_2$ ($\widetilde{\Theta}_2 \in (0,1)$) satisfying $f(\widetilde{\Theta}_1, \widetilde{\Theta}_2) = 0$; substituting $\widetilde{\Theta}_1$ and $\widetilde{\Theta}_2$ to (37), we have a positive equilibrium of system (5) $E_2(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_n, \tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$, where

$$\begin{aligned} \tilde{i}_k = {} & \left(\mu\left(p + \beta_1 k \widetilde{\Theta}_1 + \beta_2 k \widetilde{\Theta}_2\right)\right) \\ & \times \left(\left(\alpha + \mu + \beta_3 k \widetilde{\Theta}_2\right)\left(p + \beta_1 k \widetilde{\Theta}_1 + \beta_2 k \widetilde{\Theta}_2\right)\right. \\ & \left. + \mu\left(\alpha + \gamma + \beta_3 k \widetilde{\Theta}_2\right)\right)^{-1}, \quad k = 1, \ldots, n, \\ \tilde{a}_k = {} & \left(\left(\alpha + \beta_3 k \widetilde{\Theta}_2\right)\left(p + \beta_1 k \widetilde{\Theta}_1 + \beta_2 k \widetilde{\Theta}_2\right)\right) \\ & \times \left(\left(\alpha + \mu + \beta_3 k \widetilde{\Theta}_2\right)\left(p + \beta_1 k \widetilde{\Theta}_1 + \beta_2 k \widetilde{\Theta}_2\right)\right. \\ & \left. + \mu\left(\alpha + \gamma + \beta_3 k \widetilde{\Theta}_2\right)\right)^{-1}, \quad k = 1, \ldots, n. \end{aligned} \quad (42)$$

We summarize these results in the following theorem.

Theorem 6. *When $\alpha \neq 0$, system (5) has at least a positive equilibrium E_2.*

4. Numerical Simulation

In this section, we will perform a series of numerical simulations to verify the mathematical analysis on a scale-free network with power-law distribution ($p(k) = 8k^{-4}$, $k = 2, 3, \ldots, 150$, $\langle k \rangle = 9.1824$). Parameters in system (5) are chosen as $p = 0.001$, $\beta_1 = 0.0001$, $\beta_2 = 0.0004$, $\gamma = 0.0001$, and $\mu = 0.015$.

We give numerical simulations about the network model (5) with different degrees and different initial value in Figures 1 and 2. In Figure 1, when $\alpha = 0$, $\beta_3 = 0.0004$, and $R = 0.8915$, Figures 1(a)–1(c) show that system (5) has a stable trivial equilibrium; namely, new products cannot diffuse on the market; when $\alpha = 0$, $\beta_3 = 0.01$, and $R = 1.7310$, Figures 1(d)–1(f) show that system (5) has a positive equilibrium; new products can diffuse on the market. Letting $\alpha = 0.005$, Figure 2 gives time series of a_{50}, a_{100}, and a_{150}, and it indicates that system (5) has a positive equilibrium when $\alpha = 0.005$.

In Figure 3, (a) and (b) indicate that the relative density of adopters always increases with the enhancement of mass media' influence; furthermore, by comparing Figure 3(a) to Figure 3(b), we can find that the bigger the degree of adopters is, the smaller the increment of adopters with the enhancement of mass media' influence is; however, Figures 3(c) and 3(d) manifest that the enhancement of mass media' influence can increase adopters only in the early stage as the adopters' degree continuously increases; moreover, in Figure 3(d), we find that mass media is effective only in $[0, \tau]$. These simulation results can be theoretically explained as follows: i_k exponentially decreases with the increment of α and k in system (5), so $(\alpha + \beta_3 k \Theta_2(t)) i_k(t)$ decreases with the increment of α and k; namely, $da_k(t)/dt$ decreases when α or k increases. In other words, the larger α is (or the larger k), the smaller i_k is, and the smaller $da_k(t)/dt$ is, the faster a_k decrease is if $da_k(t)/dt < 0$ and the slower a_k increase is if $da_k(t)/dt > 0$. In view of economics, Figures 3(a)–3(d) imply two different propagation strategies in the decision-making stage of new products diffusion process; for those majority of nodes having small degree in scale-free networks, it is very effective and permanent to persuade them to adopt new products by mass media channels, despite the fact that the effect in the late period is less significant than the one in the early stage, so businesses should propagate chronically new products by mass media among them. For those few hubs' nodes with larger degree, mass media' effect is only very temporary; if businesses propagate and persuade people to adopt new products only in $[0, \tau]$, it will be useful to save resources for businesses; when $t > \tau$, businesses have to change advertisement strategies.

In addition, fixing the degree of nodes in (a)–(d) of Figure 3, it is found that the peak of a_k increases with increment of α; namely, the peak of adopters will increase as the advertisement is enhanced. In Figure 3(e), by fixing α, we can find that the larger the degree and the larger the relative density of adopters, the smaller the diffusion time; Figure 3(e) means that hubs' nodes are more easy to become adopters than nodes with the small degrees, so businesses should propagate timely and effectively new products among them only in some period of time τ, and they should take different advertising strategies for hubs' nodes and nodes with the small degrees.

In Figure 4, we keep track of the mean density a versus times, and we find that the mean density of networks a increases with the increment of α; that is to say, the number of adopters increases when mass media' influence is enhanced, and it is reasonable. Moreover, we can also see that the effect of mass media is more pronounced before the stable state of the adopters than after the stable state; this indicates that the enterprise should take persuasive advertisement strategy to tell consumers why to choose a particular brand; after adopters tend to stable states, namely, once the brand is popularized, the enterprise should take remindful advertisement to frequently remind consumer to have such a product, and it is useful to save advertising cost.

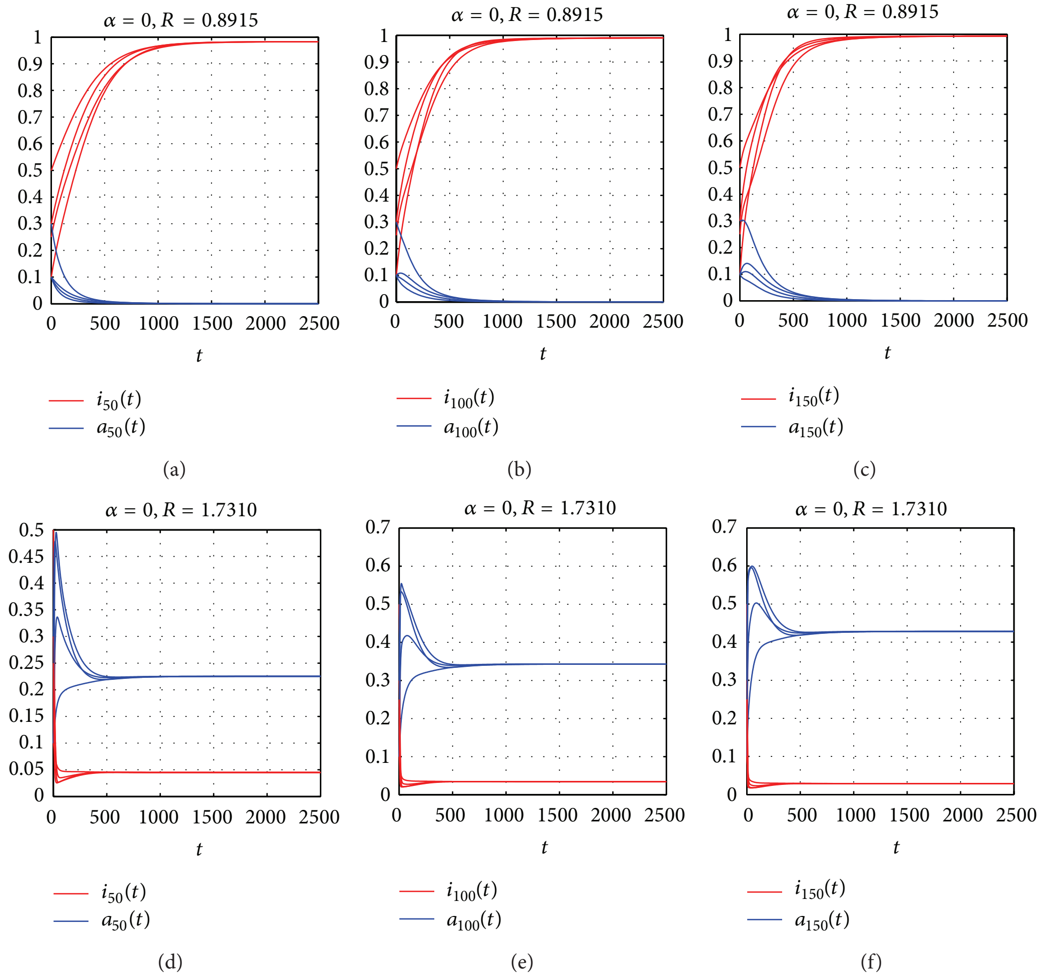

FIGURE 1: Time series when $\alpha = 0$. (a)–(c) time series with different degrees, respectively, when $R = 0.8915$; (d)–(f) time series with different degrees, respectively, when $R = 1.7310$.

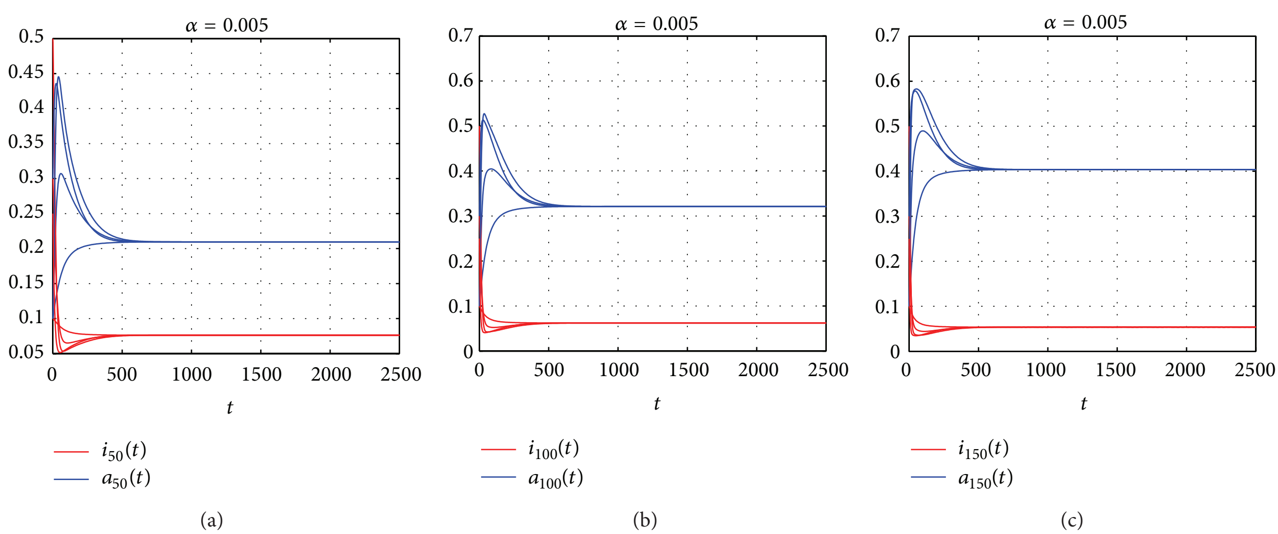

FIGURE 2: Time series with different degrees when $\alpha = 0.005$.

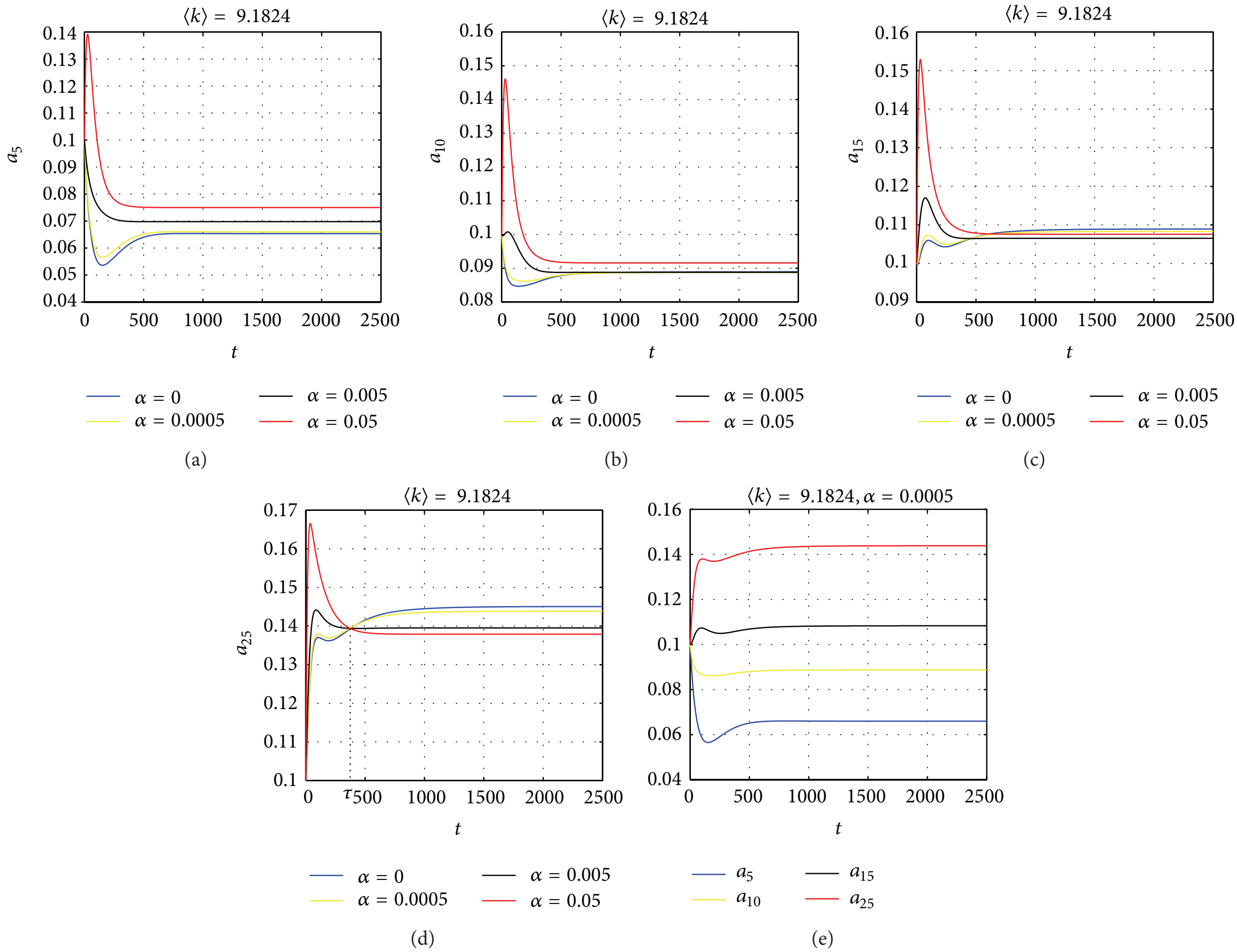

FIGURE 3: (a)–(d) time series of a_5, a_{10}, a_{15}, and a_{25}, respectively, when α changes from 0 to 0.05 ($\alpha = 0, 0.0005, 0.005, 0.05$); (e) time series of a_k ($k = 5, 10, 15, 25$) when $\alpha = 0.0005$.

5. Conclusion and Discussion

In this paper, we have proposed a complex networks model with the awareness stage and the decision-making stage to expound adoption processes. Unlike the classical diffusion model, where population contacts are homogeneous mixing, populations have complex and heterogeneous connectivity patterns. We study the existence and stability of equilibrium when the influence of mass media is neglected or considered in the decision-making stage. In simulations of networks with the power-law distribution, we analyze the effect of the network structure and mass media in the decision-making stage on the density of adopters and obtain different propagation strategies to persuade individuals to adopt the products. From the local point of view, businesses should take different propagation strategies for hub nodes and other nodes with the small degree. From the global point of view, because the effect of mass media is more significant before the arrival of the stable state of the adopters than after the arrival of the stable state, businesses should change propagation strategies after the average density of adopters in the network reaches the stable state.

It is worth noting here that we only consider the influence of nodes' degree on diffusion; however, in recent years there has been considerable interest within the physics community in the network structures, such as clusters, path length, and centrality indices [24–29]; if other network structures are considered in the model, it could have a profound influence.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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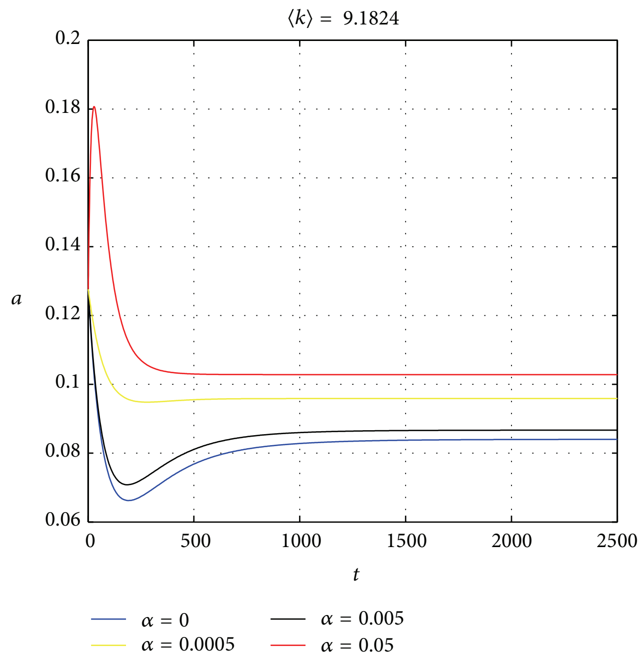

FIGURE 4: Time series of the mean density a when the change of α is as shown in the figure.

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