

Research Article

On the Starlikeness of Certain Class of Multivalent Analytic Functions

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The main purpose of this paper is to determine the conditions of starlikeness for certain class of multivalent analytic functions. Relevant connections of the results presented here with those obtained in earlier works are pointed out.

1. Introduction and Main Result

Let \mathcal{A}_p denote the class of functions of the form

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \quad (p \in \mathbb{N} := \{1, 2, \dots\}), \quad (1)$$

which are analytic in the open unit disk $\mathbb{U} := \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. For convenience, we set $\mathcal{A}_1 =: \mathcal{A}$. A function $f \in \mathcal{A}_p$ is said to be in the class $\mathcal{S}_p^*(\varrho)$ of p -valent starlike functions of order ϱ in \mathbb{U} , if it satisfies the following inequality:

$$\Re \left(\frac{z f'(z)}{f(z)} \right) > \varrho \quad (0 \leq \varrho < p; z \in \mathbb{U}). \quad (2)$$

For simplicity, we write $\mathcal{S}_1^*(0) =: \mathcal{S}^*$.

In [1], Chichra introduced the class \mathcal{R} of analytic functions $f \in \mathcal{A}$ which satisfy the condition

$$\Re (f'(z) + z f''(z)) > 0 \quad (z \in \mathbb{U}). \quad (3)$$

He proved that the members of \mathcal{R} are univalent in \mathbb{U} . Later, R. Singh and S. Singh [2] showed that $\mathcal{R} \subset \mathcal{S}^*$. Recently, Gao and Zhou [3] considered the subclass $\mathcal{R}(\beta, \gamma)$ of \mathcal{A} which is defined by

$$\mathcal{R}(\beta, \gamma) := \left\{ f \in \mathcal{A} : \Re (f'(z) + \beta z f''(z)) > \gamma \right\} \quad (4)$$

$$(\beta > 0; \gamma < 1; z \in \mathbb{U}).$$

They derived some mapping properties of this class. Moreover, several authors discussed some related analytic function classes associated with the class \mathcal{R} (see [4–7]). By using the method of differential subordination, Yang and Liu [8] generalized the above works and studied the subclass $\mathcal{T}_p(A, B, \gamma, \alpha)$ of \mathcal{A}_p which satisfies the condition

$$f'(z) + \alpha z f''(z) < h(z) \quad (z \in \mathbb{U}), \quad (5)$$

where

$$h(z) = \begin{cases} \left(\frac{1 + Az}{1 + Bz} \right)^\gamma, & (A \leq 1; 0 < \gamma < 1), \\ \frac{1 + Az}{1 + Bz}, & (\gamma = 1). \end{cases} \quad (6)$$

In [9], Owa et al. introduced a new subclass $\mathcal{R}_p(\alpha, \beta, \gamma; j)$ of \mathcal{A}_p which satisfies the inequality

$$\Re \left(\alpha \frac{f^{(j)}(z)}{z^{p-j}} + \beta \frac{f^{(j+1)}(z)}{z^{p-j-1}} \right) > \gamma \quad (7)$$

$$(j \in \{0, 1, 2, \dots, p\}; p \in \mathbb{N}; z \in \mathbb{U}),$$

where $\alpha > 0, \beta > 0$, and (throughout this paper unless otherwise mentioned) the parameters γ and δ are constrained as follows:

$$\gamma < \delta := \frac{p! [\alpha + (p - j) \beta]}{(p - j)!} \tag{8}$$

$$(\alpha > 0; \beta > 0; p \in \mathbb{N}; j \in \{0, 1, 2, \dots, p\}).$$

The extreme points, coefficient inequalities, radius of starlikeness, and inclusion relationship for the class $\mathcal{R}_p(\alpha, \beta, \gamma; j)$ are derived. By setting $p = j = \alpha = 1$, it is easy to see that the class $\mathcal{R}_p(\alpha, \beta, \gamma; j)$ reduces to the class $\mathcal{R}(\beta, \gamma)$. If we set $p = j = \alpha = \gamma = 1$ in the class $\mathcal{R}_p(\alpha, \beta, \gamma; j)$, then it reduces to the class $\mathcal{R}(\gamma)$, which was studied earlier by Silverman [10], R. Singh and S. Singh [2, 11], independently.

For some recent investigations on the starlikeness of analytic functions, one can refer to [12–20]. In the present paper, we aim at deriving the conditions of starlikeness for the class $\mathcal{R}_p(\alpha, \beta, \gamma; j)$. The main result is presented below.

Theorem 1. *Let $\beta \geq \alpha > 0$. Then*

- (1) $\mathcal{R}_p(\alpha, \beta, \gamma; 1) \subset \mathcal{S}_p^*$ for $\gamma_1 \leq \gamma < p[\alpha + (p - 1)\beta]$, where γ_1 is the solution of the following equation:

$$\begin{aligned} & -\frac{p}{2} \left(1 + 2 \{p[\alpha + (p - 1)\beta] - \gamma_1\} \right. \\ & \quad \left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n + p - 1)[\alpha + (n + p - 2)\beta]} \right) \\ & = \frac{\gamma_1}{\beta} + \frac{\beta - \alpha}{\beta} \end{aligned} \tag{9}$$

$$\times \left(p + 2 \{p[\alpha + (p - 1)\beta] - \gamma_1\} \right.$$

$$\left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n + p - 2)\beta} \right);$$

- (2) $\mathcal{R}_p(\alpha, \beta, \gamma; j) \subset \mathcal{S}_p^*(j - 1)$ ($j \in \{2, 3, \dots, p\}$) for $\gamma_2 \leq \gamma < p[\alpha + (p - 1)\beta]$, where γ_2 is the solution of the following equation:

$$\begin{aligned} & -\frac{p - j + 1}{2} \left(\frac{p!}{(p - j + 1)!} + 2(\delta - \gamma_2) \right. \\ & \quad \left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n + p - j)\beta} \right) \\ & = \frac{\gamma_2}{\beta} + \frac{\beta - \alpha}{\beta} \end{aligned} \tag{10}$$

$$\times \left(\frac{p!}{(p - j)!} + 2(\delta - \gamma_2) \right.$$

$$\left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n + p - j - 1)\beta} \right).$$

2. Preliminary Results

In order to establish our main theorem, we will require the following lemmas.

Lemma 2 (see [9]). *A function $f \in \mathcal{R}_p(\alpha, \beta, \gamma; j)$ if and only if f can be expressed as follows:*

$$\begin{aligned} f(z) & = z^p + 2(\delta - \gamma) \\ & \times \int_{|x|=1} \left(\sum_{n=p+1}^{\infty} \frac{(n - j)!}{n! [\alpha + (n - j)\beta]} x^{n-p} z^n \right) d\mu(x), \end{aligned} \tag{11}$$

where $\mu(x)$ is the probability measure on $\mathbb{X} := \{x \in \mathbb{C} : |x| = 1\}$.

The proof of the following lemma is much akin to that of Theorem 1 which was obtained by Nunokawa et al. [21] (see also Liu [22] and Yang [23]). We, therefore, choose to omit the analogous details involved.

Lemma 3. *If $f \in \mathcal{A}_p$ satisfies the inequality*

$$\Re \left(\frac{zf^{(j)}(z)}{f^{(j-1)}(z)} \right) > 0 \quad (j \in \{1, 2, \dots, p\}; z \in \mathbb{U}), \tag{12}$$

then $f \in \mathcal{S}_p^*(j - 1)$.

Lemma 4 (Jack's Lemma [24]). *Let ϕ be a nonconstant regular function in \mathbb{U} . If $|\phi|$ attains its maximum value on the circle $|z| = r < 1$ at z_0 , then*

$$z_0 \phi'(z_0) = k\phi(z_0), \tag{13}$$

where $k \geq 1$ is a real number.

We now give the lower bounds of the following continuous linear operators:

$$\begin{aligned} \mathcal{L}_1(f) & = \Re \left(\frac{f(z)}{z^p} \right) \quad (f \in \mathcal{R}_p(\alpha, \beta, \gamma; 1); z \in \mathbb{U}), \\ \mathcal{L}_2(f) & = \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} \right) \quad (f \in \mathcal{R}_p(\alpha, \beta, \gamma; j); \\ & \quad j \in \{1, 2, \dots, p\}; z \in \mathbb{U}) \end{aligned} \tag{14}$$

acting on the class $\mathcal{R}_p(\alpha, \beta, \gamma; j)$, which played crucial role in the proof of our main result.

Lemma 5. *If $f \in \mathcal{R}_p(\alpha, \beta, \gamma; j)$, then, for $|z| \leq r < 1$, one has the following.*

(1) When $j \in \{1, 2, \dots, p\}$, then

$$\begin{aligned} \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} \right) &\geq \frac{p!}{(p-j)!} \\ &\quad + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-r)^{n-1}}{\alpha + (n+p-j-1)\beta} \\ &> \frac{p!}{(p-j)!} \\ &\quad + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n+p-j-1)\beta}. \end{aligned} \tag{15}$$

This inequality is sharp.

(2) When $j = 1$, then

$$\begin{aligned} \Re \left(\frac{f(z)}{z^p} \right) &\geq 1 + 2(\delta_1 - \gamma) \sum_{n=2}^{\infty} \frac{(-r)^{n-1}}{(n+p-1)[\alpha + (n+p-2)\beta]} \\ &> 1 + 2(\delta_1 - \gamma) \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n+p-1)[\alpha + (n+p-2)\beta]}, \end{aligned} \tag{16}$$

where $\delta_1 = p[\alpha + (p-1)\beta]$. The inequality is sharp.

Proof. By Lemma 2, we know that

$$f(z) = z^p + 2(\delta - \gamma) \sum_{n=p+1}^{\infty} \frac{(n-j)!}{n! [\alpha + (n-j)\beta]} z^n \tag{17}$$

is the extreme function of the class $\mathcal{R}_p(\alpha, \beta, \gamma; j)$. Thus, we only need to consider the function f defined by (17); it follows that

$$\frac{f^{(j)}(z)}{z^{p-j}} = \frac{p!}{(p-j)!} + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-r)^{n-1}}{\alpha + (n+p-j-1)\beta} z^{n-1}. \tag{18}$$

We note that (18) can be written as follows:

$$\frac{f^{(j)}(z)}{z^{p-j}} = \frac{p!}{(p-j)!} + \frac{2(\delta - \gamma)}{\beta} \int_0^1 t^{(\alpha/\beta)+p-j} \frac{z}{1-tz} dt. \tag{19}$$

Thus, we find from (19) that

$$\begin{aligned} \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} \right) &= \frac{p!}{(p-j)!} + \frac{2(\delta - \gamma)}{\beta} \\ &\quad \times \int_0^1 t^{(\alpha/\beta)+p-j} \Re \left(\frac{z}{1-tz} \right) dt. \end{aligned} \tag{20}$$

Moreover, we observe that the function

$$k(z) = \frac{z}{1-tz} \quad (0 \leq t \leq 1) \tag{21}$$

is convex in \mathbb{U} , $k(\bar{z}) = \overline{k(z)}$, and $k(z)$ maps real axis to real axis; we have

$$-\frac{r}{1+tr} \leq \Re \left(\frac{z}{1-tz} \right) \leq \frac{r}{1-tr} \quad (|z| \leq r < 1). \tag{22}$$

Upon substituting (22) into (20) and expanding the integrand into the power series of t and integrating it, we can easily get (15). The sharpness of (15) can be seen from (18).

By similarly applying the method of proof of (15), we also can prove (16) holds true. The sharpness of (16) can be found in (17). \square

3. Proof of Theorem 1

Proof. Suppose that $f \in \mathcal{R}_p(\alpha, \beta, \gamma; j)$ with $\beta \geq \alpha > 0$. It follows from (7) that

$$\Re \left(\frac{f^{(j)}(z)}{z^{p-j}} + \frac{\beta}{\alpha} \frac{f^{(j+1)}(z)}{z^{p-j-1}} \right) > \frac{\gamma}{\alpha} \quad (z \in \mathbb{U}). \tag{23}$$

By noting that

$$\begin{aligned} \frac{\beta}{\alpha} \left(\frac{f^{(j)}(z)}{z^{p-j}} + \frac{f^{(j+1)}(z)}{z^{p-j-1}} \right) &= \frac{f^{(j)}(z)}{z^{p-j}} + \frac{\beta}{\alpha} \frac{f^{(j+1)}(z)}{z^{p-j-1}} + \left(\frac{\beta}{\alpha} - 1 \right) \frac{f^{(j)}(z)}{z^{p-j}}, \end{aligned} \tag{24}$$

we get

$$\begin{aligned} \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} + \frac{f^{(j+1)}(z)}{z^{p-j-1}} \right) &= \frac{\alpha}{\beta} \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} + \frac{\beta}{\alpha} \frac{f^{(j+1)}(z)}{z^{p-j-1}} \right) \\ &\quad + \frac{\beta - \alpha}{\beta} \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} \right). \end{aligned} \tag{25}$$

Thus, we can easily find from (15), (23), and (25) that

$$\begin{aligned} \Re \left(\frac{f^{(j)}(z)}{z^{p-j}} + \frac{f^{(j+1)}(z)}{z^{p-j-1}} \right) &> \frac{\gamma}{\beta} + \frac{\beta - \alpha}{\beta} \\ &\quad \times \left(\frac{p!}{(p-j)!} + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n+p-j-1)\beta} \right). \end{aligned} \tag{26}$$

We now set

$$\begin{aligned} \frac{f^{(j)}(z)}{f^{(j-1)}(z)} &= (p-j+1) \frac{1+\omega(z)}{1-\omega(z)} \\ &\quad (j \in \{1, 2, \dots, p\}; z \in \mathbb{U}). \end{aligned} \tag{27}$$

Then ω is analytic in \mathbb{U} with $\omega(0) = 0$. It follows from (27) that

$$\begin{aligned} & \frac{f^{(j)}(z)}{z^{p-j}} + \frac{f^{(j+1)}(z)}{z^{p-j-1}} \\ &= (p-j+1) \frac{f^{(j-1)}(z)}{z^{p-j+1}} \left[\left(\frac{1+\omega(z)}{1-\omega(z)} \right)^2 + \frac{2z\omega'(z)}{(1-\omega(z))^2} \right]. \end{aligned} \tag{28}$$

At the same time, we can claim that $|\omega(z)| < 1$. Indeed, if not, there exists a point $z_0 \in \mathbb{U}$ such that

$$\max_{|z| \leq |z_0|} |\omega(z)| = |\omega(z_0)| = 1; \tag{29}$$

by Lemma 4, we obtain

$$z_0 \omega'(z_0) = k \omega(z_0) = k e^{i\theta} \quad (0 < \theta < 2\pi; k \geq 1). \tag{30}$$

For $z = z_0$, by virtue of (28), we split it into two cases to prove the following.

(1) When $j = 1$, in view of (16), we get

$$\begin{aligned} & \Re \left(\frac{f'(z_0)}{z_0^{p-1}} + \frac{f''(z_0)}{z_0^{p-2}} \right) \\ &= p \Re \left(\frac{f(z_0)}{z_0^p} \left[\left(\frac{1+e^{i\theta}}{1-e^{i\theta}} \right)^2 + \frac{2ke^{i\theta}}{(1-e^{i\theta})^2} \right] \right) \\ &\leq -\frac{pk}{2\sin^2(\theta/2)} \Re \left(\frac{f(z_0)}{z_0^p} \right) \leq -\frac{p}{2} \Re \left(\frac{f(z_0)}{z_0^p} \right) \\ &\leq -\frac{p}{2} \left(1 + 2 \{ p[\alpha + (p-1)\beta] - \gamma \} \right. \\ &\quad \left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n+p-1)[\alpha + (n+p-2)\beta]} \right). \end{aligned} \tag{31}$$

Let $\beta \geq \alpha > 0$. If γ satisfies the condition

$$\begin{aligned} & -\frac{p}{2} \left(1 + 2 \{ p[\alpha + (p-1)\beta] - \gamma \} \right. \\ &\quad \left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n+p-1)[\alpha + (n+p-2)\beta]} \right) \\ &\leq \frac{\gamma}{\beta} + \frac{\beta - \alpha}{\beta} \\ &\quad \times \left(p + 2 \{ p[\alpha + (p-1)\beta] - \gamma \} \right. \\ &\quad \left. \times \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n+p-j-1)\beta} \right), \end{aligned} \tag{32}$$

we have a contradiction to (26) at $z = z_0$; the smallest γ satisfies (32) is solution γ_1 of (9). This implies that if $\beta \geq \alpha > 0$ and $\gamma_1 \leq \gamma < p[\alpha + (p-1)\beta]$, we have $|\omega(z)| < 1$. Thus, we conclude that $f \in \mathcal{S}_p^*$.

(2) When $j \in \{2, 3, \dots, p\}$, by means of (15), we have

$$\begin{aligned} & \Re \left(\frac{f^{(j)}(z_0)}{z_0^{p-j}} + \frac{f^{(j+1)}(z_0)}{z_0^{p-j-1}} \right) \\ &= (p-j+1) \Re \left(\frac{f^{(j-1)}(z_0)}{z_0^{p-j+1}} \left[\left(\frac{1+e^{i\theta}}{1-e^{i\theta}} \right)^2 + \frac{2ke^{i\theta}}{(1-e^{i\theta})^2} \right] \right) \\ &\leq -\frac{(p-j+1)k}{2\sin^2(\theta/2)} \Re \left(\frac{f^{(j-1)}(z_0)}{z_0^{p-j+1}} \right) \\ &\leq -\frac{p-j+1}{2} \Re \left(\frac{f^{(j-1)}(z_0)}{z_0^{p-j+1}} \right) \\ &\leq -\frac{p-j+1}{2} \\ &\quad \times \left(\frac{p!}{(p-j+1)!} + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n+p-j)\beta} \right). \end{aligned} \tag{33}$$

Let $\beta \geq \alpha > 0$. If γ satisfies the following inequality,

$$\begin{aligned} & -\frac{p-j+1}{2} \\ &\quad \times \left(\frac{p!}{(p-j+1)!} + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n+p-j)\beta} \right) \\ &\leq \frac{\gamma}{\beta} + \frac{\beta - \alpha}{\beta} \\ &\quad \times \left(\frac{p!}{(p-j)!} + 2(\delta - \gamma) \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\alpha + (n+p-j-1)\beta} \right), \end{aligned} \tag{34}$$

we have a contradiction to (26) at $z = z_0$; the smallest γ satisfies (34) is solution γ_2 of (10). This shows that if $\beta \geq \alpha > 0$ and $\gamma_2 \leq \gamma < \delta$, we have $|\omega(z)| < 1$. It follows from (27) that

$$\Re \left(\frac{zf^{(j)}(z)}{f^{(j-1)}(z)} \right) > 0 \quad (j \in \{2, 3, \dots, p\}; z \in \mathbb{U}). \tag{35}$$

Therefore, by Lemma 3, we deduce that

$$\mathcal{R}_p(\alpha, \beta, \gamma; j) \in \mathcal{S}_p^*(j-1) \quad (j \in \{2, 3, \dots, p\}). \tag{36}$$

The proof of the theorem is thus completed. \square

Remark 6. If $\beta < \alpha$ ($\alpha > 0; \beta > 0$), we cannot find the number $\gamma(\alpha, \beta)$ such that

$$\mathcal{R}_p(\alpha, \beta, \gamma; j) \in \mathcal{S}_p^*(j-1) \quad (j \in \{2, 3, \dots, p\}), \tag{37}$$

since the continuous linear operators $\mathcal{L}_1(f)$ and $\mathcal{L}_2(f)$ acting on $\mathcal{R}_p(\alpha, \beta, \gamma; j)$ do not exist in sharp upper bounds.

Putting $p = \alpha = 1$ in the first part of Theorem 1, we can get the following result.

Corollary 7. Let $\beta \geq 1$. Then $\mathcal{R}(\beta, \gamma) \subset \mathcal{S}^*$ for $\gamma_3 \leq \gamma < 1$, where γ_3 is the solution of the following equation:

$$1 - \frac{3}{2}\beta = \gamma_3 + (1 - \gamma_3) \sum_{n=2}^{\infty} \frac{(-1)^{n-1} [\beta + 2n(\beta - 1)]}{n[1 + (n-1)\beta]}. \quad (38)$$

Remark 8. Corollary 7 corrects some errors of Theorem 4 in [3].

By setting $p = \alpha = \beta = 1$ in the first part of Theorem 1, we also can get the following criterion for starlikeness obtained by Silverman [10].

Corollary 9. Consider $\mathcal{R}(\gamma) \subset \mathcal{S}^*$ for $(6 - \pi^2)/(24 - \pi^2) \leq \gamma < 1$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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