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Research Article

Soft α-Open Sets and Soft α-Continuous Functions

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We introduce soft α-sets on soft topological spaces and study some of their properties. We also investigate the concepts of soft α-continuous and soft α-open functions and discuss their relationships with soft continuous and other weaker forms of soft continuous functions. Also counterexamples are given to show the noncoincidence of these functions.

1. Introduction

Molodtsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory to several directions such as smoothness of functions, game theory, Riemann Integration, and theory of measurement. In recent years, development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parameterization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al. [3], Aygunoglu and Aygun [4], and Hussain et al. continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. He investigated soft semiopen sets in soft topological spaces and studied some properties of them. Arockiarani and Arokialancy defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space.

In the present paper, we introduce some new concepts in soft topological spaces such as soft α-open sets, soft α-closed sets, and soft α-continuous functions. We also study relationship between soft continuity [6], soft semicontinuity [7], and soft α-continuity of functions defined on soft topological spaces. With the help of counterexamples, we show the noncoincidence of these various types of mappings.

2. Preliminaries

Definition 1 (see [1]). Let X be an initial universe and let E be a set of parameters. Let $P(X)$ denote the power set of X and let A be a nonempty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2 (see [8]). A soft set (F, A) over X is called a null soft set, denoted by Φ; if $e \in A$, $F(e) = \emptyset$.

Definition 3 (see [8]). A soft set (F, A) over X is called an absolute soft set, denoted by $\widetilde{A}$; if $e \in A$, $F(e) = X$.

Definition 4 (see [8]). The union of two soft sets of (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \tilde{\cup} B$ and, for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \tilde{\cup} G(e), & \text{if } e \in A \tilde{\cap} B. \end{cases} \tag{1}$$

We write $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 5 (see [8]). The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X, denoted $(F, A) \tilde{\cap} (G, B)$, is defined as $C = A \tilde{\cap} B$ and $H(e) = F(e) \tilde{\cap} G(e)$ for all $e \in C$.

Definition 6 (see [8]). Let (F, A) and (G, B) be two soft sets over a common universe X. $(F, A)\tilde{\subset}(G, B)$, if $A \subset B$, and $H(e) = F(e) \subset G(e)$, for all $e \in A$.

Definition 7 (see [2]). Let τ be the collection of soft sets over X; then τ is said to be a soft topology on X if it satisfies the following axioms:

(1) Φ and $\widetilde{X}$ belong to τ,

(2) the union of any number of soft sets in τ belongs to τ,

(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft topological space over X; then the members of τ are said to be soft open sets in X. The relative complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \rightarrow P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in A$. Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed set in X if its relative complement $(F, E)^c$ belongs to τ. If (X, τ, E) is a soft topological space with $\tau = \{\Phi, \widetilde{X}\}$, then τ is called the soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete topological space. If (X, τ, E) is a a soft topological space with τ is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete topological space.

Definition 8. Let (X, τ, E) be a soft topological space over X and let (A, E) be a soft set over X.

(1) [3] The soft interior of (A, E) is the soft set $\operatorname{int}(A, E) = \cup\{(O, E) : (O, E)$ which is soft open and $(O, E)\tilde{\subset}(A, E)\}$.

(2) [2] The soft closure of (A, E) is the soft set $\operatorname{cl}(A, E) = \cap\{(F, E) : (F, E)$ which is soft closed and $(A, E)\tilde{\subset}(F, E)\}$.

Clearly $\operatorname{cl}(A, E)$ is the smallest soft closed set over X which contains (A, E) and $\operatorname{int}(A, E)$ is the largest soft open set over X which is contained in (A, E).

Throughout the paper, the spaces X and Y (or (X, τ, E) and (Y, ν, K)) stand for soft topological spaces assumed unless otherwise stated.

3. Soft α-Open Sets

Definition 9. A soft set (A, E) of a soft topological space (X, τ, E) is called soft α-open set if $(A, E)\tilde{\subset}\operatorname{int}(\operatorname{cl}(\operatorname{int}(A, E)))$. The complement of soft α-open set is called soft α-closed set.

Definition 10. A soft set (A, E) is called soft preopen set [9] (resp., soft semiopen [5]) in a soft topological space X if $(A, E)\tilde{\subset}\operatorname{int}(\operatorname{cl}(A, E))$ (resp., $(A, E)\tilde{\subset}\operatorname{cl}(\operatorname{int}(A, E))$).

We will denote the family of all soft α-open sets (resp., soft α-closed sets and soft preopen sets) of a soft topological space (X, τ, E) by $S\alpha OS(X, \tau, E)$ (resp., $S\alpha CS(X, \tau, E)$ and $SPO(X, \tau, E)$).

Soft open set
$\downarrow$ Soft semiopen set $\nearrow$
$\downarrow$
Soft α-open set
$\searrow$
Soft preopen set

Figure 1

Proposition 11. *(1) Arbitrary union of soft α-open sets is a soft α-open sets*

(2) Arbitrary intersection of soft α-closed sets is a soft α-closed set.

Proof. (1) Let $\{(A_i, E) : i \in \Lambda\}$ be a collection of soft α-open sets. Then, for each $i \in \Lambda$,

$$
\begin{aligned}
&(A_i, E)\tilde{\subset}\operatorname{int}(\operatorname{cl}(\operatorname{int}(A_i, E))). \text{ Now}\\
&\tilde{\cup}(A_i, E)\tilde{\subset}\tilde{\cup}\operatorname{int}(\operatorname{cl}(\operatorname{int}(A_i, E)))\\
&\tilde{\subset}\operatorname{int}\tilde{\cup}\operatorname{cl}(\operatorname{int}(A_i, E))\\
&= \operatorname{int}(\operatorname{cl}(\tilde{\cup}\operatorname{int}(A_i, E)))\\
&\tilde{\subset}\operatorname{int}(\operatorname{cl}(\operatorname{int}\tilde{\cup}(A_i, E))).
\end{aligned}
$$

Hence $\tilde{\cup}(A_i, E)$ is a soft α-open set.

(2) Follows immediately from (1) by taking complements. □

Example 12. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2, e_3\}$, $\tau = \{\Phi, \widetilde{X}, (F, E)\}$, and $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$ and let (X, τ, E) be a soft topological space. Then $(F, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$ is a soft α-open set in X.

Remark 13. It is obvious that every soft open (resp., soft closed) set is a soft α-open set (resp., soft α-closed set). Similarly, every soft α-open set is soft semiopen and soft preopen. Thus we have implications as shown in Figure 1.

The examples given below show that the converses of these implications are not true.

Example 14. Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3\}$, and $\tau = \{\Phi, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}$, where $(F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)$ are soft sets over X, defined as follows:

$(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1, x_4\})\}$,

$(F_2, E) = \{(e_1, \{x_2, x_4\}), (e_2, \{x_1, x_3, x_4\}), (e_3, \{x_1, x_2, x_4\})\}$,

$(F_3, E) = \{(e_2, \{x_3\}), (e_3, \{x_1\})\}$,

$(F_4, E) = \{(e_1, \{x_1, x_2, x_4\}), (e_2, X), (e_3, X)\}$,

$(F_5, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_4\}), (e_3, \{x_2\})\}$,

$(F_6, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$,

$(F_7, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2, x_4\})\}$,

$(F_8, E) = \{(e_2, \{x_4\}), (e_3, \{x_2\})\}$,

$(F_9, E) = \{(e_1, X), (e_2, X), (e_3, \{x_1, x_2, x_3\})\}$,

$(F_{10}, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2\})\}$,
$(F_{11}, E) = \{(e_1, \{x_2, x_3, x_4\}), (e_2, X), (e_3, \{x_1, x_2, x_3\})\}$,
$(F_{12}, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2, x_4\})\}$,
$(F_{13}, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_4\}), (e_3, \{x_2\})\}$,
$(F_{14}, E) = \{(e_1, \{x_3, x_4\}), (e_2, \{x_1, x_2\})\}$,
$(F_{15}, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1\})\}$.

Then τ defines a soft topology on X and thus (X, τ, E) is a soft topological space over X.

Clearly the soft closed sets are $\widetilde{X}, \Phi, (F_1, E)^c, (F_2, E)^c, (F_3, E)^c, \ldots, (F_{15}, E)^c$.

Then, let us take $(B, E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2\})\}$, then $\mathrm{int}((B, E)) = (F_{10}, E)$, $\mathrm{cl}(\mathrm{int}((B, E))) = \widetilde{X}$, $\mathrm{int}(\mathrm{cl}(\mathrm{int}((B, E)))) = \widetilde{X}$, and so $(B, E) \widetilde{\subset} \mathrm{int}(\mathrm{cl}(\mathrm{int}((B, E))))$; hence (B, E) is soft α-open but not soft open since $(B, E) \notin \tau$.

Now, let us take $(H, E) = \{(e_1, \{x_2\}), (e_2, \{x_3\}), (e_3, \{x_1, x_3\})\}$ then $\mathrm{int}((H, E)) = (F_3, E)$, $\mathrm{cl}(\mathrm{int}((H, E))) = (F_5, E)^c$, and so $(H, E) \widetilde{\subset} \mathrm{cl}(\mathrm{int}((B, E)))$; hence (B, E) is soft semiopen but not soft α-open.

Finally, let us take $(G, E) = \{(e_3, \{x_2\})\}$ then $\mathrm{cl}((G, E)) = (F_1, E)^c$, $\mathrm{int}(\mathrm{cl}((G, A))) = (F_8, E)$, and so $(G, E) \widetilde{\subset} \mathrm{int}(\mathrm{cl}((G, E)))$, hence (G, E) is soft preopen but not soft α-open.

Definition 15. Let (X, τ, E) be a soft topological space and let (A, E) be a soft set over X.

(1) Soft α-closure of a soft set (A, E) in X is denoted by $s\alpha\,\mathrm{cl}((A, E)) = \widetilde{\cap}\{(F, E) : (F, E)$ which is a soft α-closed set and $(A, E) \widetilde{\subset} (F, E)\}$.

(2) Soft α-interior of a soft set (A, E) in X is denoted by $s\alpha\,\mathrm{int}((A, E)) = \widetilde{\cup}\{(O, E) : (O, E)$ which is a soft α-open set and $(O, E) \widetilde{\subset} (A, E)\}$.

Clearly $s\alpha\,\mathrm{cl}((A, E))$ is the smallest soft α-closed set over X which contains (A, E) and $s\alpha\,\mathrm{int}((A, E))$ is the largest soft α-open set over X which is contained in (A, E).

Proposition 16. *Let (X, τ, E) be a soft topological space and let (A, E) be a soft set over X; then*

(1) $(A, E) \in S\alpha CS(X, \tau, E) \Leftrightarrow (A, E) = s\alpha\,\mathrm{cl}(A, E)$;

(2) $(A, E) \in S\alpha OS(X, \tau, E) \Leftrightarrow (A, E) = s\alpha\,\mathrm{int}(A, E)$.

Proof. (1) Let $(A, E) = s\alpha\,\mathrm{cl}((A, E)) = \widetilde{\cap}\{(F, E) : (F, E)$ be a soft α-closed set and $(A, E) \widetilde{\subset} (F, E)\}$.

This shows that $(A, E) \in \{(F, E) : (F, E)$ is a soft α-closed set and $(A, E) \widetilde{\subset} (F, E)\}$.

Hence (A, E) is soft α-closed.

Conversely, let (A, E) be soft α-closed set. Since $(A, E) \widetilde{\subset} (A, E)$ and (A, E) is a soft α-closed, $(A, E) \in \{(F, E) : (F, E)$ is a soft α-closed set and $(A, E) \widetilde{\subset} (F, E)\}$.

Further, $(A, E) \widetilde{\subset} (F, E)$ for all such (F, E)'s.

$(A, E) = \widetilde{\cap}\{(F, E) : (F, E)$ is a soft α-closed set and $(A, E) \widetilde{\subset} (F, E)\}$.

(2) Similar to (1). □

Proposition 17. *In a soft space (X, τ, E), the folowing hold for soft α-closure.*

(1) $s\alpha\,\mathrm{cl}(\Phi) = \Phi$.

(2) *$s\alpha\,\mathrm{cl}((A, E))$ is soft α-closed in (X, τ, E) for each soft subset (A, E) of X.*

(3) $s\alpha\,\mathrm{cl}((A, E)) \widetilde{\subset} s\alpha\,\mathrm{cl}((B, E))$, *if* $(A, E) \widetilde{\subset} (B, E)$.

(4) $s\alpha\,\mathrm{cl}(s\alpha\,\mathrm{cl}(A, E)) = s\alpha\,\mathrm{cl}((A, E))$.

Proof. Easy. □

Theorem 18. *Let (X, τ, E) be a soft topological space and let (G, E) and (K, E) be two soft sets over X; then*

(1) $(s\alpha\,\mathrm{cl}(G, E))^c = s\alpha\,\mathrm{int}(G, E)^c$;

(2) $(s\alpha\,\mathrm{int}(G, E))^c = s\alpha\,\mathrm{cl}(G, E)^c$;

(3) $(G, E) \widetilde{\subset} (K, A) \Rightarrow s\alpha\,\mathrm{int}(G, E) \widetilde{\subset} s\alpha\,\mathrm{int}(K, E)$;

(4) $s\alpha\,\mathrm{cl}(\Phi) = \Phi$ *and* $s\alpha\,\mathrm{cl}(\widetilde{X}) = \widetilde{X}$;

(5) $s\alpha\,\mathrm{int}(\Phi) = \Phi$ *and* $s\alpha\,\mathrm{int}(\widetilde{X}) = \widetilde{X}$;

(6) $s\alpha\,\mathrm{cl}((G, E) \widetilde{\cup} (K, E)) = s\alpha\,\mathrm{cl}(G, E) \widetilde{\cup} s\alpha\,\mathrm{cl}(K, E)$;

(7) $s\alpha\,\mathrm{int}((G, E) \widetilde{\cap} (K, E)) = s\alpha\,\mathrm{int}(G, E) \widetilde{\cap} s\alpha\,\mathrm{int}(K, E)$;

(8) $s\alpha\,\mathrm{cl}((G, E) \widetilde{\cap} (K, E)) \widetilde{\subset} s\alpha\,\mathrm{cl}(G, E) \widetilde{\cap} s\alpha\,\mathrm{cl}(K, E)$;

(9) $s\alpha\,\mathrm{int}((G, E) \widetilde{\cup} (K, E)) \widetilde{\subset} s\alpha\,\mathrm{int}(G, E) \widetilde{\cup} s\alpha\,\mathrm{int}(K, E)$;

(10) $s\alpha\,\mathrm{cl}((s\alpha\,\mathrm{cl}(G, E))) = s\alpha\,\mathrm{cl}(G, E)$;

(11) $s\alpha\,\mathrm{int}(s\alpha\,\mathrm{int}(G, E)) = s\alpha\,\mathrm{int}(G, E)$.

Proof. Let (G, E) and (K, E) be two soft sets over X.

(1) $(s\alpha\,\mathrm{cl}(G, E))^c = (\widetilde{\cap}\{(F, E) \mid (G, E) \widetilde{\subset} (F, E)$ and $(F, E) \in S\alpha CS(X, \tau, E)\})^c$

$= \widetilde{\cup}\{(F, E)^c \mid (G, E) \widetilde{\subset} (F, E)$ and $(F, E) \in S\alpha CS(X, \tau, E)\}$

$= \widetilde{\cup}\{(F, E)^c \mid (F, E)^c \widetilde{\subset} (G, E)^c$ and $(F, E)^c \in S\alpha OS(X, \tau, E)\} = s\alpha\,\mathrm{int}(G, E)^c$.

(2) Similar to (1).

(3) It follows from Definition 15.

(4) Since Φ and $\widetilde{X}$ are soft α-closed sets so, $s\alpha\,\mathrm{cl}(\Phi) = \Phi$ and $s\alpha\,\mathrm{cl}(\widetilde{X}) = \widetilde{X}$.

(5) Since Φ and $\widetilde{X}$ are soft α-open sets so, $s\alpha\,\mathrm{int}(\Phi) = \Phi$ and $s\alpha\,\mathrm{int}(\widetilde{X}) = \widetilde{X}$.

(6) We have $(G, E) \widetilde{\subset} ((G, E) \widetilde{\cup} (K, E))$ and $(K, E) \widetilde{\subset} ((G, E) \widetilde{\cup} (K, E))$.

Then by Proposition 17(3), $s\alpha\,\mathrm{cl}(G, E) \widetilde{\subset} s\alpha\,\mathrm{cl}((G, E) \widetilde{\cup} (K, E))$ and $s\alpha\,\mathrm{cl}(K, E) \widetilde{\subset} s\alpha\,\mathrm{cl}((G, E) \widetilde{\cup} (K, E)) \Rightarrow s\alpha\,\mathrm{cl}(K, E) \widetilde{\subset} s\alpha\,\mathrm{cl}(G, E) \widetilde{\subset} s\alpha\,\mathrm{cl}((G, E) \widetilde{\cup} (K, E))$.

Now, $s\alpha\,\mathrm{cl}(G, E)$, $s\alpha\,\mathrm{cl}(K, E) \in S\alpha CS(X, \tau, E) \Rightarrow s\alpha\,\mathrm{cl}(G, E) \widetilde{\cup} s\alpha\,\mathrm{cl}(K, E) \in S\alpha CS(X, \tau, E)$.

Then $(G, E) \widetilde{\subset} s\alpha\,\mathrm{cl}(G, E)$ and $(K, E) \widetilde{\subset} s\alpha\,\mathrm{cl}(K, E)$ imply $(G, E) \widetilde{\cup} (K, E) \widetilde{\subset} s\alpha\,\mathrm{cl}(G, E) \widetilde{\cup} s\alpha\,\mathrm{cl}(K, E)$. That is, $s\alpha\,\mathrm{cl}(G, E) \widetilde{\cup} s\alpha\,\mathrm{cl}(K, E)$ is a soft α-closed set containing $(G, E) \widetilde{\cup} (K, E)$.

But $s\alpha\,\mathrm{cl}((G,E)\tilde{\cup}(K,E))$ is the smallest soft α-closed set containing $(G,E)\tilde{\cup}(K,E)$.

Hence $s\alpha\,\mathrm{cl}((G,E)\tilde{\cup}(K,E))\tilde{\subset}s\alpha\,\mathrm{cl}(G,E)\tilde{\cup}s\alpha\,\mathrm{cl}(K,E)$.

So, $s\alpha\,\mathrm{cl}((G,E)\tilde{\cup}(K,E)) = s\alpha\,\mathrm{cl}(G,E)\tilde{\cup}s\alpha\,\mathrm{cl}(K,E)$.

(7) Similar to (6)

(8) We have $((G,E)\tilde{\cap}(K,E))\tilde{\subset}(G,E)$ and $((G,E)\tilde{\cap}(K,E))\tilde{\subset}(K,E)$

$\Rightarrow s\alpha\,\mathrm{cl}((G,E)\tilde{\cap}(K,E))\tilde{\subset}s\alpha\,\mathrm{cl}(G,E)$ and $s\alpha\,\mathrm{cl}((G,E)\tilde{\cap}(K,E))\tilde{\subset}s\alpha\,\mathrm{cl}(K,E)$.

$\Rightarrow s\alpha\,\mathrm{cl}((G,E)\tilde{\cap}(K,E))\tilde{\subset}s\alpha\,\mathrm{cl}(G,E)\tilde{\subset}s\alpha\,\mathrm{cl}(K,E)$.

(9) Similar to (8).

(10) Since $(s\alpha\,\mathrm{cl}(G,E)) \in S\alpha CS(X,\tau,E)$ so by Proposition 16(1), $s\alpha\,\mathrm{cl}((s\alpha\,\mathrm{cl}(G,E))) = (s\alpha\,\mathrm{cl}(G,E))$.

(11) Since $s\alpha\,\mathrm{int}(G,E) \in S\alpha OS(X,\tau,E)$ so by Proposition 16(2), $s\alpha\,\mathrm{int}(s\alpha\,\mathrm{int}(G,E)) = s\alpha\,\mathrm{int}(G,E)$.

□

Theorem 19. *If (G,E) is any soft set in a soft topological space (X,τ,E), then following are equivalent:*

(1) *(G,E) is soft α-closed set;*

(2) $\mathrm{int}(\mathrm{cl}(\mathrm{int}(G,E)^c))\tilde{\supset}(G,E)^c$;

(3) $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(G,E)))\tilde{\subset}(G,E)$;

(4) *(G,E) is soft α-open set.*

Proof. (1)⇒(2) If (G,E) is soft α-closed set, then $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(G,E))\tilde{\subset}(G,E)) \Rightarrow (G,E)^c\tilde{\subset}\,\mathrm{int}(\mathrm{cl}(\mathrm{int}(G,E)^c))$;

(2)⇒(3)$(\mathrm{int}(\mathrm{cl}(\mathrm{int}(G,E)^c))^c\tilde{\supset}((G,E)^c))^c \Rightarrow \mathrm{cl}(\mathrm{int}(\mathrm{cl}(G,E)))\tilde{\subset}(G,E)$;

(3)⇒(4) It is obvious from Definition 9;

(4)⇒(1) It is obvious from Definition 9. □

4. Soft α-Continuity

Definition 20 (see [10]). Let (X,E) and (Y,K) be soft classes. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then a mapping $f : (X,E) \to (Y,K)$ is defined as follows: for a soft set (F,A) in (X,E), $(f(F,A),B)$, $B = p(A) \subseteq K$ is a soft set in (Y,K) given by $f(F,A)(\beta) = u\left(\underset{\alpha\in p^{-1}(\beta)\cap A}{\cup F(\alpha)}\right)$ for $\beta \in K$. $(f(F,A),B)$ is called a soft image of a soft set (F,A). If $B = K$, then we will write $(f(F,A),K)$ as $f(F,A)$.

Definition 21 (see [10]). Let $f : (X,E) \to (Y,K)$ be a mapping from a soft class (X,E) to another soft class (Y,K) and (G,C) a soft set in soft class (Y,K), where $C \subseteq K$. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then $(f^{-1}(G,C),D)$, $D = p^{-1}(C)$, is a soft set in the soft classes (X,E), defined as $f^{-1}(G,C)(\alpha) = u^{-1}(G(p(\alpha)))$ for $\alpha \in D \subseteq E$. $(f^{-1}(G,C),D)$ is called a soft inverse image of (G,C). Hereafter we will write $(f^{-1}(G,C),E)$ as $f^{-1}(G,C)$.

Theorem 22 (see [10]). *Let $f : (X,E) \to (Y,K)$, $u : X \to Y$, and $p : E \to K$ be mappings. Then for soft sets (F,A) and (G,B) and a family of soft sets (F_i,A_i) in the soft class (X,E), one has:*

(1) $f(\Phi) = \Phi$,

(2) $f(\widetilde{X}) = \widetilde{Y}$,

(3) $f((F,A)\tilde{\cup}(G,B)) = f(F,A)\tilde{\cup}f(G,B)$ *in general* $f(\cup_i(F_i,A_i)) = \cup_i f(F_i,A_i)$,

(4) $f((F,A)\tilde{\cap}(G,B))\tilde{\supseteq}f(F,A)\tilde{\cap}f(G,B)$ *in general* $f(\cap_i(F_i,A_i))\tilde{\subseteq}\cap_i f(F_i,A_i)$,

(5) *if* $(F,A)\tilde{\subseteq}(G,B)$, *then* $f(F,A)\tilde{\subseteq}f(G,B)$,

(6) $f^{-1}(\Phi) = \Phi$,

(7) $f^{-1}(Y) = \widetilde{X}$,

(8) $f^{-1}((F,A)\tilde{\cup}(G,B)) = f^{-1}(F,A)\tilde{\cup}f^{-1}(G,B)$ *in general* $f^{-1}(\cup_i(F_i,A_i)) = \cup_i f^{-1}(F_i,A_i)$,

(9) $f^{-1}((F,A)\tilde{\cap}(G,B)) = f^{-1}(F,A)\tilde{\cap}f^{-1}(G,B)$ *in general* $f^{-1}(\cap_i(F_i,A_i)) = \cap_i f^{-1}(F_i,A_i)$,

(10) *if* $(F,A)\tilde{\subseteq}(G,B)$, *then* $f^{-1}(F,A)\tilde{\subseteq}f^{-1}(G,B)$.

Definition 23. A mapping $f : (X,\tau,E) \to (Y,\upsilon,K)$ is said to be soft mapping if (X,τ,E) and (Y,υ,K) are soft topological spaces and $u : X \to Y$ and $p : E \to K$ are mappings.

Throughout the paper, the spaces X and Y (or (X,τ,E) and (Y,υ,K)) stand for soft topological spaces assumed unless otherwise stated.

Definition 24. A soft mapping $f : X \to Y$ is said to be soft α-continuous if the inverse image of each soft open subset of Y is a soft α-open set in X.

Example 25. Let $X = \{x_1,x_2,x_3\}$, $Y = \{y_1,y_2,y_3\}$, $E = \{e_1,e_2,e_3\}$, $K = \{k_1,k_2,k_3\}$, $\tau = \{\Phi,\widetilde{X},(F,E)\}$, $(F,E) = \{(e_1,\{x_1\}),(e_2,\{x_2\}),(e_3,\{x_1,x_3\})\}$, $\upsilon = \{\Phi,\widetilde{Y},(G,K)\}$, and $(G,K) = \{(k_1,\{y_1,y_3\}),(k_2,\{y_1\}),(k_3,\{y_3\})\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces.

Define $u : X \to Y$ and $p : E \to K$ as

$$u(x_1) = \{y_1\},\ u(x_2) = \{y_3\},\ u(x_3) = \{y_2\},$$
$$p(e_1) = \{k_2\},\ p(e_2) = \{k_3\},\ p(e_3) = \{k_1\}.$$

Let $f_{up} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft mapping. Then (G,K) is a soft open in Y and $f_{pu}^{-1}((G,K)) = (F,E)$ is a soft α-open in X. Therefore, f_{pu} is a soft α-continuous function.

Theorem 26. *Let $f : X \to Y$ be a mapping from a soft space X to soft space Y. Then the following statements are true:*

(1) *f is soft α-continuous;*

(2) *for each soft singleton (P,E) in X and each soft open set (O,K) in Y and $f((P,E))\tilde{\subset}(O,K)$, there exists a soft α-open set (U,E) in X such that $(P,E)\tilde{\subset}(U,E)$ and $f((U,E))\tilde{\subset}(O,K)$;*

(3) *the inverse image of each soft closed set in Y is soft α-closed in X;*

(4) $f(\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A,E))))\tilde{\subset}\,\mathrm{cl}(f(A,E))$, *for each soft set (A,E) in X;*

(5) $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(B,K))))\tilde{\subset}f^{-1}(\mathrm{cl}(B,K))$, *for each soft set (B,K) in Y.*

Proof. (1)⇒(2) Since (O,K) is soft open in Y and $f((P,E))\widetilde{\subset}(O,K)$, so $(P,E)\widetilde{\subset}f^{-1}((O,K))$ and $f^{-1}((O,K))$ is a soft α-open set in X. Put $(U,E)=f^{-1}((O,K))$. Then $(P,E)\widetilde{\subset}(U,E)$ and $f((U,E))\widetilde{\subset}(O,K)$.

(2)⇒(1) Let (O,K) be a soft open set in Y such that $(P,E)\widetilde{\subset}f^{-1}((O,K))$ and thus there exists $(U,E) \in SaOS(X)$ such that $(P,E)\widetilde{\subset}(U,E)$ and $f((U,E))\widetilde{\subset}(O,K)$. Then $(P,E)\widetilde{\subset}(U,E)\widetilde{\subset}f^{-1}((O,K)) = \widetilde{\cup}(U,E) \in S_a(X)$. Hence $f^{-1}((O,K)) \in SaOS(X)$ and therefore f is soft α-continuous.

(1)⇒(3) Let (G,K) be a soft closed set in Y. Then $(G,K)^c$ is soft open in Y. Thus $f^{-1}((G,K)^c) \in SaOS(X)$; that is, $\widetilde{X}-f^{-1}((G,K)) \in SaOS(X)$. Hence $f^{-1}((G,K))$ is a soft α-set in X.

(3)⇒(4) Let (S,K) be a soft set in Y. Then $\mathrm{cl}(f(S,K))$ is a soft closed set in Y, so that $f^{-1}(\mathrm{cl}(S,K))$ is soft α-closed in X.

Therefore, we have $f^{-1}(\mathrm{cl}(S,K))\widetilde{\supset}\,\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{cl}(S,K)))))\widetilde{\supset}\,\mathrm{cl}(\mathrm{int}(\mathrm{cl}(\mathrm{cl}(S,K)))) = \mathrm{cl}(\mathrm{int}(\mathrm{cl}(S,K)))$.

(4)⇒(5) Since (B,K) be a soft set in Y, then $f^{-1}((B,K))$ is a soft set in X; thus by hypothesis we have $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f(f^{-1}((B,K))))))\widetilde{\subset}\,\mathrm{cl}(f(f^{-1}((B,K))))$ or $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f(f^{-1}((B,K))))))\widetilde{\subset}\,\mathrm{cl}(B,K)$; that is, $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}((B,K)))))\widetilde{\subset}f^{-1}(\mathrm{cl}(B,K))$.

(5)⇒(1) Let (O,K) be soft open in Y. Let $(U,K) = (O,K)^c$, and $(D,E) = f^{-1}((U,K))$. By (5) we have $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f(f^{-1}((U,K))))))\widetilde{\subset}\,\mathrm{cl}(U,K) = (U,K)$; that is, $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}((O,K)^c))))\widetilde{\subset}f^{-1}((O,K)^c)$. Therefore $f^{-1}((O,K))$ is a soft α-open set in X; hence f is a soft α-continuous function. □

Corollary 27. *Let $f : X \to Y$ be a soft α-continuous mapping. Then*

(1) $f(\mathrm{cl}(A,E))\widetilde{\subset}\,\mathrm{cl}(f((A,E)))$, *for each* $(A,E) \in SPO(X)$;

(2) $\mathrm{cl}(f^{-1}((B,K)))\widetilde{\subset}f^{-1}(\mathrm{cl}(B,K))$, *for each* $(B,K) \in SPO(Y)$.

Proof. Since for each $(A,E) \in SPO(X), \mathrm{cl}((A,E)) = \mathrm{cl}(\mathrm{int}(\mathrm{cl}((A,E))))$, therefore the proof follows directly from statements (4) and (5) of Theorem 26. □

Definition 28. A soft mapping $f : X \to Y$ is called soft precontinuous (resp., soft semicontinuous [7]) if the inverse image of each soft open set in Y is soft preopen (resp., soft semiopen) in X.

Remark 29. It is clear that every soft α-continuous map is soft semicontinuous and soft precontinuous. Every soft continuous map is soft α-continuous. Thus we have implications as shown in Figure 2.

The converses of these implications are not true, which is clear from the following examples.

Example 30. Let $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3,y_4\}$, $E=\{e_1,e_2,e_3\}$, and $K=\{k_1,k_2,k_3\}$ and (X,τ,E) and let (Y,υ,K) be soft topological spaces.

$$\begin{tikzcd}
\text{Soft continuity} \arrow[d] & \\
\text{Soft } \alpha\text{-continuity} \arrow[r] \arrow[dr] & \text{Soft semicontinuity} \\
 & \text{Soft precontinuity}
\end{tikzcd}$$

Figure 2

Define $u : X \to Y$ and $p : E \to K$ as

$u(x_1)=\{y_2\}$, $u(x_2)=\{y_4\}$, $u(x_3)=\{y_1\}$, $u(x_4)=\{y_3\}$,

$p(e_1)=\{k_2\}$, $p(e_2)=\{k_1\}$, $p(e_3)=\{k_3\}$.

Let us consider the soft topology τ given in Example 14; that is,

$\tau=\{\Phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),\ldots,(F_{15},E)\}$, $\upsilon=\{\Phi,\widetilde{Y},(F,K)\}$, and $(F,K) = \{(k_1,\{y_1,y_3,y_4\}),(k_2,\{y_1,y_2,y_4\}),(k_3,\{y_2,y_4\})\}$ and let mapping $f_{up} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft mapping. Then (F,K) is a soft open in Y and $f_{pu}^{-1}((F,K)) = \{(e_1,\{x_1,x_2,x_3\}),(e_2,\{x_2,x_3,x_4\}),(e_3,\{x_1,x_2\})\}$ is a soft α-open but not soft open in X. Therefore, f_{pu} is a soft α-continuous function but not soft continuous function.

Example 31. Let $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3,y_4\}$, $E=\{e_1,e_2,e_3\}$, and $K=\{k_1,k_2,k_3\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces.

Define $u : X \to Y$ and $p : E \to K$ as

$u(x_1)=\{y_2\}$, $u(x_2)=\{y_4\}$, $u(x_3)=\{y_1\}$, $u(x_4)=\{y_3\}$,

$p(e_1)=\{k_2\}$, $p(e_2)=\{k_1\}$, $p(e_3)=\{k_3\}$.

Let us consider the soft topology τ given in Example 14; that is,

$\tau=\{\Phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),\ldots,(F_{15},E)\}$, $\upsilon=\{\Phi,Y,(G,K)\}$, and $(G,K)=\{(k_3,\{y_4\})\}$ and be mapping $f_{up} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft mapping. Then (G,K) is a soft open in Y and $f_{pu}^{-1}((G,K))=\{(e_3,\{x_2\})\}$ is a soft preopen but not soft α-open in X. Therefore, f_{pu} is a soft precontinuous function but not soft α-continuous function.

Example 32. Let $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3,y_4\}$, $E=\{e_1,e_2,e_3\}$, and $K=\{k_1,k_2,k_3\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces.

Define $u : X \to Y$ and $p : E \to K$ as

$u(x_1)=\{y_2\}$, $u(x_2)=\{y_4\}$, $u(x_3)=\{y_1\}$, $u(x_4)=\{y_3\}$,

$p(e_1)=\{k_2\}$, $p(e_2)=\{k_1\}$, $p(e_3)=\{k_3\}$.

Let us consider the soft topology τ given in Example 14; that is,

$\tau=\{\Phi,\widetilde{X},(F_1,E),(F_2,E),(F_3,E),\ldots,(F_{15},E)\}$ and $\upsilon=\{\Phi,Y,(H,K)\}$, $(H,K) = \{(k_1,\{y_1\}),(k_2,\{y_4\}),(k_3,\{y_2,y_4\})\}$ and be mapping $f_{up} : (X,\tau,E) \to (Y,\upsilon,K)$ be a soft mapping. Then (H,K) is a soft open in Y and $f_{pu}^{-1}((H,K)) = \{(e_1,\{x_2\}),(e_2,\{x_3\}),(e_3,\{x_1,x_3\})\}$ is a soft semiopen but not soft α-open in X.

Therefore, f_{pu} is a soft semicontinuous function but not soft α-continuous function.

Theorem 33. *Let $(A,E) \in SPO(X)$ and $(B,E) \in S\alpha OS(X)$. Then $(A,E)\tilde{\cap}(B,E) \in S\alpha OS(X)$.*

Proof. Since $(A,E)\tilde{\cap}(B,E)\tilde{\subset}\operatorname{int}(\operatorname{cl}(A,E))\tilde{\cap}\operatorname{int}(\operatorname{cl}(B,E))$

$= \operatorname{int}(\operatorname{int}(\operatorname{cl}(A,E)))\tilde{\cap}\operatorname{cl}(\operatorname{int}(B,E))$

$\tilde{\subset}\operatorname{int}(\operatorname{cl}((A,E)\tilde{\cap}\operatorname{int}(\operatorname{cl}(B,E))))$, it follows that $(A,E)\tilde{\cap}(B,E)\tilde{\subset}\operatorname{int}(\operatorname{cl}((A,E)\tilde{\cap}\operatorname{int}(B,E)))\tilde{\cap}(A,E)$

$= s\alpha\operatorname{int}(\operatorname{int}(\operatorname{cl}((A,E)\tilde{\cap}\operatorname{int}(B,E)))\tilde{\cap}(A,E))$

$\tilde{\subset} s\alpha\operatorname{int}(\operatorname{cl}((A,E)\tilde{\cap}\operatorname{int}(B,E))\tilde{\cap}(A,E))$

$= s\alpha\operatorname{int}(s\alpha\operatorname{cl}((A,E)\tilde{\cap}\operatorname{int}(B,E)))$

$= s\alpha\operatorname{int}(s\alpha\operatorname{cl}(s\alpha\operatorname{int}((A,E)\tilde{\cap}\operatorname{int}(B,E))))$

$\tilde{\subset} s\alpha\operatorname{int}(s\alpha\operatorname{cl}(s\alpha\operatorname{int}((A,E)\tilde{\cap}(B,E))))$.

Therefore $(A,E)\tilde{\cap}(B,E)$ is a soft α-open set of (A,E). □

Theorem 34. *If $f : X \to Y$ is a soft α-continuous mapping and $(A,E) \in SPO(X)$, then $f|_{(A,E)}$ is a soft α-continuous mapping.*

Proof. Let (B,E) in Y be a soft open set. Then $f^{-1}((B,E)) \in S_a(X)$ and since (A,E) is a soft preopen set in X, by Theorem 26, we have $(A,E)\tilde{\cap}f^{-1}((B,E)) = (f|_{(A,E)})^{-1}((B,E)) \in SaOS(X)((A,E))$. Therefore $f|_{(A,E)}$ is a soft α-continuous mapping. □

Theorem 35. *A soft function $f : X \to Y$ is soft α-continuous if and only if $f(s\alpha\operatorname{cl}(F,E))\tilde{\subset}\operatorname{cl}(f((F,E)))$ for every soft set (F,E) of X.*

Proof. Let $f : X \to Y$ be soft α-continuous. Now $\operatorname{cl}(f((F,E)))$ is a soft closed set of Y;

so by soft α-continuity of f, $f^{-1}(\operatorname{cl}(f((F,E))))$ is soft α-closed and $(F,E)\tilde{\subset}f^{-1}(\operatorname{cl}(f((F,E))))$.

But $s\alpha\operatorname{cl}(F,E)$ is the smallest α-closed set containing (F,E);

hence $s\alpha\operatorname{cl}(F,E)\tilde{\subset}f^{-1}(\operatorname{cl}(f((F,E))))$

$\Rightarrow f(s\alpha\operatorname{cl}(F,E))\tilde{\subset}\operatorname{cl}(f((F,E)))$.

Conversely, let (F,K) be any soft closed set of Y

$\Rightarrow f^{-1}((F,K)) \in X$ and by hypothesis

$\Rightarrow f(s\alpha\operatorname{cl}(f^{-1}((F,K))))\tilde{\subset}\operatorname{cl}(f(f^{-1}((F,K))))$

$\Rightarrow f(s\alpha\operatorname{cl}(f^{-1}((F,K))))\tilde{\subset}\operatorname{cl}(F,K) = (F,K)$

$\Rightarrow s\alpha\operatorname{cl}(f^{-1}((F,K))) = f^{-1}((F,K))$; hence is soft α-closed. Consequently, f is soft α-continuous.

□

Theorem 36. *A soft function $f : X \to Y$ is soft α-continuous if and only if $f^{-1}(\operatorname{int}(H,K))\tilde{\subset}s\alpha\operatorname{int} f^{-1}((H,K))$ for every soft set (H,K) of Y.*

Proof. Let $f : X \to Y$ be soft α-continuous. Now for any soft set (G,E) in X, $\operatorname{int}(f((G,E)))$ is a soft open set in Y; since f is soft α-continuity, then $f^{-1}(\operatorname{int}(f(G,E)))$ is soft α-open and $f^{-1}(\operatorname{int}(f(G,E)))\tilde{\subset}(G,E)$. As $s\alpha\operatorname{int}(G,E)$ is the largest soft α-open set contained in (G,E), $f^{-1}(\operatorname{int}(f(G,E)))\tilde{\subset}s\alpha\operatorname{int}(G,E)$.

Conversely, take a soft open set (G,K) in Y. Then $f^{-1}(\operatorname{int}(G,K))\tilde{\subset}s\alpha\operatorname{int} f^{-1}(G,K) \Rightarrow f^{-1}(G,K)\tilde{\subset}s\alpha\operatorname{int} f^{-1}(G,K) \Rightarrow f^{-1}(G,K)$ is soft α-open. □

5. Soft α-Open and Soft α-Closed Mappings

Definition 37. A soft mapping $f : X \to Y$ is called soft α-open (resp., soft α-closed) mapping if the image of each soft open (resp., soft closed) set in X is a soft α-open set (resp., soft α-closed set) in Y.

Definition 38. A soft mapping $f : X \to Y$ is called soft preopen (resp., soft semiopen [7]) if the image of each soft open set in X is soft preopen (resp., soft semiopen) in Y.

Clearly a soft open map is soft α-open and every soft α-open map is soft preopen as well as soft α-open. Similar implications hold for soft closed mappings.

Theorem 39. *A soft mapping $f : X \to Y$ is soft α-closed if and only if $s\alpha\operatorname{cl}(f(A,E))\tilde{\subset}f(\operatorname{cl}(A,E))$ for each soft set (A,E) in X.*

Proof. Let $s\alpha\operatorname{cl}(f(A,E))\tilde{\subset}f(\operatorname{cl}(A,E))$. By the definition of soft α-closure, we have $f((A,E)) = f(\operatorname{cl}(A,E))$ and so $f(\operatorname{cl}(A,E))$ is a soft α-closed set and f is a soft α-closed mapping.

Conversely, if f is soft α-closed, then $f(\operatorname{cl}(A,E))$ is a soft α-closed set containing $f((A,E))$ and therefore $s\alpha\operatorname{cl}(f(A,E))\tilde{\subset}f(\operatorname{cl}(A,E))$. □

Theorem 40. *A soft function $f : X \to Y$ is soft α-open if and only if $f(\operatorname{int}(F,E))\tilde{\subset}s\alpha\operatorname{int}(f((F,E)))$ for every soft set (F,A) in X.*

Proof. If $f : X \to Y$ is soft α-open, then $f(\operatorname{int}(F,E)) = s\alpha\operatorname{int} f(\operatorname{int}(F,E))\tilde{\subset}s\alpha\operatorname{int} f(F,E)$.

On the other hand, take a soft open set (G,E) in X. Then by hypothesis, $f((G,E)) = f(\operatorname{int}(G,E))\tilde{\subset}s\alpha\operatorname{int}(f((G,E))) \Rightarrow f((G,E))$ is soft α-open in Y. □

Theorem 41. *Let $f : X \to Y$ be a soft α-open (resp., soft α-closed) mapping. If (B,K) is a soft set in Y and (A,E) is a soft closed (resp., soft open) set in X, containing $f^{-1}((B,K))$; then there exists a soft α-closed (resp., soft α-open) set (C,K) in Y, such that $(B,K)\tilde{\subset}(C,K)$ and $f^{-1}((C,K))\tilde{\subset}(A,E)$.*

Proof. Let $(C,K) = (f(A,E)^c)^c$. Since $f^{-1}((B,K))\tilde{\subset}(A,E)$, we have $f((A,E)^c)\tilde{\subset}(B,K)^c$. Since f is soft α-open (resp., soft α-closed), then (C,K) is a soft α-closed set (resp., soft α-open set) if $f^{-1}((C,K)) = (f^{-1}(f((A,E)^c)))^c\tilde{\subset}((A,E)^c)^c = (A,E)$ and hence $(B,K)\tilde{\subset}(C,K)$ and $f^{-1}((C,K))\tilde{\subset}(A,E)$. □

Corollary 42. *If $f : X \to Y$ is a soft α-open mapping, then*

(1) $f^{-1}(\mathrm{cl}(\mathrm{int}(\mathrm{cl}((B,K)))))\tilde{\subseteq}\,\mathrm{cl}(f^{-1}((B,K)))$, *for every soft set (B,K) in Y;*

(2) $f^{-1}(\mathrm{cl}((C,K)))\tilde{\subseteq}\,\mathrm{cl}(f^{-1}((C,K))$, $(C,K) \in SPO(Y)$.

Proof. (1) $\mathrm{cl}(f^{-1}((B,K))$ is a soft closed set in X, containing $f^{-1}((B,K))$, for a soft set (B,E) in Y.

By Theorem 44, there exists a soft α-closed set (F,K) in Y, and $(B,K)\tilde{\subseteq}(F,K)$ such that $f^{-1}((F,K))\tilde{\subseteq}\,\mathrm{cl}(f^{-1}((B,K))$.

Thus $f^{-1}(\mathrm{cl}(\mathrm{int}(\mathrm{cl}((B,E)))))\tilde{\subseteq}\,\mathrm{cl}(f^{-1}(\mathrm{cl}(\mathrm{int}(\mathrm{cl}((F,E)))))\tilde{\subseteq}\,f^{-1}((F,K))\tilde{\subseteq}\,\mathrm{cl}(f^{-1}((B,K))$.

(2) follows easily from (1). □

Theorem 43. *If $f : X \to Y$ is a soft precontinuous and soft α-open mapping, then $f^{-1}((B,K)) \in SPO(X)$ for each $(B,K) \in SPO(Y)$.*

Proof. We have $f^{-1}((B,K))\tilde{\subseteq}f^{-1}(\mathrm{int}(\mathrm{cl}((B,K))))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{int}(\mathrm{cl}((B,K))))))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{cl}((B,K)))))$.

Since f is a soft α-open map, we have, by Corollary 42, $f^{-1}((B,K))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{cl}((B,K)))))\tilde{\subseteq}\,\mathrm{cl}(\mathrm{int}(f^{-1}(\mathrm{cl}((B,K))))) = \mathrm{cl}(\mathrm{int}(f^{-1}((B,K))))$.

Therefore $f^{-1}((B,K))$ is a soft preopen set in X. □

Theorem 44. *If $f : X \to Y$ is a soft precontinuous and soft semicontinuous, then f is soft α-continuous.*

Proof. Let (B,K) be any soft open set in Y. Then $f^{-1}((B,K))$ is a soft preopen set as well as a soft semiopen set in X.

We have $f^{-1}((B,K))\tilde{\subseteq}\,\mathrm{cl}(\mathrm{int}(f^{-1}((B,K))))$ and $f^{-1}((B,K))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(B,K))))) = \mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(B,K))))$.

Hence f is a soft α-continuous mapping. □

Theorem 45. *If $f : X \to Y$ is a soft preopen mapping, then, for each soft set (B,K) in Y, $f^{-1}(\mathrm{int}(\mathrm{cl}(B,K)))\tilde{\subseteq}\,\mathrm{cl}(f^{-1}((B,K)))$.*

Proof. It follows immediately from Corollary 42. □

Theorem 46. *If $f : X \to Y$ is soft α-continuous and soft preopen, then the inverse image of each soft α-set is a soft α-open set.*

Proof. Let (B,K) be any soft α-open set in Y.

Then $f^{-1}((B,K))\tilde{\subseteq}f^{-1}(\mathrm{int}(\mathrm{cl}(\mathrm{int}((B,K)))))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(\mathrm{int}(\mathrm{cl}(\mathrm{int}((B,K))))))))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(\mathrm{cl}(\mathrm{int}(B,K))))))$.

By Theorem 44 we have $f^{-1}((B,E))\tilde{\subseteq}\,\mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{int}((B,E)))))$.

Since f is a soft α-continuous mapping, by Theorem 22(5), $f^{-1}((B,K))\tilde{\subseteq}f^{-1}(\mathrm{int}(\mathrm{cl}(\mathrm{int}((B,K)))))$.

Hence $f^{-1}((B,K))$ is a soft α-open set. □

Corollary 47. *If $f : X \to Y$ is soft α-continuous and soft preopen mapping, then one has the following:*

(1) *the inverse image of each soft α-closed set is soft α-closed,*

(2) $f(\alpha\,\mathrm{cl}(A,E))\tilde{\subseteq}\alpha\,\mathrm{cl}\,f((A,E))$ *for each soft set (A,E) in X.*

Proof. It follows immediately from the previous Theorem 45. □

Theorem 48. *Let $f : X \to Y$ and $g : Y \to Z$ be two soft mappings. If f is soft preopen and soft α-continuous and g is soft α-continuous, then $g \circ f$ is soft α-continuous.*

Proof. It follows immediately from Theorem 45. □

Conflict of Interests

There is no conflict of interests regarding the publication of this paper.

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