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Research Article

Finite-Time H_∞ Control for Time-Delayed Stochastic Systems with Markovian Switching

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This paper studies the problem of finite-time H_∞ control for time-delayed Itô stochastic systems with Markovian switching. By using the appropriate Lyapunov-Krasovskii functional and free-weighting matrix techniques, some sufficient conditions of finite-time stability for time-delayed stochastic systems with Markovian switching are proposed. Based on constructing new Lyapunov-Krasovskii functional, the mode-dependent state feedback controller for the finite-time H_∞ control is obtained. Simulation results illustrate the effectiveness of the proposed method.

1. Introduction

Finite-time stability is different from the usual Lyapunov stability. Lyapunov stability is always used to deal with the asymptotic pattern of system trajectories by applying the steady-state behavior of control dynamics over an infinite-time interval [1]. Often Lyapunov asymptotic stability is not enough for practical applications, because there are some cases where large values of the state are not acceptable, for instance, in the presence of saturations [2]. Lyapunov asymptotic stability depicts steady-state performance of a dynamic system, and it could not reflect transient state performance [3]. A finite-time stable system may not be Lyapunov stable, and a Lyapunov stable system may not be finite-time stable. To study the transient performances of a system, the concept of finite-time stability was introduced by Dorato in [4]. Finite-time stability (or short-time stability) is also called finite-time boundness. A system is said to be finite-time stable if, once a time interval is fixed, its state does not exceed some bounds during this time interval. Because the working time of many systems such as communication network system, missile system, and robot control system is short, people are more interested in finite-time stability of these systems.

Early results on finite-time stability are mostly confined to the stability analysis and lack of design and comprehensiveness of control systems (see [5–9]). During the nineteen seventies, scholars began to discuss the control design method of finite-time stabilization (see [10–13]). In recent years, the development of the theory of linear matrix inequalities promotes the research on finite-time stability and makes this research field a new breakthrough [14–20].

In particular, for systems with time delay or Markov switching or random disturbance, there are some significant research results on finite-time stability and stabilization. For example, finite-time stability and stabilization problem for Itô stochastic systems was studied in [21–27], finite-time stability and stabilization problem for Markovian jump systems was studied in [28–31], and finite-time stability and stabilization problem for time-delay systems was studied in [2, 32].

With the development of finite-time [33] stability, the problem of finite-time H_∞ control has received a lot of attention [1, 3, 34–39]. For example, using the average dwell time method and the multiple Lyapunov-like function technique, some sufficient conditions are proposed to guarantee the finite-time properties for the switched Itô stochastic systems in the form of matrix inequalities and a state feedback controller for the finite-time H_∞ control problem is also obtained in [36]. Delay-dependent observer-based H_∞ finite-time control for switched systems with time-varying delay was investigated in [34]. The robust finite-time H_∞ control problem for a class of uncertain switched neutral systems with unknown time-varying disturbance was developed in [3]. The problem of robust finite-time H_∞ control of singular Itô stochastic systems via static output feedback was addressed in [38]. However, the systems discussed in [3, 34, 36] are general switched systems rather than Markovian jump systems. Markovian jump systems [40–45] (also called systems with Markovian switching) are frequently used to model the dynamics behavior of the process in which variable parameters or structures subject to random abrupt changes occur, for example, sudden environment changes, system noises, subsystem switching, and failures that occurred in interconnections or components and executor faults [46]. On the other hand, most work on the problem of finite-time control focused on the determination of linear or nonlinear system. As is known, stochastic modeling plays an important role in many branches of science and engineering (see [47, 48]). At present, the research of finite-time control for Itô stochastic system is still at the beginning stage. To the best of the authors' knowledge, the problem of finite-time H_∞ control for time-delayed Itô stochastic systems with Markovian switching has not been investigated, which motivated our study.

In this paper, we will focus on the finite-time H_∞ state feedback control problem for time-delayed Itô stochastic systems with Markovian switching. The aim is to find a state feedback controller

$$u_i(t) = K_i x(t), \quad t \in [0, T], \ i = 1, 2, \ldots, N \tag{1}$$

for system (2) such that the corresponding closed-loop system is finite-time stochastically bounded with a weighted H_∞ performance γ. The rest of the paper is organized as follows. In Section 2, problem description and some definitions are given. In Section 3, finite-time stochastic stability and bounded conditions for time-delayed Itô stochastic systems with Markovian switching are presented. The corresponding results of finite-time stochastic H_∞ control problem for time-delayed Itô stochastic systems with Markovian switching are proposed in Section 4. An illustrative example is given in Section 5, and conclusions are given in Section 6.

Notation. Throughout this paper, if not explicit, matrices are assumed to have compatible dimensions. The notation $M > (\geq, <, \leq) 0$ means that the symmetric matrix M is positive-definite (positive-semidefinite, negative, and negative-semidefinite). $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix. $\|\cdot\|$ represents the Euclidean norm for vector or the spectral norm of matrices. I refers to an identity matrix of appropriate dimensions. $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation. The symbol "$*$" within a matrix denotes a term that is induced by symmetry.

2. Problem Description

In this paper, we consider the following time-delayed stochastic systems with Markovian switching:

$$\begin{aligned} dx(t) &= [A(r_t)x(t) + A_1(r_t)x(t-\tau(t)) + B_1(r_t)u(t) \\ &\quad + E_1(r_t)v(t)]\,dt \\ &\quad + [H(r_t)x(t) + H_1(r_t)x(t-\tau(t)) \\ &\quad + B_2(r_t)u(t) + E_2(r_t)v(t)]\,dw(t), \\ z(t) &= C(r_t)x(t) + C_1(r_t)x(t-\tau(t)) + D_1(r_t)u(t), \\ x(t) &= \varphi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^l$ is the control input, $z(t) \in \mathbb{R}^p$ is the control output, and $v(t) \in \mathbb{R}^q$ is exogenous disturbance that satisfies $\int_0^T v^T(t)v(t)dt \leq d(d \geq 0)$. $A(r_t)$, $A_1(r_t)$, $B_1(r_t)$, $E_1(r_t)$, $H(r_t)$, $H_1(r_t)$, $B_2(r_t)$, $E_2(r_t)$, $C(r_t)$, $C_1(r_t)$, and $D_1(r_t)$ are known mode-dependent constant matrices with appropriate dimensions. $w(t)$ is a zero-mean real scalar Wiener process on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where Ω is the sample space, $\mathcal{F}$ is the σ-algebras of sets of the sample space, and P is the probability measure on $\mathcal{F}$. $\varphi(t)$ is an initial condition. It is known that system (2) has a unique solution, denoted by $x(t) = x(t, \varphi)$. $\tau(t)$ is the time-varying delay and satisfies $0 \leq \tau(t) < \tau$, $\dot{\tau}(t) \leq h$, where τ, h are constants.

The jump parameter $r_t (t \geq 0)$ is a continuous-time discrete-state Markov stochastic process taking values on a finite set $\Lambda = \{1, 2, \ldots, N\}$ with transition rate matrix $\Pi = \{\Pi_{ij}\}$ given by

$$P_r = P_r\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \Pi_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1 + \Pi_{ij}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{3}$$

where $\lim_{\Delta t \to 0^+}(o(\Delta t)/\Delta t) = 0$, $\Pi_{ij} \geq 0$, for $i \neq j$, and $\sum_{j=1, j\neq i}^{N} \Pi_{ij} = -\Pi_{ii}$, for $i, j \in \Lambda$.

Definition 1. For given time-constant $T > 0$, system (2) with $u(t) = 0$ and $v(t) = 0$ is said to be stochastically finite-time stable with respect to (c_1, c_2, T, R_i), where $c_1 < c_2$, $R_i > 0$, if

$$\sup_{t\in[-\tau,0]} \varphi^T(t) R_i \varphi(t) \leq c_1 \Longrightarrow \mathbf{E}\{x^T(t) R_i x(t)\} < c_2, \quad \forall t \in [0, T], \ i \in \Lambda. \tag{4}$$

Definition 2. For given time-constant $T > 0$, system (2) with $u(t) = 0$ is said to be finite-time stochastically bounded with respect to (c_1, c_2, T, R_i, d), where $c_1 < c_2$, $R_i > 0$, if

$$\sup_{t\in[-\tau,0]} \varphi^T(t) R_i \varphi(t) \leq c_1 \Longrightarrow \mathbf{E}\{x^T(t) R_i x(t)\} < c_2, \quad \forall t \in [0, T], \ i \in \Lambda, \quad \forall v(t): \int_0^T v^T(t) v(t)\,dt \leq d. \tag{5}$$

Definition 3. For given time-constant $T > 0$, $\gamma > 0$, system (2) with $u(t) = 0$ is said to be H_∞ finite-time stochastically bounded with respect to (c_1, c_2, T, R_i, d), where $c_1 < c_2$, $R_i > 0$, if

(i) system (2) is finite-time stochastically bounded with respect to (c_1, c_2, T, R_i, d);

(ii) under zero-initial condition, the output $z(t)$ satisfies

$$\mathbf{E}\left\{\int_0^T z^T(t) z(t)\,\mathrm{d}t\right\} < \gamma^2 \int_0^T v^T(t) v(t)\,\mathrm{d}t. \tag{6}$$

Definition 4. For given time-constant $T > 0$, $\gamma > 0$, systems (2) are said to be finite-time stabilizable with H_∞ disturbance attenuation level γ, if there exists a controller $u_i(t) = K_i x(t)$ such that

(i) the corresponding closed-loop system is finite-time stochastically bounded with respect to (c_1, c_2, T, R_i, d);

(ii) under zero-initial condition, (6) holds for any $v(t)$ satisfying $\int_0^T v^T(t)v(t)\mathrm{d}t \le d$.

Lemma 5. *Given constant matrices Ω_1, Ω_2, and Ω_3 with appropriate dimensions, where $\Omega_1 = \Omega_1^T$, $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0. \tag{7}$$

3. Finite-Time Stochastic Stability and Bounded Analysis

In this section, we consider the systems (2) with $u(t) = 0$:

$$\begin{aligned} \mathrm{d}x(t) &= \left[A(r_t)x(t) + A_1(r_t)x(t-\tau(t)) + E_1(r_t)v(t)\right]\mathrm{d}t \\ &\quad + \left[H(r_t)x(t) + H_1(r_t)x(t-\tau(t))\right. \\ &\quad \left. + E_2(r_t)v(t)\right]\mathrm{d}w(t), \\ z(t) &= C(r_t)x(t) + C_1(r_t)x(t-\tau(t)), \\ x(t) &= \varphi(t), \quad t \in [-\tau, 0]. \end{aligned} \tag{8}$$

Let $V(x(t), r_t, t)$ be the stochastic Lyapunov Krasovskii functional; define its weak infinitesimal operator as

$$\begin{aligned} &\mathscr{L}V(x(t), r_t, t) \\ &= \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left[\mathbf{E}\left\{V(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t) \mid x(t), r_t\right\}\right. \\ &\quad \left. - V(x(t), r_t, t)\right]. \end{aligned} \tag{9}$$

Theorem 6. *System (2) with $u(t) = 0$ is finite-time stochastically bounded with respect to (c_1, c_2, T, R_i, d), where $c_1 < c_2$, $R_i > 0$, if there exist positive-definite symmetric matrices P_i, N_i, Q, and W and positive scalars α, λ_1, λ_2, and λ_3, such that the following conditions hold:*

$$\begin{bmatrix} A_i^T P_i + P_i A_i + H_i^T P_i H_i - \alpha P_i + Q + \sum_{j=1}^N \Pi_{ij} P_j & P_i A_{1i} + H_i^T P_i H_{1i} - A_i^T N_i & P_i E_{1i} + H_i^T P_i E_{2i} & A_i^T N_i^T \\ * & H_{1i}^T P_i H_{1i} - \Phi(h) Q - N_i A_{1i} & H_{1i}^T P_i E_{2i} - N_i E_{1i} & N_i + A_{1i}^T N_i \\ * & * & -W & E_{1i}^T N_i^T \\ * & * & * & -N_i \end{bmatrix} < 0, \tag{10}$$

$$\lambda_1 R_i \le P_i \le \lambda_2 R_i, \tag{11}$$

$$0 < Q \le \lambda_3 R_i, \tag{12}$$

$$e^{\alpha T}\lambda_2 c_1 + e^{\alpha T}\lambda_3 \tau e^{\alpha\tau} c_1 + \lambda_{\max}(W) e^{\alpha T} d < \lambda_1 c_2. \tag{13}$$

Proof. We denote that $r_t = i$. For convenience, we also denote $A(r_t), A_1(r_t), B_1(r_t), E_1(r_t), H(r_t), H_1(r_t), B_2(r_t), E_2(r_t), C(r_t)$, $C_1(r_t)$, and $D_1(r_t)$ as A_i, A_{1i}, B_{1i}, E_{1i}, H_i, H_{1i}, B_{2i}, E_{2i}, C_i, C_{1i}, and D_{1i}. Take the Lyapunov-Krasovskii functional for systems (8) as

$$\begin{aligned} V(x(t), i, t) &= x^T(t) P_i x(t) + \int_{t-\tau(t)}^t e^{\alpha(t-s)} x^T(s) Q x(s)\,\mathrm{d}s \\ &\triangleq V_{1i}(t) + V_{2i}(t), \end{aligned} \tag{14}$$

where $P_i > 0$ is the given mode-dependent symmetric positive-definite matrix for each mode $i \in \Lambda$ and Q is the symmetric positive-definite matrix.

Along the trajectory of system (8), we have

$$\begin{aligned} &\mathscr{L}V_{1i}(t) \\ &= x^T(t)\left(A_i^T P_i + P_i A_i + H_i^T P_i H_i \right. \\ &\quad \left. - \alpha P_i + \sum_{j=1}^N \Pi_{ij} P_j\right) x(t) \end{aligned}$$

$$\begin{aligned}&+2x^T(t)\left(P_iA_{1i}+H_i^TP_iH_{1i}\right)x(t-\tau(t))\\&+2x^T(t)P_iE_{1i}v(t)+2x^T(t)H_i^TP_iE_{2i}v(t)\\&+x^T(t-\tau(t))H_{1i}^TP_iH_{1i}x(t-\tau(t))\\&+2x^T(t-\tau(t))H_{1i}^TP_iE_{2i}v(t)-v^T(t)Wv(t)\\&+\alpha x^T(t)P_ix(t)+v^T(t)Wv(t),\end{aligned}\tag{15}$$

where $W>0$.

Consider the following:

$$\begin{aligned}\mathscr{L}V_{2i}(t)&=x^T(t)Qx(t)-(1-\dot{\tau}(t))e^{\alpha\tau(t)}\\&\quad\times x^T(t-\tau(t))Qx(t-\tau(t))\\&\quad+\alpha\int_{t-\tau(t)}^{t}e^{\alpha(t-s)}x^T(s)Qx(s)\,\mathrm{d}s\\&\le x^T(t)Qx(t)-\Phi(h)x^T\\&\quad\times(t-\tau(t))Qx(t-\tau(t))\\&\quad+\alpha\int_{t-\tau(t)}^{t}e^{\alpha(t-s)}x^T(s)Qx(s)\,\mathrm{d}s,\end{aligned}\tag{16}$$

where

$$\Phi(h)=\begin{cases}1-h, & h\le 1\\(1-h)e^{\alpha\tau}, & h>1.\end{cases}\tag{17}$$

Set $y(t)=A_ix(t)+A_{1i}x(t-\tau(t))+E_{1i}v(t)$, $N_i>0$; we have

$$\begin{aligned}&2x^T(t-\tau(t))N_i\left[y(t)-A_ix(t)\right.\\&\qquad\left.-A_{1i}x(t-\tau(t))-E_{1i}v(t)\right]=0,\end{aligned}\tag{18}$$

$$2y^T(t)N_i\left[A_ix(t)+A_{1i}x(t-\tau(t))+E_{1i}v(t)-y(t)\right]=0.\tag{19}$$

From (15) to (19), we obtain

$$\begin{aligned}\mathscr{L}V(x(t),i,t)&<\xi^T(t)\Omega\xi(t)\\&\quad+\alpha V(x(t),i,t)+v^T(t)Wv(t),\end{aligned}\tag{20}$$

where

$$\xi^T(t)=\left[x^T(t),x^T(t-\tau(t)),v^T(t),y^T(t)\right],$$
$$\Omega=\begin{bmatrix}A_i^TP_i+P_iA_i+H_i^TP_iH_i-\alpha P_i+Q+\sum_{j=1}^{N}\Pi_{ij}P_j & P_iA_{1i}+H_i^TP_iH_{1i}-A_i^TN_i & P_iE_{1i}+H_i^TP_iE_{2i} & A_i^TN_i^T\\ * & H_{1i}^TP_iH_{1i}-\Phi(h)Q-N_iA_{1i} & H_{1i}^TP_iE_{2i}-N_iE_{1i} & N_i+A_{1i}^TN_i\\ * & * & -W & E_{1i}^TN_i^T\\ * & * & * & -N_i\end{bmatrix}.\tag{21}$$

Using weak infinitesimal operator and (8), we can get

$$\begin{aligned}&\mathrm{d}\left[e^{-\alpha t}V(x(t),i,t)\right]\\&\quad=-\alpha e^{-\alpha t}V(x(t),i,t)\,\mathrm{d}t+e^{-\alpha t}\mathrm{d}V(x(t),i,t)\\&\quad=e^{-\alpha t}(\mathscr{L}V(x(t),i,t)-\alpha V(x(t),i,t))\,\mathrm{d}t\\&\qquad+2e^{-\alpha t}x^T(t)P_i\left[H_ix(t)+H_{1i}x(t-\tau(t))\right.\\&\qquad\left.+E_{2i}v(t)\right]\mathrm{d}w(t).\end{aligned}\tag{22}$$

By integrating both sides of (22) from 0 to t, taking expectations, and by (10)–(12), it follows that

$$\begin{aligned}\mathbf{E}\{V(x(t),i,t)\}&<e^{\alpha t}\mathbf{E}\{V(x(0),r_0,0)\}\\&\quad+\int_0^t e^{\alpha(t-s)}v^T(s)Wv(s)\,\mathrm{d}s\\&\le e^{\alpha t}x^T(0)P_ix(0)+e^{\alpha t}\int_{-\tau}^{0}e^{\alpha s}x^T(s)Qx(s)\,\mathrm{d}s\\&\quad+e^{\alpha T}\int_0^t e^{-\alpha s}v^T(s)Wv(s)\,\mathrm{d}s\\&\le e^{\alpha T}\lambda_2x^T(0)R_ix(0)\\&\quad+e^{\alpha T}\lambda_3\int_{-\tau}^{0}e^{\alpha s}x^T(s)R_ix(s)\,\mathrm{d}s\\&\quad+e^{\alpha T}\int_0^t v^T(s)Wv(s)\,\mathrm{d}s\\&\le e^{\alpha T}\lambda_2c_1+e^{\alpha T}\lambda_3\tau e^{\alpha\tau}c_1+\lambda_{\max}(W)e^{\alpha T}d.\end{aligned}\tag{23}$$

On the other hand, by (11), it is easy to see that

$$\mathbf{E}\{V(x(t),i,t)\}>\mathbf{E}\left\{x^T(t)P_ix(t)\right\}\ge\lambda_1\mathbf{E}\left\{x^T(t)R_ix(t)\right\}.\tag{24}$$

Now, (24) together with (13) and (23) implies that

$$\mathbf{E}\left\{x^T(t)R_ix(t)\right\} < c_2. \tag{25}$$

The proof is completed. □

Remark 7. It should be pointed out that the upper bound h of the derivative of time-varying delay $\tau(t)$ in this paper allows $h \le 1$ or $h > 1$. When $h \le 1$, we have $(\dot{\tau}(t)-1)e^{\alpha\tau(t)} \le h-1$. When $h > 1$, we have $(\dot{\tau}(t)-1)e^{\alpha\tau(t)} < (h-1)e^{\alpha\tau}$ whether $1 < \dot{\tau}(t) < h$ or $\dot{\tau}(t) < 1 < h$. So the function $\Phi(h)$ in (16) is introduced. It should be noted that the upper bound h in [49] only allows $h < 1$. Moreover, as explained above, the inequality amplification result on (14) in [49] is not true. So our results can be applied to more general systems.

Remark 8. From (13), we can obtain the upper bound $\tau_{\max}$ of the delay $\tau(t)$; that is,

$$\tau_{\max} = \frac{\lambda_1 c_2/e^{\alpha T} - \lambda_2 c_1 - d\lambda_{\max}(W)}{\lambda_3 c_1}. \tag{26}$$

Remark 9. Assuming that $W \le \lambda_4 I$, for certain τ and α, by Lemma 5, we can obtain the following linear matrix inequalities (LMIs) that are equivalent to condition (13):

$$\begin{bmatrix} -\lambda_1 c_2 e^{-\alpha T} & \lambda_2\sqrt{c_1} & \lambda_3\sqrt{\tau c_1 e^{\alpha\tau}} & \lambda_4\sqrt{d} \\ * & -\lambda_2 & 0 & 0 \\ * & * & -\lambda_3 & 0 \\ * & * & * & -\lambda_4 \end{bmatrix} < 0. \tag{27}$$

Corollary 10. *System (8) with $v(t)=0$ is stochastically finite-time stable with respect to (c_1, c_2, T, R_i), where $c_1 < c_2$, $R_i > 0$, if there exist positive-definite symmetric matrices P_i, Q, and N_i and positive scalars α, λ_1, λ_2, and λ_3, such that the following conditions hold:*

$$\begin{gathered} \begin{bmatrix} A_i^T P_i + P_i A_i + H_i^T P_i H_i - \alpha P_i + \sum_{j=1}^{N}\Pi_{ij}P_j & P_i A_{1i} + H_i^T P_i H_{1i} - A_i^T N_i & A_i^T N_i^T \\ * & H_{1i}^T P_i H_{1i} - \Phi(h)Q - N_i A_{1i} & N_i + A_{1i}^T N_i \\ * & * & -N_i \end{bmatrix} < 0, \\ \lambda_1 R_i \le P_i \le \lambda_2 R_i, \\ 0 < Q \le \lambda_3 R_i, \\ e^{\alpha T}\lambda_2 c_1 + e^{\alpha T}\lambda_3 \tau e^{\alpha\tau} c_1 < \lambda_1 c_2. \end{gathered} \tag{28}$$

4. Finite-Time Stochastic H_∞ Control

In this section, we consider the problem of finite-time stochastic H_∞ control for time-delayed Itô stochastic systems with Markovian switching. We consider the mode-dependent controller $u(t) = K_i x(t)$, $t \in [0, T]$, where K_i is the state feedback gain that has to be determined. Applying the state feedback controller into system (2) and denoting $r_t = i$, we can obtain the corresponding closed-loop system as follows:

$$\begin{aligned} \mathrm{d}x(t) &= \left[\widetilde{A}_i x(t) + A_{1i}x(t-\tau(t)) + E_{1i}v(t)\right]\mathrm{d}t \\ &\quad + \left[\widetilde{H}_i x(t) + H_{1i}x(t-\tau(t)) + E_{2i}v(t)\right]\mathrm{d}w(t), \\ z(t) &= \widetilde{C}_i x(t) + C_{1i}x(t-\tau(t)), \\ x(t) &= \varphi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{29}$$

where $\widetilde{A}_i = A_i + B_{1i}K_i$, $\widetilde{H}_i = H_i + B_{2i}K_i$, and $\widetilde{C}_i = C_i + D_{1i}K_i$.

Theorem 11. *System (29) is finite-time stabilizable with H_∞ disturbance attenuation level $\overline{\gamma}$, if there exist positive-definite symmetric matrices P_i, Q, and $\widetilde{N}_i$ and positive scalars α, λ_1, λ_2, and λ_3, such that conditions (11)-(12) and the following conditions hold:*

$$\begin{bmatrix} \widetilde{A}_i^T P_i + P_i\widetilde{A}_i - \alpha P_i + Q + \sum_{j=1}^{N}\Pi_{ij}P_j & P_i A_{1i} - \widetilde{A}_i^T\widetilde{N}_i^T & P_i E_{1i} & \widetilde{A}_i^T\widetilde{N}_i^T & \widetilde{C}_i^T & \widetilde{H}_i^T \\ * & -\Phi(h)Q - \widetilde{N}_i A_{1i} & -\widetilde{N}_i E_{1i} & \widetilde{N}_i + A_{1i}^T\widetilde{N}_i & C_{1i}^T & H_{1i}^T \\ * & * & -\gamma^2 I & E_{1i}^T\widetilde{N}_i^T & 0 & E_{2i}^T \\ * & * & * & -\widetilde{N}_i & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -P_i^T \end{bmatrix} < 0, \tag{30}$$

$$e^{\alpha T}\lambda_2 c_1 + e^{\alpha T}\lambda_3 \tau e^{\alpha\tau} c_1 + \gamma^2 d e^{\alpha T} < \lambda_1 c_2. \tag{31}$$

Proof. Choose the Lyapunov-Krasovskii functional for systems (29) as

$$\begin{aligned} V(x(t), i, t) &= x^T(t) P_i x(t) + \int_{t-\tau(t)}^{t} e^{\alpha(t-s)} x^T(s) Q x(s)\, ds \\ &\quad + \int_0^t e^{\alpha(t-s)} v^T(s) E_{2i}^T P_i E_{2i} v(s)\, ds \\ &\triangleq V_{1i}(t) + V_{2i}(t) + V_{3i}(t), \end{aligned} \tag{32}$$

where $P_i > 0$ is the given mode-dependent symmetric positive-definite matrix for each mode $i \in \Lambda$ and Q is the symmetric positive-definite matrix.

Along the trajectory of system (29), we have

$$\begin{aligned} \mathscr{L}V_{1i}(t) &= x^T(t) \left(\widetilde{A}_i^T P_i + P_i \widetilde{A}_i + \widetilde{H}_i^T P_i \widetilde{H}_i - \alpha P_i + \sum_{j=1}^{N} \Pi_{ij} P_j \right) x(t) \\ &\quad + 2x^T(t) \left(P_i A_{1i} + \widetilde{H}_i^T P_i H_{1i} \right) x(t-\tau(t)) \\ &\quad + 2x^T(t) P_i E_{1i} v(t) \\ &\quad + 2x^T(t) \widetilde{H}_i^T P_i E_{2i} v(t) \\ &\quad + x^T(t-\tau(t)) H_{1i}^T P_i H_{1i} x(t-\tau(t)) \\ &\quad + 2x^T(t-\tau(t)) H_{1i}^T P_i E_{2i} v(t) - \gamma^2 v^T(t) v(t) \\ &\quad + \alpha x^T(t) P_i x(t) + \gamma^2 v^T(t) v(t), \end{aligned}$$

$$\begin{aligned} \mathscr{L}V_{2i}(t) &\le x^T(t) Q x(t) \\ &\quad - \Phi(h) x^T(t-\tau(t)) Q x(t-\tau(t)) + \alpha V_{2i}(t), \\ \mathscr{L}V_{3i}(t) &= v^T(t) E_{2i}^T P_i E_{2i} v(t) \\ &\quad + \alpha \int_0^t e^{\alpha(t-s)} v^T(s) E_{2i}^T P_i E_{2i} v(s)\, ds. \end{aligned} \tag{33}$$

Set $\widetilde{y}(t) = \widetilde{A}_i x(t) + A_{1i} x(t-\tau(t)) + E_{1i} v(t)$, $\widetilde{N}_i > 0$; we have

$$\begin{aligned} &2x^T(t-\tau(t)) \widetilde{N}_i \left[\widetilde{y}(t) - \widetilde{A}_i x(t) - A_{1i} x(t-\tau(t)) - E_{1i} v(t) \right] = 0, \\ &2\widetilde{y}^T(t) \widetilde{N}_i \left[\widetilde{A}_i x(t) + A_{1i} x(t-\tau(t)) + E_{1i} v(t) - \widetilde{y}(t) \right] = 0. \end{aligned} \tag{34}$$

From (33) to (34), we obtain

$$\begin{aligned} \mathscr{L}V(x(t), i, t) &< \widetilde{\xi}^T(t) \widetilde{\Omega} \widetilde{\xi}(t) \\ &\quad + \alpha V(x(t), i, t) + \gamma^2 v^T(t) v(t) - z^T(t) z(t), \end{aligned} \tag{35}$$

where

$$\widetilde{\xi}^T(t) = \left[x^T(t), x^T(t-\tau(t)), v^T(t), \widetilde{y}^T(t) \right],$$

$$\widetilde{\Omega} = \begin{bmatrix} \widetilde{A}_i^T P_i + P_i \widetilde{A}_i + \widetilde{H}_i^T P_i \widetilde{H}_i - \alpha P_i + \sum_{j=1}^{N} \Pi_{ij} P_j + \widetilde{C}_i^T \widetilde{C}_i & P_i A_{1i} + \widetilde{H}_i^T P_i H_{1i} - \widetilde{A}_i^T \widetilde{N}_i + \widetilde{C}_i^T \widetilde{C}_{1i} & P_i E_{1i} + \widetilde{H}_i^T P_i E_{2i} & \widetilde{A}_i^T \widetilde{N}_i^T \\ * & H_{1i}^T P_i H_{1i} - \Phi(h) Q - \widetilde{N}_i A_{1i} + C_{1i}^T C_{1i} & H_{1i}^T P_i E_{2i} - \widetilde{N}_i E_{1i} & \widetilde{N}_i + A_{1i}^T \widetilde{N}_i \\ * & * & -\gamma^2 E_{2i}^T P_i E_{2i} & E_{1i}^T \widetilde{N}_i^T \\ * & * & * & -\widetilde{N}_i \end{bmatrix}. \tag{36}$$

Using Lemma 5, we have that (30) is equivalent to $\widetilde{\Omega} < 0$. Then (35) becomes

$$\begin{aligned} \mathscr{L}V(x(t), i, t) &< \alpha V(x(t), i, t) \\ &\quad + \gamma^2 v^T(t) v(t) - z^T(t) z(t). \end{aligned} \tag{37}$$

Under zero initial condition, we have

$$\begin{aligned} 0 &< e^{-\alpha T} \mathbf{E} \{ V(x(t), i, t) \} \\ &< \mathbf{E} \left\{ \int_0^T e^{-\alpha s} \left(\gamma^2 v^T(s) v(s) - z^T(s) z(s) \right) ds \right\}. \end{aligned} \tag{38}$$

Thus

$$\begin{aligned}
&\mathbf{E}\left\{\int_0^T e^{-\alpha s} z^T(s)\, z(s)\, \mathrm{d}s\right\} \\
&\quad < \gamma^2 \mathbf{E}\left\{\int_0^T e^{-\alpha s} v^T(s)\, v(s)\, \mathrm{d}s\right\}, \\
&\mathbf{E}\left\{\int_0^T z^T(s)\, z(s)\, \mathrm{d}s\right\} \\
&\quad < \gamma^2 e^{\alpha T} \mathbf{E}\left\{\int_0^T e^{-\alpha s} v^T(s)\, v(s)\, \mathrm{d}s\right\} \\
&\quad < \gamma^2 e^{\alpha T} \mathbf{E}\left\{\int_0^T v^T(s)\, v(s)\, \mathrm{d}s\right\}.
\end{aligned} \tag{39}$$

Let $\overline{\gamma} = \sqrt{e^{\alpha T}}\gamma$; then $\overline{\gamma}$ is H_∞ performance index. When $z(t) = 0$, similar to the proof of Theorem 6, it can be obtained that

$$\mathbf{E}\left\{x^T(t)\, R_i x(t)\right\} \le \frac{e^{\alpha T}\lambda_2 c_1 + e^{\alpha T}\lambda_3 \tau e^{\alpha\tau} c_1 + \gamma^2 e^{\alpha T} d}{\lambda_1}. \tag{40}$$

From (31), we can get

$$\mathbf{E}\left\{x^T(t)\, R_i x(t)\right\} < c_2. \tag{41}$$

The proof is completed. □

Theorem 12. *System* (29) *is finite-time stabilizable with* H_∞ *disturbance attenuation level* $\overline{\gamma}$, *if there exist positive-definite symmetric matrices* X_i, $\widetilde{Q}_i$, $\widehat{Q}_i$, *and* $\widehat{N}_i$, *appropriate dimensions matrices* Y_i, *and positive scalars* α, λ_1, λ_2, *and* λ_3, *such that conditions* (11)-(12), (31) *and the following conditions hold:*

$$\begin{bmatrix}
A_iX_i + X_iA_i^T + B_{1i}Y_i + Y_i^TB_{1i}^T - \alpha X_i + \widehat{Q}_i + \sum_{j=1}^{N}\Pi_{ij}X_j & A_{1i}\widehat{N}_i - X_iA_i^T - Y_i^TB_{1i}^T & E_{1i} & X_iA_i^T + Y_i^TB_{1i}^T & X_iC_i^T + Y_i^TD_{1i}^T & X_iH_i^T + Y_i^TB_{2i}^T \\
* & -\Phi(h)\,\widetilde{Q}_i - A_{1i}\widehat{N}_i & -E_{1i} & \widehat{N}_i + \widehat{N}_iA_{1i}^T & \widehat{N}_iC_{1i}^T & \widehat{N}_iH_{1i}^T \\
* & * & -\gamma^2\widehat{N}_i & \widehat{N}_iE_{1i}^T & 0 & E_{2i}^T \\
* & * & * & -\widehat{N}_i & 0 & 0 \\
* & * & * & * & -I & 0 \\
* & * & * & * & * & -X_i
\end{bmatrix} < 0. \tag{42}$$

Moreover, a state feedback controller gain is given by $K_i = Y_iX_i^{-1}$.

Proof. Replacing $\widetilde{A}_i$, $\widetilde{H}_i$, and $\widetilde{C}_i$ in (30) with $A_i + B_{1i}K_i$, $H_i + B_{2i}K_i$, and $C_i + D_{1i}K_i$, then premultiplying and postmultiplying it by $\mathrm{diag}\{P_i^{-1}, \widetilde{N}_i^{-1}, I, \widetilde{N}_i^{-1}, I, I\}$, and denoting $P_i^{-1} = X_i$, $Y_i = KX_i$, $X_i^TQX_i = \widehat{Q}_i$, $\widetilde{N}_i^{-1} = \widehat{N}_i$, and $\widetilde{N}_i^{-1}Q\widetilde{N}_i^{-1} = \widetilde{Q}_i$, we can obtain (42).

The proof is completed. □

Remark 13. Replacing λ_4 in (27) with γ^2, then it is equivalent to (31). For certain λ_1 and λ_2, all the conditions of Theorem 12 can be expressed as linear matrix inequalities. In this way, finite-time H_∞ state feedback stabilization conditions for time-delayed Itô stochastic systems with Markovian switching are based entirely on linear matrix inequalities. In the practical application of dynamical systems, we can obtain the controller effectively with the help of LMI toolbox in MATLAB.

Remark 14. In order to obtain the finite-time H_∞ stabilization conditions based on LMIs for time-delayed Itô stochastic systems with Markovian switching, new Lyapunov-Krasovskii functional (32) is introduced.

Remark 15. In the sense of Lyapunov stability, the problem of H_∞ control for systems with Markovian switching and time delay has attracted a lot of research (e.g., see [40, 41]). Different from these studies, this paper focuses on this problem under the sense of finite-time stability. The latter is suitable for transient performance of actual systems such as communication network system, missile system, and robot control system.

5. Illustrative Example

In this section, we will discuss one example to illustrate our results.

Example 16. Consider time-delayed Itô stochastic systems with Markovian switching (29) with the following parameters:

$$A_1 = \begin{bmatrix} -0.1 & 2 \\ 2 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

$$C_1 = C_{11} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix}, \qquad C_2 = C_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \qquad H_{12} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix},$$

$$E_{21} = \begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0.4 \end{bmatrix}, \qquad E_{22} = \begin{bmatrix} 0.1 & 0.3 \\ -0.1 & 0.1 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} 16 & 12 \\ 2 & 13 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} 12 & 8 \\ 10 & 3 \end{bmatrix},$$

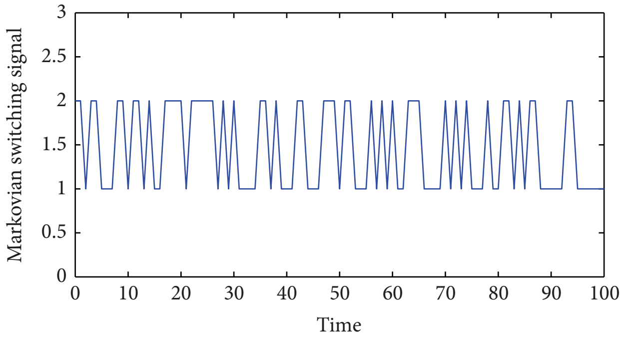

FIGURE 1: Markovian switching signal.

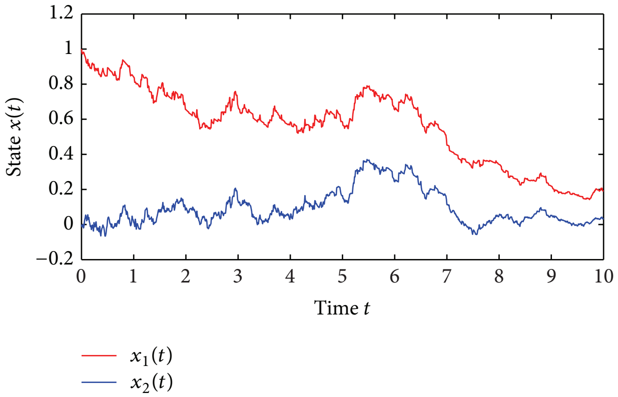

FIGURE 2: State trajectory of the closed-loop system.

$$B_{21} = \begin{bmatrix} 0.1 & 0.3 \\ -2 & 0.4 \end{bmatrix}, \qquad B_{22} = \begin{bmatrix} 0.1 & 0.3 \\ 0 & 0.1 \end{bmatrix},$$
$$E_{11} = \begin{bmatrix} 0.01 & 0.02 \\ 0.1 & 0.2 \end{bmatrix}, \qquad E_{12} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0 \end{bmatrix},$$
$$D_{11} = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0.2 & 0.2 \\ 1 & 0.1 \end{bmatrix},$$
$$\Pi = \begin{bmatrix} -6 & 6 \\ 8 & -8 \end{bmatrix}. \tag{43}$$

Denote transition probabilities by p_1 and p_2. By using $[p_1 \quad p_2]\Pi = 0$ and $p_1 + p_2 = 1$, we can obtain $p_1 = 4/7$ and $p_2 = 3/7$. Figure 1 shows the Markovian switching signal within 100 times according to the above transition probabilities.

Choose $\alpha = 0.1$, $\tau = 0.1$, $h = 1$, $c_1 = 1$, $c_2 = 4$, $T = 10$, $R_1 = R_2 = I$, $\gamma = \sqrt{3}$, and $d = 0.1$. Then, solving conditions (41), (11), (12), and (31) in Theorem 12 for $\lambda_1 = 2$ and $\lambda_2 = 2.01$ yields

$$K_1 = \begin{bmatrix} 0.1209 & -0.1828 \\ -0.1743 & 0.0778 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.1106 & 0.3890 \\ 0.3831 & -0.7077 \end{bmatrix},$$
$$\lambda_3 = 0.0012. \tag{44}$$

The state trajectories of the closed-loop system are shown in Figure 2. It is easy to see that the system is finite-time stochastically bounded.

6. Conclusions

In this paper, finite-time stochastic stability and finite-time stochastic H_∞ control problem for time-delayed Itô stochastic systems with Markovian switching are investigated with Lyapunov-Krasovskii functional approach and free-weighting matrix techniques. Some criteria are established. One example is given for illustration.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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