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Research Article

Exact Boundary Controller Design for a Kind of Enhanced Oil Recovery Models

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The exact boundary controllability of a class of enhanced oil recovery systems is discussed in this paper. With a simple transformation, the enhanced oil recovery model is first affirmed to be neither genuinely nonlinear nor linearly degenerate. It is then shown that the enhanced oil recovery system with nonlinear boundary conditions is exactly boundary controllable by applying a constructed method. Moreover, an interval of the control time is presented to not only give the optimal control time but also show the time for avoiding the blowup of the controllable solution. Finally, an example is given to illustrate the effectiveness of the proposed criterion.

1. Introduction

In recent years, the economy has developed rapidly over the years requiring a lot of energy sources in China, but it is impossible to largely import oil required. Many oil fields in China are developed by water flooding, but now, the recovery efficiency is low and water cut is over 80% because of the heterogeneity of reservoirs and high viscosity of oil. It is essential to increase the oil production of oil fields. As a result, enhanced oil recovery (EOR) has been a challenging field for different scientific disciplines. A mathematical model in [1] is developed to describe surfactant-enhanced solubilization of nonaqueous-phase liquids (NAPLs) in porous media. The goal in [2] is to find an optimal viscosity profile of the intermediate layer that almost eliminates the growth of the interfacial disturbances induced by mild perturbation of the permeability field. The mechanism of enhanced oil recovery using lipophobic and hydrophilic polysilicon (LHP) nanoparticles ranging in size from 10 to 500 nm for changing the wettability of porous media is analyzed theoretically in [3]. It is shown in [4] that water-soluble hydrophobically associating polymers are reviewed with particular emphasis on their application in improved oil recovery (IOR). The solution properties of enhanced oil recovery are provided in [5, 6]. In order to enhance oil recovery and stabilize oil production, the study on EOR has been carried out for more than 20 years (also see [7–11]).

One of the strategies used in EOR is to use polymer flooding. Polymer flooding involves using a polymer additive to increase water viscosity, improve the water-oil mobility ratio, and enhance the displacement efficiency. Polymer flooding has been widely applied as an effective tertiary oil-recovery method in Daqing, Shengli, and other oilfields in China.

However, Different polymer flooding units have different static conditions and development status before polymer flooding. The production performance and behavior are also different. The quantitative characterization and prediction of polymer flooding performance have important guiding significance for polymer flooding scheme programming, performance evaluation, and adjustment. Hence, it is necessary to construct some mathematical models to illustrate the properties of polymer flooding. In [12–15], a 2×2 nonlinear system model is presented to describe the polymer flooding of an oil recovery:

$$\begin{aligned} s_t + (sf(s, cs))_x &= 0, \\ (sc)_t + (scf(s, cs))_x &= 0, \end{aligned} \tag{1}$$

where $s = s(x,t)$ is the saturation of the aqueous phase (i.e., the solution of polymer and water, $0 \le s \le 1$), $c = c(x,t)$ is the concentration of polymer in the water ($0 \le c \le 1$). $f = f(s, cs)$ [16] is the particle velocity of the aqueous phase. x denotes the position in the reservoir and t denotes the time. In the polymer flooding, water thickened with polymer is injected into the reservoir.

Let $u = (s, cs)$; system (1) can be written as

$$u_t + (uf(u))_x = 0, \tag{2}$$

where $u \triangleq (u_1, u_2) = (s, sc)$, $f(u) = f(s, cs)$ is a scalar function and usually referred to be the flow function. In this paper, we consider f to be rotationally invariant; namely, define that $f(u) = f(\|u\|)$ with $\|u\| = \sqrt{u_1^2 + u_2^2}$. As a result, system (2) can be described as

$$u_t + (uf(\|u\|))_x = 0. \tag{3}$$

To the authors' knowledge, seldom researchers discussed the optimal control problem of system (3) (or (1)), but it is really interesting. Actually, in the last forty years, different optimal control schemes such as pinning control and impulsive control have been presented on all kinds of mathematical models of the engineering and physical application [17–19]. It is worth noting that almost all of the discussed models in [17–19] are ordinary differential but system (3) is partial differential. A problem is arisen: how to discuss the control problem of the partial differential model (3)? By the constructive method, the authors in [20, 21] discuss the global exact boundary controllability of a class of quasilinear hyperbolic systems of conservation laws with linearly degenerate characteristics. Inspired by [20, 21], we will discuss the exact boundary control problem of system (3) by using a constructed method. Hence, the main concern of this paper is to design an exact boundary controller for the EOR model (3).

The remainder of this paper is organized as follows. In Section 2, the exact boundary control problem and some Lemmas are presented. In Section 3, the main result is completed by a constructive method. Moreover, some important lemmas are also proposed in this section. In Section 4, an example is carried out to illustrate the effectiveness of the main result. Finally, conclusions are drawn in Section 5.

2. Problem Description

Let $u = R\cdot\vec{\theta}$ ($0 \le R \le 1$ and $\vec{\theta} = (\cos\theta, \sin\theta)$ is a unit vector); system (3) can be written as

$$(R\vec{\theta})_t + (f(R)R\vec{\theta})_x = 0; \tag{4}$$

one has

$$(R_t + (f(R)R)_x)\vec{\theta} + R(\vec{\theta}_t + f(R)\vec{\theta}_x) = 0. \tag{5}$$

According to (5), one has

$$(R_t + (f(R)R)_x)\cos\theta - R\sin\theta(\theta_t + f(R)\theta_x) = 0, \tag{6}$$

$$(R_t + (f(R)R)_x)\sin\theta + R\cos\theta(\theta_t + f(R)\theta_x) = 0. \tag{7}$$

Then, one has from (6) and (7)

$$R_t + (f(R)R)_x = 0. \tag{8}$$

Inserting (8) into (6) or (7), one gets

$$\theta_t + f(R)\theta_x = 0. \tag{9}$$

Hence, one has

$$\begin{aligned} R_t + A(R)R_x &= 0, \\ \theta_t + f(R)\theta_x &= 0, \end{aligned} \tag{10}$$

where $A(R) = f'(R)R + f(R)$.

In the following, we will investigate the *exact boundary control problem* for system (10) (or system (3)). Consider system (10) posed on the domain

$$D = \{(x,t) \mid 0 \le x \le 1, t \ge 0\}, \tag{11}$$

with the initial data

$$R(x,0) = R_0(x), \qquad \theta(x,0) = \theta_0(x), \tag{12}$$

and the nonlinear boundary conditions

$$\begin{aligned} R &= g_1(\theta, t) + h_1(t), \quad \text{at } x = 0, \\ \theta &= g_2(R, t) + h_2(t), \quad \text{at } x = 1, \end{aligned} \tag{13}$$

where $g_i(R, \theta, t)$ ($i = 1, 2$) are given smooth functions. systems (10) and (13) can be viewed as boundary control systems when boundary value functions h_1 and h_2 are considered as control inputs. Hence, we only need to study the following problem.

Exact Boundary Control Problem: given $R_0, \theta_0 \in C^1[0,1]$, and $R_T, \theta_T \in C^1[0,1]$, can one find a time $T > 0$ and control inputs $h_1, h_2 \in C^1[0,T]$ such that the boundary control systems (10) and (13) have a C^1 solution (R, θ) satisfying the initial conditions (12) and the terminal conditions

$$R(x,T) = R_T(x), \qquad \theta(x,T) = \theta_T(x)? \tag{14}$$

In order to solve the above boundary control problem, we need the following assumptions and Lemmas.

Assumption 1. For any $R \in [0,1]$, there exists

$$f(R) < 0 < A(R). \tag{15}$$

Assumption 2. For simplification, we assume that

$$\frac{\partial A(R)}{\partial R} > 0, \quad \forall R \in [0,1]. \tag{16}$$

As a result, when $R_0'(x) \geq 0$, the Cauchy problem (10) and (12) has a C^1 global solution. Moreover, $R_x \geq 0$.

Remark 3. Assumption 1 denotes that system (10) is strongly strictly hyperbolic.

Remark 4. The discussed models in [20, 21] are both linearly degenerate. However, model (10) in this paper is neither genuinely nonlinear nor linearly degenerate, which is more difficult and complicated to be discussed.

Lemma 5 (see [22]). *Consider the Cauchy problem* (10) *and* (12). *Suppose that* $f(R)$, $A(R)$, R_0, θ_0 *are all* C^1 *functions and the* C^1 *norm of* $R_0(x)$ *and* $\theta_0(x)$ *are bounded. Under Assumption 1, if*

$$\frac{\partial A(R_0(\alpha))}{\partial \alpha} \geq 0, \quad \forall \alpha \in [0,1], \tag{17}$$

the Cauchy problem (10) *and* (12) *has a unique global* C^1 *solution* $(R,\theta) = (R(t,x),\theta(t,x))$ *on the domain* $\Gamma = \{(t,x) \mid x \in \mathbb{R}, t \geq 0\}$.

Remark 6. For the Cauchy problem (10) and (14), we need to modify (17) to be

$$\frac{\partial A(R_T(\beta))}{\partial \beta} \leq 0, \quad \forall \beta \in [0,1]. \tag{18}$$

When $R_T'(x) \leq 0$, the Cauchy problem (10) and (14) has a unique global C^1 solution $(R,\theta) = (R(t,x),\theta(t,x))$ on the domain $\Gamma_1 = \{(t,x) \mid x \in [0,1], t \in (-\infty, T]\}$. Moreover, $R_x \leq 0$.

In the following, we consider the Goursat problem of system (10) on the angular domain

$$D_1 = \{(x,t) \mid x_1(t) \leq x \leq x_2(t), t \geq 0\}. \tag{19}$$

We prescribe boundary conditions

$$\begin{aligned} R(t, x_1(t)) &= \widetilde{R}(t), \quad \text{on } x = x_1(t), \\ \theta(t, x_2(t)) &= \widetilde{\theta}(t), \quad \text{on } x = x_2(t), \end{aligned} \tag{20}$$

where $x_1(t)$ and $x_2(t)$ are the characteristics passing through the origin point $O = (0,0)$, on which it holds

$$\begin{aligned} \frac{dx_1(t)}{dt} &= f(\widetilde{R}(t)), \quad x_1(0) = 0, \\ \frac{dx_2(t)}{dt} &= A(R_0(0)), \quad x_2(0) = 0. \end{aligned} \tag{21}$$

Lemma 7 (see [22]). *Suppose that* $f(R)$, $A(R)$, $\widetilde{R}$, $\widetilde{\theta}$ *are all* C^1 *functions. Under Assumptions 1 and 2, if*

$$\widetilde{R}'(t) \leq 0, \tag{22}$$

the Goursat problem (10) *and* (20) *has a unique global* C^1 *solution* $(\widetilde{R}, \widetilde{\theta})$ *on the domain* D_1.

First, we need to discuss the lifespan of the Cauchy problem and Goursat problem. From the Cauchy problem (10) and (12), we have the following lemma.

Lemma 8. *If there exists* $\alpha_0 \in \mathbb{R}$ *such that* $dR_0(\alpha)/d\alpha|_{\alpha=\alpha_0} < 0$, *the Cauchy problem* (10) *and* (12) *must blow up in a finite time and the lifespan is dependent on the initial data.*

Proof. For the first equation of system (10), the characteristic $x_2(t)$ can be defined by

$$\frac{dx_2(t)}{dt} = A(R(x_2(t),t)), \quad x_2(0) = \alpha. \tag{23}$$

One has $dR/dt = 0$; that is, $R(x_2(t),t) = R_0(\alpha)$. That is, the C^0 norm of R is finite. Hence, we need to show that the first derivative of R must blow up in a finite time.

Clearly, $R_x = (dR_0(\alpha)/d\alpha)/(dx/d\alpha)$. From (23), one has $x_2 = \alpha + A(R_0(\alpha))t$. Then,

$$\frac{dx_2}{d\alpha} = 1 + \frac{\partial A(R)}{\partial R}\frac{dR_0(\alpha)}{d\alpha}t. \tag{24}$$

According to (16), if there exists $\alpha_0 \in \mathbb{R}$ such that $dR_0(\alpha)/d\alpha|_{\alpha=\alpha_0} < 0$, one has

$$R_x \longrightarrow \infty, \quad \text{at } t = \frac{-1}{(\partial A(R)/\partial R)(dR_0(\alpha)/d\alpha)|_{\alpha=\alpha_0}}. \tag{25}$$

That is, the Cauchy problem (10) and (12) must blow up in a finite time. Moreover, from (24), one has $t = 1/O(\varepsilon)$ when $dR_0(\alpha)/d\alpha = O(\varepsilon)$, which means that the lifespan t is dependent on the initial data. The proof is completed. □

Remark 9. Note that, in the second equation of system (10), according to [22], θ_x will always be bounded. Moreover, θ_x will blow up if the following holds:

$$\frac{\partial f(R_0(\alpha), \theta_0(\beta))}{\partial \beta} = \frac{\partial f}{\partial \theta} \cdot \frac{\partial \theta_0(\beta)}{\partial \beta} < 0. \tag{26}$$

Obviously, this does not hold since $f(\cdot)$ is independent of the function θ. As a result, θ_x will never blow up in a finite time.

For the Goursat problem (10) and (20), we have the following result.

Lemma 10. *If there exists* $\alpha_0 \in \mathbb{R}$ *such that* $\widetilde{R}'(\alpha_0) > 0$, *the solution of the Goursat problem* (10) *and* (20) *must blow up in a finite time and the lifespan depends on the initial data.*

Proof. For $\forall (x,t) \in D_1$, its two characteristics have two intersect points with the curves $x_1(t)$ and $x_2(t)$, which are, respectively, defined as $(x_1(\alpha),\alpha)$ and $(x_2(\beta),\beta)$. Along the two characteristics, one has $R(x,t) = \widetilde{R}(\alpha)$ and $\theta(x,t) = \widetilde{\theta}(\beta)$, respectively. As a result, $R(x,t), \theta(x,t)$ are bounded.

In the following, we will calculate R_x and θ_x. Clearly, one has $R_x = (d\widetilde{R}(\alpha)/d\alpha)/(dx/d\alpha)$, and $x(t) =$

$x_1(\alpha) + A(\widetilde{R}(\alpha))(t - \alpha)$. Hence, $x_\alpha = f(\widetilde{R}(\alpha)) + (\partial A(\widetilde{R}(\alpha))/\partial R)(d\widetilde{R}(\alpha)/d\alpha)(t-\alpha) - A(\widetilde{R}(\alpha))$. According to (15) and (16), one has the fact that $f(\widetilde{R}(\alpha)) - A(\widetilde{R}(\alpha)) < 0$ and $\partial A(\widetilde{R}(\alpha))/\partial R > 0$. As a result, if there exists $\alpha_0 \in \mathbb{R}$ such that $\widetilde{R}'(\alpha_0) > 0$, $R_x \to \infty$ in a finite time t. Moreover, similar to Lemma 8, one knows that the lifespan t depends on the initial data.

Similarly, one has $\theta(x,t) = \widetilde{\theta}(\beta)$ with $x = x_2(\beta) + f(\widetilde{R}(\alpha))(t-\beta)$. As a result, $dx/d\beta = A(\widetilde{R}(\alpha)) - f(\widetilde{R}(\alpha)) > 0$, which means that θ_x will never blow up in a finite time. □

Remark 11. According to Lemmas 8 and 10 and Remark 9, one knows that the blowup of the Goursat problem and the Cauchy problem only occurs in the solution R.

3. Main Results

In this section, the boundary controllers will be designed.

Theorem 12. *With Assumptions 1 and 2, and conditions* (17)-(18), *for given R_0, θ_0, R_T, θ_T in the space $C^1[0,1]$ with their C^0 norms bounded by $\Xi > 0$ and the small norms of their first derivatives, and for any T satisfying $T_0 < T < T_1 < +\infty$, there exist $h_1(t), h_2(t) \in C^1[0,T]$ such that systems* (10) *and* (12) *have a C^1 solution $(R(x,t), \theta(x,t))$ on the domain*

$$D_T = \{(x,t) \mid 0 \le x \le 1, 0 \le t \le T\}, \tag{27}$$

satisfying $R(x,0) = R_0(x)$, $\theta(x,0) = \theta_0(x)$, $R(x,T) = R_T(x)$, $\theta(x,T) = \theta_T(x)$, for $0 \le x \le 1$, where

$$T_0 = \max\left\{-\frac{1}{f_{\max}}, \frac{1}{A_{\min}}\right\}, \tag{28}$$

and $f_{\max} = \max_{|R|\le\Xi} f(R)$, $A_{\min} = \min_{|R|\le\Xi} A(R)$. Moreover, T_1 is the lifespan of the Cauchy problem (10) *with the initial data on $t = T$.*

Proof. One has the following.

Step 1. Discuss T_0. Let $O = (0,0)$, $A(1,0)$, $B = (1,T)$, $C = (0,T)$ (see Figure 1). Let $J = (x_j, t_j)$ be the intersection point of the lines

$$l_1 : x(t) = A_{\min} t, \qquad l_2 : x(t) = 1 + f_{\max} t. \tag{29}$$

Let $G = (x_g, t_g)$ be the intersection point of the lines

$$\bar{l}_1 : x(t) = 1 + A_{\min}(t - T), \qquad \bar{l}_2 : x(t) = f_{\max}(t - T). \tag{30}$$

Let the curves OPM and APF be, respectively, described by the characteristics $x_1(t)$ and $x_2(t)$ (see Figure 1), which satisfy

$$\begin{aligned} \frac{dx_1(t)}{dt} &= f(R(x_1(t)), t), \quad t = 0 : x_1(0) = 1, \\ \frac{dx_2(t)}{dt} &= A(R_0(0)), \quad t = 0 : x_2(0) = 0. \end{aligned} \tag{31}$$

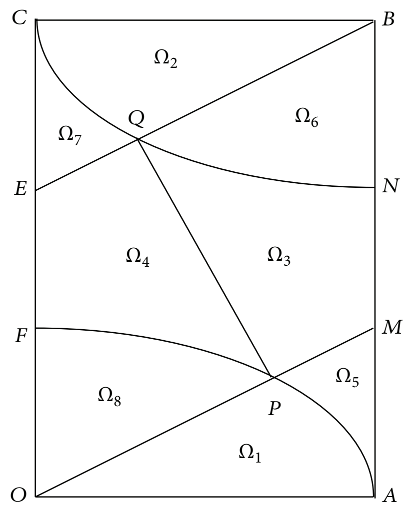

Figure 1: Domains Ω_i $(i = 1, 2, \ldots, 8)$, characteristics: OPM: $x = x_2(t)$, APF: $x = x_1(t)$, CQN: $x = \overline{x}_1(t)$, BQE: $x = \overline{x}_2(t)$, straight line: PQ.

Define

$$\widetilde{R}(t) \equiv R(x_2(t), t), \qquad \widetilde{\theta}(t) \equiv \theta(x_1(t), t). \tag{32}$$

Let $P = (x_p, t_p)$ be the intersection point of the curves OPM and APF. Similarly, let $Q = (x_q, t_q)$ be the intersection point of the curves CQN and BQE. Here, curves CQN and BQE are, respectively, described by the characteristics $\overline{x}_1(t)$ and $\overline{x}_2(t)$ (see Figure 1), which satisfy

$$\begin{aligned} \frac{d\overline{x}_1(t)}{dt} &= f(R(\overline{x}_1(t), t)), \quad t = T : \overline{x}_1(T) = 0, \\ \frac{d\overline{x}_2(t)}{dt} &= A(R_T(1)), \quad t = T : \overline{x}_2(T) = 1. \end{aligned} \tag{33}$$

Define

$$\overline{R}(t) \equiv R(\overline{x}_2(t), t), \qquad \overline{\theta}(t) \equiv \theta(\overline{x}_1(t), t). \tag{34}$$

Here, we have to satisfy two conditions.

(1) The lines l_1 and BC have no intersection point in $x \in [0,1]$.

(2) The lines l_2 and BC have no intersection point in $x \in [0,1]$.

Otherwise, if condition (1) is not satisfied, there exists a characteristic $x = \widehat{x}_2(t)$ which passes through two points $O = (0,0)$ and $C_1 = (\alpha_1, T)$, $0 \le \alpha_1 \le 1$. According to the characteristic property, one has $R(x,t) = R_0(0) = R_T(\alpha_1)$, for $\forall (t,x) \in \widehat{\Omega}$ and $\widehat{\Omega}$ is enclosed by the characteristics $x = x_1(t)$, $x = \widehat{x}_2(t)$, and the x axis.

Note that the initial and terminal conditions are usually to be arbitrarily chosen. If one choose that $R_0(0) \ne R_T(\alpha_1)$, the system will not go from the given initial state to the desired terminal state no matter what control inputs are given. As a result, condition (1) should be satisfied. Also, with the similar analysis, condition (2) should be satisfied.

Let M_1 be the intersection point of the lines l_1 and AB; one has $M_1 = (1, 1/A_{\min})$. From condition (1), one has the fact that $1/A_{\min} < T$. Let N_1 be the intersection point of the lines l_2 and OC; one has $N_1 = (0, -1/f_{\max})$. From condition (2), one has that $-1/f_{\max} < T$. As a result, $T > \max\{1/A_{\min}, -1/f_{\max}\}$. Hence, $T > T_0$ and $T_0 = \max\{1/A_{\min}, -1/f_{\max}\}$.

Remark 13. (i) In [20, 21], the authors require that $T_0 = \max\{1/A_{\min}, -1/f_{\max}, 2/(A_{\min} - f_{\max})\}$. Actually, we only need that $T_0 = \max\{1/A_{\min}, -1/f_{\max}\}$. Here, $J = (x_j, t_j) = (A_{\min}/(A_{\min} - f_{\max}), 1/(A_{\min} - f_{\max}))$, and $G = (x_g, t_g) = (-f_{\max}/(A_{\min} - f_{\max}), T - (1/(A_{\min} - f_{\max})))$.

If $1/A_{\min} < 1/-f_{\max}$, then $A_{\min} > -f_{\max}$, $T_0 = 1/-f_{\max}$ and $T > T_0$. One has the fact that

$$\begin{aligned} t_g - t_j &= T - \frac{2}{A_{\min} - f_{\max}} > \frac{1}{-f_{\max}} - \frac{2}{A_{\min} - f_{\max}} \\ &= -\frac{A_{\min} + f_{\max}}{f_{\max}(A_{\min} - f_{\max})} > 0. \end{aligned} \tag{35}$$

From the above inequality, one has $1/-f_{\max} > 2/(A_{\min} - f_{\max})$. Similarly, if $1/A_{\min} > 1/-f_{\max}$, one can obtain that $1/A_{\min} > 2/(A_{\min} - f_{\max})$.

Hence, $T_0 = \max\{1/A_{\min}, -1/f_{\max}, 2/(A_{\min} - f_{\max})\}$ can be written as $T_0 = \max\{1/A_{\min}, -1/f_{\max}\}$. (ii) $T < T_1$ denotes that the solution of the Cauchy problem does not blow up in the domain Ω_2.

Step 2. Discuss the Cauchy problem in the domains Ω_1 and Ω_2. Let the domain Ω_1 be enclosed by the characteristics $x_1(t)$ and $x_2(t)$ and the x axis (see Figure 1). Let the domain Ω_2 be enclosed by the characteristics $\overline{x}_1(t)$ and $\overline{x}_2(t)$ and the horizontal line $t = T$ (see Figure 1). From Lemma 5 and Remark 6, the Cauchy problem (10) and (12) (or (10) and (14)) has a unique global C^1 solution $(R, \theta) = (R(t, x), \theta(t, x))$ on the domain Ω_1 (or Ω_2). Moreover, one has

(1) on the straight line OPM, $R = R_0(0)$, and on the curve APF, $\theta = \theta_0(1)$;

(2) on the straight line BQE, $R = R_T(1)$, and on the curve CQN, $\theta = \theta_T(0)$.

Step 3. Let Ω_3 be the domain enclosed by the characteristic PM, the characteristic QN, the straight line MN, and the straight line PQ, where PQ is denoted by

$$x(t) = x_3(t) = x_p + k(t - t_p), \quad t_p \le t \le t_q, \tag{36}$$

where $k = (x_q - x_p)/(t_q - t_p)$ is the slope dx/dt of the straight line PQ. Consider the following system on the domain Ω_3:

$$\begin{aligned} R_x + \frac{1}{A(R)} R_t &= 0, \\ \theta_x + \frac{1}{f(R)} \theta_t &= 0, \end{aligned} \tag{37}$$

with initial conditions

$$\begin{aligned} \widetilde{R}_0(t) &= R(x_3(t), t), \\ \widetilde{\theta}_0(t) &= \theta(x_3(t), t). \end{aligned} \tag{38}$$

For points P and Q, it is required that

$$\begin{aligned} \widetilde{R}_0(t_q) &= R(\overline{x}_2(t_q), t_q) = R_T(1), \\ \widetilde{\theta}_0(t_q) &= \theta(\overline{x}_1(t_q), t_q) = \theta_T(0), \\ \widetilde{R}_0(t_p) &= R(x_2(t_p), t_p) = R_0(0), \\ \widetilde{\theta}_0(t_p) &= \theta(x_1(t_p), t_p) = \theta_0(1). \end{aligned} \tag{39}$$

Along the characteristic $QN : x = \overline{x}_1(t)$, one has

$$\begin{aligned} \overline{R}'(t_q) &= \left.\frac{dR(\overline{x}_1(t), t)}{dt}\right|_{t=t_q} = \left.\left(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial x} f(R)\right)\right|_{t=t_q} \\ &= (f(R) - A(R)) \left.\frac{\partial R}{\partial x}\right|_{t=t_q} \\ &= (f(R_T(1)) - A(R_T(1))) \frac{\partial R}{\partial x}. \end{aligned} \tag{40}$$

Along the straight line PQ, one has

$$\begin{aligned} \widetilde{R}_0'(t_q) &= \left.\frac{dR(x_3(t), t)}{dt}\right|_{t=t_q} = \left.\left(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial x} k\right)\right|_{t=t_q} \\ &= (k - A(R)) \left.\frac{\partial R}{\partial x}\right|_{t=t_q} = (k - A(R_T(1))) \frac{\partial R}{\partial x}. \end{aligned} \tag{41}$$

So, it is required that

$$\widetilde{R}_0'(t_q) = \overline{R}'(t_q) \frac{k - A(R_T(1))}{f(R_T(1)) - A(R_T(1))}. \tag{42}$$

Similarly, along the characteristic $QE : x = \overline{x}_2(t)$, one has

$$\begin{aligned} \overline{\theta}'(t_q) &= \left.\frac{d\theta(\overline{x}_2(t), t)}{dt}\right|_{t=t_q} = \left.\left(\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} A(R)\right)\right|_{t=t_q} \\ &= (A(R) - f(R)) \left.\frac{\partial \theta}{\partial x}\right|_{t=t_q} \\ &= (A(R_T(1)) - f(R_T(1))) \frac{\partial \theta}{\partial x}. \end{aligned} \tag{43}$$

Along the straight line PQ, one has

$$\begin{aligned} \widetilde{\theta}_0'(t_q) &= \left.\frac{d\theta(x_3(t), t)}{dt}\right|_{t=t_q} = \left.\left(\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} k\right)\right|_{t=t_q} \\ &= (k - f(R)) \left.\frac{\partial \theta}{\partial x}\right|_{t=t_q} = (k - f(R_T(1))) \frac{\partial \theta}{\partial x}. \end{aligned} \tag{44}$$

It is therefore required that

$$\widetilde{\theta}_0'(t_q) = \widetilde{\theta}'(t_q)\frac{k - f(R_T(1))}{A(R_T(1)) - f(R_T(1))}. \quad (45)$$

In addition, along the characteristic $PF : x = x_1(t)$, one has

$$\begin{aligned}\widetilde{R}'(t_p) &= \left.\frac{dR(x_1(t), t)}{dt}\right|_{t=t_p} = \left.\left(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial x} f(R)\right)\right|_{t=t_p} \\ &= \left.(f(R) - A(R))\frac{\partial R}{\partial x}\right|_{t=t_p} \\ &= (f(R_0(0)) - A(R_0(0)))\frac{\partial R}{\partial x}.\end{aligned} \quad (46)$$

Along the straight line PQ, one has

$$\begin{aligned}\widetilde{R}_0'(t_p) &= \left.\frac{dR(x_3(t), t)}{dt}\right|_{t=t_p} = \left.\left(\frac{\partial R}{\partial t} + \frac{\partial R}{\partial x} k\right)\right|_{t=t_p} \\ &= \left.(k - A(R))\frac{\partial R}{\partial x}\right|_{t=t_p} = (k - A(R_0(0)))\frac{\partial R}{\partial x}.\end{aligned} \quad (47)$$

So, it is required that

$$\widetilde{R}_0'(t_p) = \widetilde{R}'(t_p)\frac{k - A(R_0(0))}{f(R_0(0)) - A(R_0(0))}. \quad (48)$$

Similarly, along the characteristic $PM : x = x_2(t)$, one has

$$\begin{aligned}\widetilde{\theta}'(t_p) &= \left.\frac{d\theta(x_2(t), t)}{dt}\right|_{t=t_p} = \left.\left(\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} A(R)\right)\right|_{t=t_p} \\ &= \left.(A(R) - f(R))\frac{\partial \theta}{\partial x}\right|_{t=t_p} \\ &= (A(R_0(0)) - f(R_0(0)))\frac{\partial \theta}{\partial x}.\end{aligned} \quad (49)$$

Along the straight line PQ, one has

$$\begin{aligned}\widetilde{\theta}_0'(t_p) &= \left.\frac{d\theta(x_3(t), t)}{dt}\right|_{t=t_p} = \left.\left(\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} k\right)\right|_{t=t_p} \\ &= \left.(k - f(R))\frac{\partial \theta}{\partial x}\right|_{t=t_p} = (k - f(R_0(0)))\frac{\partial \theta}{\partial x}.\end{aligned} \quad (50)$$

It is therefore required that

$$\widetilde{\theta}_0'(t_p) = \widetilde{\theta}'(t_p)\frac{k - f(R_0(0))}{A(R_0(0)) - f(R_0(0))}. \quad (51)$$

Moreover, the following compatibility conditions of points O, A, B, C are also required:

$$\begin{aligned}R_0(0) &= g_1(\theta_0(0), 0) + h_1(0), \\ \theta_0(1) &= g_2(R_0(1), 0) + h_2(0), \\ R_T(0) &= g_1(\theta_T(0), T) + h_1(T), \\ \theta_T(1) &= g_2(R_T(1), T) + h_2(T), \\ -A(R_0(0)) R_0'(0) &= \frac{\partial g_1(\theta_0(0), 0)}{\partial t} - f(R_0(0))\frac{\partial g_1(\theta_0(0), 0)}{\partial \theta}\theta_0'(0) + h_1'(0), \\ -A(R_T(0)) R_T'(0) &= \frac{\partial g_1(\theta_T(0), T)}{\partial t} - f(R_T(0))\frac{\partial g_1(\theta_T(0), T)}{\partial \theta}\theta_T'(0) + h_1'(T), \\ -f(R_0(1))\theta_0'(1) &= \frac{\partial g_2(R_0(1), 0)}{\partial t} - A(R_0(1))\frac{\partial g_2(R_0(1), 0)}{\partial R}R_0'(1) + h_2'(0), \\ -f(R_T(1))\theta_T'(1) &= \frac{\partial g_2(R_T(1), T)}{\partial t} - A(R_T(1))\frac{\partial g_2(R_T(1), T)}{\partial R}R_T'(1) + h_2'(T).\end{aligned} \quad (52)$$

According to Proposition 2.1 of [21], one has the fact that

$$f(R) < k < A(R), \quad t_p \le t \le t_q, \quad (53)$$

when $T > T_0 = \max\{-1/f_{\max}, 1/A_{\min}\}$. Then, from Assumption 2 and Remark 6, one has $\widetilde{R}_0'(t_q) > 0$, $\widetilde{R}_0'(t_p) < 0$. With Lemma 8, the Cauchy problem (37) and (38) with prescribed data on $x_3(t)$ must blow up in a finite time. Here, we have interchanged the role of x and t variables. Hence, the time means the x-axis. □

In what follows we will prove that we can choose an appropriate vector $(\widetilde{R}_0(t), \widetilde{\theta}_0(t))$ to satisfy that the first blowup point is out of $(0, 1)$.

Lemma 14. *One can choose $(\widetilde{R}_0(t), \widetilde{\theta}_0(t))$ such that the first blowup point (x, t) satisfying $x > 1$; that is, the Cauchy problem* (37) *and* (38) *has C^1 solution on the domain Ω_3.*

Proof. $\forall(x,t)\in\Omega_3$, two characteristics passing by (x,t) can be defined by

$$\frac{dt_1(x)}{dx}=\frac{1}{A(R(x,t_1(x)))},\quad t_1\left(x_3\left(\widetilde{\beta}\right)\right)=\widetilde{\beta},\tag{54}$$

$$\frac{dt_2(x)}{dx}=\frac{1}{f(R(x,t_2(x)))},\quad t_2\left(x_3\left(\widetilde{\alpha}\right)\right)=\widetilde{\alpha},\tag{55}$$

where $(x_3(\widetilde{\alpha}),\widetilde{\alpha})$ and $(x_3(\widetilde{\beta}),\widetilde{\beta})$ are the intersection points of the two characteristics and the straight line PQ. Along the characteristic $t_1(x)$, one has the fact that $dR/dx=0$. As a result,

$$R(x,t)=\widetilde{R}_0\left(\widetilde{\beta}\right).\tag{56}$$

From (54), one has $t_1=\widetilde{\beta}+(1/A(\widetilde{R}_0(\widetilde{\beta})))(x-x_3(\widetilde{\beta}))$. According to Lemma 10 and Remark 9, one knows that $\theta(x,t)$ never blow up. Hence, we only need to estimate R_t. From (56), one has $R_t=\widetilde{R}_0'(\widetilde{\beta})/(dt_1/d\widetilde{\beta})$ and $dt_1/d\widetilde{\beta}=1-(x-x_3(\widetilde{\beta}))(\partial A(\widetilde{R}_0(\widetilde{\beta}))/\partial\widetilde{\beta})/A^2(\widetilde{R}_0(\widetilde{\beta}))-k/A(\widetilde{R}_0(\widetilde{\beta}))=1-k/A(\widetilde{R}_0(\widetilde{\beta}))-(x-x_3(\widetilde{\beta}))(\partial A(\widetilde{R}_0(\widetilde{\beta}))/\partial\widetilde{R})\widetilde{R}_0'(\widetilde{\beta})/A^2(\widetilde{R}_0(\widetilde{\beta}))$. From (53), one can find $\delta>0$ such that $1-k/A(\widetilde{R}_0(\widetilde{\beta}))\geq\delta>0$. Note that $x>x_3(\widetilde{\beta})$ in Ω_3, $R_t\nrightarrow\infty$ if $\widetilde{R}_0'(\widetilde{\beta})<0$. So, we only need to discuss $\widetilde{R}_0'(\widetilde{\beta})>0$.

When $\widetilde{R}_0'(\widetilde{\beta})>0$, there exists x such that $R_t\to\infty$. As a result, if we choose $\widetilde{R}_0'(\widetilde{\beta})$ satisfy that

$$0<\widetilde{R}_0'\left(\widetilde{\beta}\right)<\min_{t_p\leq\widetilde{\beta}\leq t_q}\left\{\frac{A^2\left(\widetilde{R}_0\left(\widetilde{\beta}\right)\right)-kA\left(\widetilde{R}_0\left(\widetilde{\beta}\right)\right)}{\partial A\left(\widetilde{R}_0\left(\widetilde{\beta}\right)\right)/\partial\widetilde{R}}\right\},\tag{57}$$

then, the blowup point satisfies that $x>x_3(\widetilde{\beta})$ and $x>1$. □

Remark 15. For convenience, conditions (17) and (18) are chosen to guarantee that there have been no blowup points in Ω_1 and Ω_2. Actually, without conditions (17) and (18), we can also get the C^1 solution in the domains Ω_1 and Ω_2 if we use the above method to obtain a similar condition of (57).

Similarly, we have the following lemma for the domain Ω_4.

Lemma 16. *One can choose $(\widetilde{R}_0(t),\widetilde{\theta}_0(t))$ such that the first blowup point (x,t) satisfying $x<0$, that is, the Cauchy problem* (37) *and* (38)*, has C^1 solution on the domain Ω_4.*

Proof. Using the similar proof with Lemma 14, if we choose $\widetilde{R}_0'(\widetilde{\beta_1})$ satisfy that

$$-\max_{t_p\leq\widetilde{\beta_1}\leq t_q}\left\{\frac{A^2\left(\widetilde{R}_0\left(\widetilde{\beta_1}\right)\right)-kA\left(\widetilde{R}_0\left(\widetilde{\beta_1}\right)\right)}{\partial A\left(\widetilde{R}_0\left(\widetilde{\beta_1}\right)\right)/\partial\widetilde{R}}\right\}<\widetilde{R}_0'\left(\widetilde{\beta_1}\right),\tag{58}$$

where $(x_3(\widetilde{\beta_1}),\widetilde{\beta_1})$ is the intersection point of the straight line PQ and the following characteristic

$$\frac{dt_1^1(x)}{dx}=\frac{1}{A(R(x,t_1^1(x)))},\quad t_1^1\left(x_3\left(\widetilde{\beta_1}\right)\right)=\widetilde{\beta_1},\tag{59}$$

then, the blowup point satisfies that $x<x_3(\widetilde{\beta_1})$ and $x<0$.

Until now, we have got C^1 solutions in the domains Ω_1, Ω_2, Ω_3, and Ω_4 and the solutions can be defined

$$(R(x,t),\theta(x,t))=\begin{cases}(R_1(x,t),\theta_1(x,t)), & \text{for } (x,t)\in\Omega_1,\\(R_2(x,t),\theta_2(x,t)), & \text{for } (x,t)\in\Omega_2,\\(R_3(x,t),\theta_3(x,t)), & \text{for } (x,t)\in\Omega_3,\\(R_4(x,t),\theta_4(x,t)), & \text{for } (x,t)\in\Omega_4.\end{cases}\tag{60}$$

Step 4. Consider the Goursat problem in Ω_5, Ω_6, Ω_7, and Ω_8. In Ω_8, we prescribe data as follows:

$$\begin{aligned}\widetilde{R}(t)&=R_4(x_1(t),t),\quad\text{on } x_1(t),\\\widetilde{\theta}(t)&=\theta_1(x_2(t),t),\quad\text{on } x_2(t).\end{aligned}\tag{61}$$

For $\forall(x,t)\in\Omega_8$, a characteristic passing by (x,t) can be defined by

$$\frac{d\widetilde{t}_1(x)}{dx}=\frac{1}{A(R(x,\widetilde{t}_1(x)))},\quad\widetilde{t}_1(x_1(\zeta))=\zeta,\tag{62}$$

where $(x_1(\zeta),\zeta)$ is the intersection point of this characteristic and the characteristic $x_1(t)$. Along the characteristic $\widetilde{t}_1(x)$, one has $dR/dx=0$; that is, $R=\widetilde{R}(\zeta)$. Clearly, the C^0 norm of R is bounded. In order to estimate R_t, we need to discuss $d\widetilde{t}_1(x)/d\zeta$. From (62), one has $\widetilde{t}_1(x)=\zeta+(1/A(R))(x-x_1(\zeta))$. Then, $d\widetilde{t}_1(x)/d\zeta=1-(f(R)/A(R))-(x-x_1(\zeta))(\partial A/\partial R)(dR/d\zeta)/A^2(R)$. According to Assumption 2, there at least exists a point $(x(\zeta_1),\zeta_1)$ such that $R_x(\zeta_1)>0$. Then, $dR/d\zeta|_{\zeta=\zeta_1}=R_t+R_xf(R)|_{\zeta=\zeta_1}=(f(R)-A(R))R_x(\zeta_1)<0$. Note that $x-x_1(\zeta)<0$, there exists x such that $R_t\to\infty$. As a result, we need to choose $\widetilde{R}(\zeta)$ satisfy that

$$-\max_{0<\zeta<t_f}\left\{\frac{A^2\left(\widetilde{R}(\zeta)\right)-f\left(\widetilde{R}(\zeta)\right)A\left(\widetilde{R}(\zeta)\right)}{\partial A\left(\widetilde{R}(\zeta)\right)/\partial R}\right\}<\widetilde{R}'(\zeta)<0,\tag{63}$$

where $F=(x_f,t_f)$. With (63), the Goursat problem has a unique global C^1 solution in Ω_8.

Similarly, in Ω_6, we prescribe data as follows:

$$\begin{aligned}\overline{R}(t)&=R_3(\overline{x}_1(t),t),\quad\text{on } \overline{x}_1(t),\\\overline{\theta}(t)&=\theta_2(\overline{x}_2(t),t),\quad\text{on } \overline{x}_2(t).\end{aligned}\tag{64}$$

For $\forall(x,t)\in\Omega_6$, a characteristic passing by (x,t) can be defined by

$$\frac{d\widetilde{t}_2(x)}{dx}=\frac{1}{A(R(x,\widetilde{t}_2(x)))},\quad\widetilde{t}_2(\overline{x}_1(v))=v,\tag{65}$$

where $(\overline{x}_1(v),v)$ is the intersection point of this characteristic and the characteristic $\overline{x}_1(t)$. Along the characteristic $\widetilde{t}_2(x)$, one has $dR/dx=0$, that is, $R=\overline{R}(v)$. Clearly, the C^0 norm of R is bounded. In order to estimate R_t, we need to discuss $d\widetilde{t}_2(x)/dv$. From (65), one has

$\tilde{t}_2(x) = \upsilon + (1/A(R))(x - \overline{x}_1(\upsilon))$. Then, $d\tilde{t}_2(x)/d\upsilon = 1 - (f(R)/A(R)) - (x - \overline{x}_1(\upsilon))(\partial A/\partial R)(dR/d\upsilon)/A^2(R)$. According to Remark 6, there at least exists a point $(x(\upsilon_1), \upsilon_1)$ such that $R_x(\upsilon_1) < 0$. Then, $dR/d\upsilon|_{\upsilon=\upsilon_1} = R_t + R_x f(R)|_{\upsilon=\upsilon_1} = (f(R) - A(T))R_x(\upsilon_1) > 0$. Note that $x - \overline{x}_1(\upsilon) > 0$, there exists x such that $R_t \to \infty$. As a result, we need to choose $\overline{R}(\upsilon)$ satisfy that

$$0 < \overline{R}'(\upsilon) < \min_{t_n < \upsilon < T} \left\{ \frac{A^2(\overline{R}(\upsilon)) - f(\overline{R}(\upsilon)) A(\overline{R}(\upsilon))}{\partial A(\overline{R}(\upsilon))/\partial R} \right\}, \tag{66}$$

where $N = (x_n, t_n)$. With (66), the Goursat problem has a unique global C^1 solution in Ω_6.

In addition, with the same analysis, the Goursat problem always has a unique global C^1 solution in Ω_5, Ω_7. Therefore, under the conditions (63) and (66), we have got C^1 solutions in the domains Ω_5, Ω_6, Ω_7, Ω_8 and the solutions can be defined

$$(R(x,t), \theta(x,t)) = \begin{cases} (R_5(x,t), \theta_5(x,t)), & \text{for } (x,t) \in \Omega_5, \\ (R_6(x,t), \theta_6(x,t)), & \text{for } (x,t) \in \Omega_6, \\ (R_7(x,t), \theta_7(x,t)), & \text{for } (x,t) \in \Omega_7, \\ (R_8(x,t), \theta_8(x,t)), & \text{for } (x,t) \in \Omega_8. \end{cases} \tag{67}$$

We have constructed solutions in the domain D_T. Let $h_1(t) = R(0,t) - g_1(\theta(0,t), t)$, and $h_2(t) = \theta(1,t) - g_2(R(1,t), t)$; then there exist $h_1, h_2 \in C^1([0,T])$ such that systems (10) (12) have a C^1 solution $(R(x,t), \theta(x,t))$ on the domain D_T satisfying $R(x,0) = R_0(x)$, $\theta(x,0) = \theta_0(x)$, $R(x,T) = R_T(x)$, $\theta(x,T) = \theta_T(x)$, for $0 \le x \le 1$. The proof is completed. □

Remark 17. In [10], an optimal control model of distributed parameter systems (DPSs) is presented to discuss the polymer injection strategies. Compared with [10], the differences of our paper are (1) the considered model is a hyperbolic system and the maximum principle does not hold here; (2) the desired outputs can be achieved by controlling the boundary inputs.

4. An Example

In this section, an example is presented to demonstrate the effectiveness of our results.

Example 18. For system (10), we define that

$$\begin{aligned} R(x,0) = R_0(x) = C + \varepsilon\phi_1(x), \quad \phi_1'(x) > 0, \\ R(x,T) = R_T(x) = C + \varepsilon\phi_2(x), \quad \phi_2'(x) < 0, \end{aligned} \tag{68}$$

where $C > 0$ is a constant, which denotes the equilibrium of the initial and terminal states. $m_1 \le \phi_1(x) \le M_1$, $m_2 \le \phi_2(x) \le M_2$, and m_1, M_1, m_2, M_2 are all constants. $\varepsilon > 0$ is also a constant, which will be chosen in the following.

As a result, one has $C_1 \le R(x,t) \le C_2$ with $C_1 = C + \varepsilon \min\{m_1, m_2\}$ and $C_2 = C + \varepsilon \max\{M_1, M_2\}$. Let $f(R) = R - C_2 - \eta$, $A(R) = f'(R)R + f(R) = 2R - C_2 - \eta$, and η depends on ε and is also decided later. In the following, we will choose appropriate ε to satisfy conditions (57), (58), (63), and (66).

According to Theorem 12, one has $\tilde{R}_0'(t_p) = (k - A(R_0(0)))(\partial R/\partial x) = (k - A(R_0(0)))(\partial R_0/\partial t)/(\partial x/\partial t) = (k - A(R_0(0)))(\varepsilon\phi_1'(x)/A(R_0(0)))$. Note that $\tilde{R}_0'(t_p) < 0$, an appropriate ε can be chosen to satisfy condition (58). Applying the same method, it is also easy to choose an appropriate ε to satisfy condition (57).

Let $(x_1(\xi_1), \xi_1)$ be the intersection point of the characteristic (65) and the straight line PQ. Because the straight line PQ is constructed, we can choose that $|\tilde{R}_0'(\xi_1)| < \min\{|\tilde{R}_0'(t_p)|, |\tilde{R}_0'(t_q)|\}$. As a result, (66) will be satisfied if condition (57) is satisfied.

Similarly, let $(x_1(\nu_1), \nu_1)$ be the intersection point of the characteristic (62) and the straight line PQ. Because the straight line PQ is constructed, we can choose that $|\tilde{R}_0'(\nu_1)| < \min\{|\tilde{R}_0'(t_p)|, |\tilde{R}_0'(t_q)|\}$. As a result, (63) will be satisfied if condition (58) is satisfied. So, there exists a positive constant ε to satisfy conditions (57), (58), (63), and (66). In addition, for the chosen ε, it is easy to find out a constant η $(0 < \eta < C - 3\varepsilon)$ to satisfy $f(R) = R - C_2 - \eta < 0$ and $A(R) = f'(R)R + f(R) = 2R - C_2 - \eta > 0$. Therefore, all the conditions in Theorem 12 can be satisfied by using the constructed method. That is, the conclusion of Theorem 12 is valid.

Remark 19. Compared with [20, 21], the difference in this paper is the fact that one cannot avoid the phenomenon of blowup. Hence, one important goal for EORs is to find the controllers $h_1(t)$ and $h_2(t)$ to make sure that the blowup points of EORs are beyond the domain we fixed. As a result, there exist more complications in the controlling process as described in Section 3. The example we give here is to illustrate that the controllers are achievable and it can be applied in real problems.

5. Conclusions

In this paper, we have discussed the exact boundary controllability of a class of enhanced oil recovery models. By using a constructed method, it has been shown that the enhanced oil recovery systems with nonlinear boundary conditions is exactly boundary controllable. Moreover, an interval of the control time has also been presented to be optimal. Finally, an example has been provided to illustrate the effectiveness of the obtained criterion.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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