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Research Article

Dynamics of Almost Periodic BAM Neural Networks with Neutral Delays

Yaqin Li

Department of Mathematics, Kunming University, Kunming 650031, China

Correspondence should be addressed to Yaqin Li; yqlikm@163.com

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The paper investigates the almost periodic oscillatory properties of neutral-type BAM neural networks with time-varying delays. By employing the contracting mapping principle and constructing suitable Lyapunov functional, several sufficient conditions are established for the existence, uniqueness, and global exponential stability of almost periodic solution of the system. The results of this paper are new and a simple example is given to illustrate the effectiveness of the new results.

1. Introduction

Recent years have witnessed rapid development of bidirectional associative memory (BAM) neural networks due to the vast applications in pattern recognition, artificial intelligence, automatic control engineering, and optimization because of its better abilities of information memory and information association [1–6]. It is well known that studies on neural dynamical systems not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory behavior, almost periodic oscillatory properties, chaos, and bifurcation. In applications, almost periodic oscillatory is more accordant with fact [7–9]. A great number of results for BAM neural networks concerning the existence and global stability of (almost) periodic solution have been derived (see, e.g., [10–16]).

In addition, owing to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions. This new type of neural networks is called neutral neural networks or neural networks of neutral type. The motivation for us to study neural networks of neutral type comes from three aspects. First, based on biochemistry experiments, neural information may transfer across chemical reactivity, which results in a neutral-type process. Second, in view of electronics, it has been shown that neutral phenomena exist in large-scale integrated (LSI) circuits. Last, the key point is that cerebra can be considered as a super LSI circuit with chemical reactivity, which reasonably implies that the neutral dynamic behaviors should be included in neural dynamic systems [17, 18]. However, up to date, there are hardly any articles concerning the almost periodic oscillation analysis for neural networks of neutral type.

Motivated by the above reason, in this paper, we consider the following neutral-type BAM neural networks with time-varying delays:

$$\begin{aligned}\dot{x}_i(t) &= -a_i(t)x_i(t) + \sum_{j=1}^{m} w_{1ji}(t) f_j\left(y_j\left(t-\alpha_j(t)\right)\right) \\ &\quad + \sum_{j=1}^{n} w_{2ij}(t) h_j\left(\dot{x}_j\left(t-\mu_j(t)\right)\right) + I_i(t), \\ \dot{y}_j(t) &= -b_j(t)y_j(t) + \sum_{i=1}^{n} v_{1ij}(t) g_i\left(x_i\left(t-\beta_i(t)\right)\right) \\ &\quad + \sum_{i=1}^{m} v_{2ji}(t) k_i\left(\dot{y}_i\left(t-\nu_i(t)\right)\right) + J_i(t),\end{aligned} \tag{1}$$

where a_i and b_j denote the rate with which the cells i and j reset their potential to the resting states when isolated from the other cells and inputs, w_{1ji}, $w_{2ij'}$, v_{1ij}, and $v_{2ji'}$ are the connection weights at the time t, I_i and J_j denote the constant external inputs, $\alpha_j \geq 0$ and $\beta_i \geq 0$ are time delays in the state, $\mu_i \geq 0$ and $\nu_j \geq 0$ are neutral delays, f_j, g_i, h_i, and k_j are the activation functions, and a_i, b_j, w_{1ji}, w_{2ij}, v_{1ij}, v_{2ji}, f_j, g_i, h_i, k_j, α_j, β_i, μ_i, ν_j, I_i, and J_j are all almost periodic functions, $i, j = 1, 2, \ldots, n$, $j, i = 1, 2, \ldots, m$.

Recently, there are many papers concerning the existence and exponential stability of (almost) periodic solution for BAM neural networks [10–16]. However, the research on the neutral-type BAM neural networks is few. Therefore, the main purpose of this paper is to establish some new sufficient conditions on the existence, uniqueness, and exponential stability of almost periodic solution of neutral-type BAM neural networks (1). First, by using the exponential dichotomy and the contracting mapping principle, the existence and uniqueness of almost periodic solution of system (1) is considered. Besides, by constructing a new Lyapunov functional, the stability criterion with system (1) is introduced. The methods used in this paper provide a possible method to study the existence and exponential stability of almost periodic solutions of neutral-type neural networks.

Let $C(\mathbb{X}, \mathbb{Y})$ and $C^1(\mathbb{X}, \mathbb{Y})$ be the space of continuous functions and continuously differential functions which map $\mathbb{X}$ into $\mathbb{Y}$, respectively. In particular, $C(\mathbb{X}) := C(\mathbb{X}, \mathbb{X})$ and $C^1(\mathbb{X}) := C^1(\mathbb{X}, \mathbb{X})$. For any bounded function $f \in C(\mathbb{R})$, $f^u = \sup_{s\in\mathbb{R}} |f(s)|$ and $f^l = \inf_{s\in\mathbb{R}} |f(s)|$.

Let $\sigma := \max_{1\leq i\leq n, 1\leq j\leq m} \{\alpha_j^u, \beta_i^u, \mu_i^u, \nu_j^u\}$. The initial conditions associated with system (1) are of the form

$$\begin{gathered} x_i(s) = \varphi_i^*(s), \qquad \dot{x}_i(s) = \dot{\varphi}_i^*(s), \\ \forall s \in [-\sigma, 0], \quad \varphi_i^* \in C^1([-\sigma, 0], \mathbb{R}), \quad i = 1, 2, \ldots, n, \\ y_j(s) = \phi_j^*(s), \qquad \dot{y}_j(s) = \dot{\phi}_j^*(s), \\ \forall s \in [-\sigma, 0], \quad \phi_j^* \in C^1([-\sigma, 0], \mathbb{R}), \quad j = 1, 2, \ldots, m. \end{gathered} \tag{2}$$

Now we list some assumptions which will be used in this paper.

(H_1) $\min_{1\leq i\leq n, 1\leq j\leq m} \{a_i^l, b_j^l\} > 0$.

(H_2) There exist some positive constants L_j^f, L_i^g, L_i^h, and L_j^k such that

$$\begin{gathered} \left|f_j(u) - f_j(v)\right| \leq L_j^f |u - v|, \\ \left|k_j(u) - k_j(v)\right| \leq L_j^k |u - v|, \\ \left|g_i(u) - g_i(v)\right| \leq L_i^g |u - v|, \\ \left|h_i(u) - h_i(v)\right| \leq L_i^h |u - v|, \\ \forall u, v \in \mathbb{R}, \end{gathered} \tag{3}$$

where $i = 1, \ldots, n$ and $j = 1, \ldots, m$.

(H_3) $\theta = \max\{\theta_1, \theta_2\} < 1$, where

$$\begin{aligned} \theta_1 &= \max_{1\leq i\leq n, 1\leq j\leq m} \left\{ \sum_{j=1}^m \left(a_i^l\right)^{-1} w_{1ji}^u L_j^f + \sum_{j=1}^n \left(a_i^l\right)^{-1} w_{2ij}^u L_j^h, \right. \\ &\qquad \left. \sum_{i=1}^n \left(b_j^l\right)^{-1} v_{1ij}^u L_i^g + \sum_{i=1}^m \left(b_j^l\right)^{-1} v_{2ji}^u L_i^k \right\}, \\ \theta_2 &= \max_{1\leq i\leq n, 1\leq j\leq m} \left\{ \left[1 + \frac{a_i^u}{a_i^l}\right] \sum_{j=1}^m \left[w_{1ji}^u L_j^f + \sum_{j=1}^n w_{2ij}^u L_j^h \right], \right. \\ &\qquad \left. \left[1 + \frac{b_j^u}{b_j^l}\right] \left[\sum_{i=1}^n v_{1ij}^u L_i^g + \sum_{i=1}^m v_{2ji}^u L_i^k \right] \right\}. \end{aligned} \tag{4}$$

By the basic theory of neutral functional differential equations in [17], the initial value problems (1) and (2) have a unique solution on interval $[\sigma, +\infty)$.

The organization of this paper is as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Sections 3 and 4, by using a fixed point theorem and constructing suitable Lyapunov functional, we obtain some sufficient conditions ensuring existence, uniqueness, and global exponential stability of almost periodic solution of system (1). Finally, an example is given to illustrate that our results are feasible.

2. Preliminaries

Now, let us state the following definitions and lemmas, which will be useful in proving our main result.

Definition 1 (see, [7]). $x \in C(\mathbb{R})$ is called almost periodic, if, for any $\epsilon > 0$, it is possible to find a real number $l = l(\epsilon) > 0$ and, for any interval with length $l(\epsilon)$, there exists a number $\tau = \tau(\epsilon)$ in this interval such that $|x(t + \tau) - x(t)| < \epsilon$, for all $t \in \mathbb{R}$. The collection of those functions is denoted by $AP(\mathbb{R})$.

Definition 2 (see, [7]). Let $y \in C(\mathbb{R}, \mathbb{R}^n)$ and $P(t)$ be a $n \times n$ continuous matrix defined on $\mathbb{R}$. The linear system

$$\dot{y}(t) = P(t) y(t) \tag{5}$$

is said to be an exponential dichotomy on $\mathbb{R}$ if there exist constants $k, \lambda > 0$, projection S, and the fundamental matrix $Y(t)$ satisfying

$$\begin{gathered} \left\|Y(t) S Y^{-1}(s)\right\| \leq k e^{-\lambda(t-s)}, \quad \forall t \geq s, \\ \left\|Y(t)(I - S) Y^{-1}(s)\right\| \leq k e^{-\lambda(s-t)}, \quad \forall t \leq s. \end{gathered} \tag{6}$$

Lemma 3 (see, [7]). *If the linear system $\dot{y}(t) = P(t)y(t)$ has an exponential dichotomy, then almost periodic system*

$$\dot{y}(t) = P(t) y(t) + g(t) \tag{7}$$

has a unique almost periodic solution $y(t)$ which can be expressed as follows:

$$y(t) = \int_{-\infty}^{t} Y(t) SY^{-1}(s) g(s) \,ds - \int_{t}^{\infty} Y(t)(I-S) Y^{-1}(s) g(s) \,ds. \quad (8)$$

Lemma 4 (see, [7]). *Let $a_i(t)$ be an almost periodic function and*

$$M[a_i] = \lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} a_i(s)\,ds > 0, \quad i = 1,2,\ldots,n. \quad (9)$$

Then the linear system $\dot{y}(t) = -A(t)y(t)$ admits an exponential dichotomy, where $A(t) = \mathrm{diag}\{a_1(t), a_2(t), \ldots, a_n(t)\}$.

Definition 5. The almost periodic solution $z = (x_1,\ldots,x_n,y_1,\ldots,y_m)^T$ of system (1) with the initial value $z^* = (x_1^*,\ldots,x_n^*,y_1^*,\ldots,y_m^*)^T$ is said to be globally exponentially stable, if there exist constants $\omega > 0$ and $M \geq 1$, for any solution $\phi = (\varphi_1,\ldots,\varphi_n,\psi_1,\ldots,\psi_m)^T$ of system (1) with initial value $\phi^* = (\varphi_1^*,\ldots,\varphi_n^*,\psi_1^*,\ldots,\psi_m^*)^T$ such that

$$\sum_{i=1}^{n} \left|x_i(t) - \varphi_i(t)\right| + \sum_{j=1}^{m} \left|y_j(t) - \psi_j(t)\right| \leq M\left\|z^* - \phi^*\right\|_\sigma e^{-\omega t}, \quad \forall t > 0, \quad (10)$$

where

$$\left\|z^* - \phi^*\right\|_\sigma := \max_{1\leq i\leq n, 1\leq j\leq m} \sup_{s\in[-\sigma,0]} \left\{ \left|x_i^*(s) - \varphi_i^*(s)\right| + \left|\dot{x}_i^*(s) - \dot{\varphi}_i^*(s)\right|, \left|y_j^*(s) - \psi_j^*(s)\right| + \left|\dot{y}_j^*(s) - \dot{\psi}_j^*(s)\right| \right\}. \quad (11)$$

Lemma 6 (see, [19]). *Assume that $\mathbb{X}$ is a Banach space with norm $\|\cdot\|$; $T : \mathbb{X} \to \mathbb{X}$ is a contraction mapping; that is, there exists $k \in (0,1)$, such that*

$$\|Tx - Ty\| \leq k\|x - y\|, \quad \forall x, y \in \mathbb{X}. \quad (12)$$

Then T has a unique fixed point in $\mathbb{X}$.

Let $\mathbb{E} = \{x \in AP(\mathbb{R}) \cap C^1(\mathbb{R}) : \dot{x} \in AP(\mathbb{R})\}$ and

$$\mathbb{X} = \left\{ z = (x_1,\ldots,x_n,y_1,\ldots,y_m)^T : x_i, y_j \in \mathbb{E}, i = 1,\ldots,n, j = 1,\ldots,m \right\}, \quad (13)$$

with the norm

$$\|z\| = \max\{\|z\|_0, \|\dot{z}\|_0\} = \max\left\{ \max_{1\leq i\leq n,1\leq j\leq m}\left\{|x_i|_0, |y_j|_0\right\}, \max_{1\leq i\leq n,1\leq j\leq m}\left\{|\dot{x}_i|_0, |\dot{y}_j|_0\right\} \right\}, \quad (14)$$

where $\|z\|_0 = \max_{1\leq i\leq n,1\leq j\leq m}\{|x_i|_0, |y_j|_0\}$, $\|\dot{z}\|_0 = \max_{1\leq i\leq n,1\leq j\leq m}\{|\dot{x}_i|_0, |\dot{y}_j|_0\}$, and $|f|_0 = \sup_{s\in\mathbb{R}}|f(s)|$, for all $f \in AP(\mathbb{R})$. Then $\mathbb{X}$ is a Banach space with the norm $\|\cdot\|$.

By Lemmas 3 and 4, system (1) has a unique almost periodic solution

$$z^\phi = \left(x_1^{\varphi_1},\ldots,x_n^{\varphi_n},y_1^{\psi_1},\ldots,y_m^{\psi_n}\right)^T, \quad (15)$$

which can be expressed as follows:

$$x_i^{\varphi_i} = \int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} F_i(s,\phi(s))\,ds, \quad y_j^{\psi_j} = \int_{-\infty}^{t} e^{-\int_s^t b_j(u)du} G_j(s,\phi(s))\,ds, \quad (16)$$

where $\phi = (\varphi_1,\ldots,\varphi_n,\psi_1,\ldots,\psi_m)^T$,

$$F_i(t,\phi(t)) = \sum_{j=1}^{m} w_{1ji}(t) f_j\left(\psi_j\left(t-\alpha_j(t)\right)\right) + \sum_{j=1}^{n} w_{2ij}(t) h_j\left(\dot{\varphi}_j\left(t-\mu_j(t)\right)\right) + I_i(t),$$
$$G_j(t,\phi(t)) = \sum_{i=1}^{n} v_{1ij}(t) g_i\left(\varphi_i\left(t-\beta_i(t)\right)\right) + \sum_{i=1}^{m} v_{2ji}(t) k_i\left(\dot{\psi}_i\left(t-\nu_i(t)\right)\right) + J_i(t), \quad (17)$$

where $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$, and $t \in \mathbb{R}$.

Let $T : \mathbb{X} \to \mathbb{X}$ be defined by

$$T(\phi) = (\Phi_1(\phi),\ldots,\Phi_n(\phi),\Psi_1(\phi),\ldots,\Psi_m(\phi))^T = \left(x_1^{\varphi_1},\ldots,x_n^{\varphi_n},y_1^{\psi_1},\ldots,y_m^{\psi_n}\right)^T, \quad \forall\phi = (\varphi_1,\ldots,\varphi_n,\psi_1,\ldots,\psi_m)^T \in \mathbb{X}. \quad (18)$$

3. Existence and Uniqueness

In this section, we study the existence and uniqueness of almost periodic solution of system (1).

Theorem 7. *Assume that (H_1)–(H_3) hold, then system* (1) *has a unique almost periodic solution.*

Proof. Consider the following nonlinear operator:

$$T(\phi) = (\Phi_1(\phi),\ldots,\Phi_n(\phi),\Psi_1(\phi),\ldots,\Psi_m(\phi))^T, \quad \forall\phi = (\varphi_1,\ldots,\varphi_n,\psi_1,\ldots,\psi_m)^T \in \mathbb{X}. \quad (19)$$

For all $\phi^p = (\varphi_1^p, \ldots, \varphi_n^p, \psi_1^p, \ldots, \psi_m^p)^T \in \mathbb{X}$, $p = 1, 2$, it follows from the definitions of F_i and G_j that

$$\begin{aligned}
&\left|F_i\left(t,\phi^1\right) - F_i\left(t,\phi^2\right)\right|_0 \\
&\quad = \sup_{s\in\mathbb{R}} \left|F_i\left(s,\phi^1\right) - F_i\left(s,\phi^2\right)\right| \\
&\quad \le \sum_{j=1}^{m} w_{1ji}^u L_j^f \left|\psi_j^1 - \psi_j^2\right|_0 + \sum_{j=1}^{n} w_{2ij}^u L_j^h \left|\dot{\varphi}_j^1 - \dot{\varphi}_j^2\right|_0, \\
&\left|G_j\left(t,\phi^1\right) - G_j\left(t,\phi^2\right)\right|_0 \\
&\quad = \sup_{s\in\mathbb{R}} \left|G_j\left(s,\phi^1\right) - G_j\left(s,\phi^2\right)\right| \\
&\quad \le \sum_{i=1}^{n} v_{1ij}^u L_i^g \left|\varphi_i^1 - \varphi_i^2\right|_0 + \sum_{i=1}^{m} v_{2ji}^u L_i^k \left|\dot{\psi}_i^1 - \dot{\psi}_i^2\right|_0.
\end{aligned} \tag{20}$$

Then

$$\begin{aligned}
&\left|\Phi_i\left(\phi^1\right) - \Phi_i\left(\phi^2\right)\right|_0 \\
&\quad = \sup_{t\in\mathbb{R}} \left|\Phi_i\left(\phi^1(t)\right) - \Phi_i\left(\phi^1(t)\right)\right| \\
&\quad \le \sup_{t\in\mathbb{R}} \left|\int_{-\infty}^{t} e^{-\int_s^t a_i(u)\mathrm{d}u} \times \left[F_i\left(s,\phi^1(s)\right) - F_i\left(s,\phi^2(s)\right)\right] \mathrm{d}s\right| \\
&\quad \le \sum_{j=1}^{m} \left(a_i^l\right)^{-1} w_{1ji}^u L_j^f \left|\psi_j^1 - \psi_j^2\right|_0 \\
&\qquad + \sum_{j=1}^{n} \left(a_i^l\right)^{-1} w_{2ij}^u L_j^h \left|\dot{\varphi}_j^1 - \dot{\varphi}_j^2\right|_0, \\
&\left|\Psi_j\left(\phi^1\right) - \Psi_j\left(\phi^2\right)\right|_0 \\
&\quad = \sup_{t\in\mathbb{R}} \left|\Psi_j\left(\phi^1(t)\right) - \Psi_j\left(\phi^2(t)\right)\right| \\
&\quad \le \sup_{t\in\mathbb{R}} \left|\int_{-\infty}^{t} e^{-\int_s^t b_j(u)\mathrm{d}u} \times \left[G_j\left(s,\phi^1(s)\right) - G_j\left(s,\phi^2(s)\right)\right] \mathrm{d}s\right| \\
&\quad \le \sum_{i=1}^{n} \left(b_j^l\right)^{-1} v_{1ij}^u L_i^g \left|\varphi_i^1 - \varphi_i^2\right|_0 \\
&\qquad + \sum_{i=1}^{m} \left(b_j^l\right)^{-1} v_{2ji}^u L_i^k \left|\dot{\psi}_i^1 - \dot{\psi}_i^2\right|_0,
\end{aligned} \tag{21}$$

where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

Further, we also obtain that

$$\begin{aligned}
&\left|\dot{\Phi}_i\left(\phi^1\right) - \dot{\Phi}_i\left(\phi^2\right)\right|_0 \\
&\quad = \sup_{t\in\mathbb{R}} \left|\dot{\Phi}_i\left(\phi^1(t)\right) - \dot{\Phi}_i\left(\phi^2(t)\right)\right| \\
&\quad = \sup_{t\in\mathbb{R}} \left|-a_i(t)\left[\Phi_i\left(\phi^1\right) - \Phi_i\left(\phi^2\right)\right] + \left[F_i\left(t,\phi^1\right) - F_i\left(t,\phi^2\right)\right]\right| \\
&\quad \le \left[1 + \frac{a_i^u}{a_i^l}\right] \left[\sum_{j=1}^{m} w_{1ji}^u L_j^f \left|\psi_j^1 - \psi_j^2\right|_0 + \sum_{j=1}^{n} w_{2ij}^u L_j^h \left|\dot{\varphi}_j^1 - \dot{\varphi}_j^2\right|_0\right] \\
&\quad \le \left[1 + \frac{a_i^u}{a_i^l}\right] \left[\sum_{j=1}^{m} w_{1ji}^u L_j^f + \sum_{j=1}^{n} w_{2ij}^u L_j^h\right] \left\|\phi^1 - \phi^2\right\|, \\
&\left|\dot{\Psi}_j\left(\phi^1\right) - \dot{\Psi}_j\left(\phi^2\right)\right|_0 \\
&\quad = \sup_{t\in\mathbb{R}} \left|\dot{\Psi}_j\left(\phi^1(t)\right) - \dot{\Psi}_j\left(\phi^2(t)\right)\right| \\
&\quad = \sup_{t\in\mathbb{R}} \left|-b_j(t)\left[\Psi_j\left(\phi^1\right) - \Psi_j\left(\phi^2\right)\right] + \left[G_j\left(t,\phi^1\right) - G_j\left(t,\phi^2\right)\right]\right| \\
&\quad \le \left[1 + \frac{b_j^u}{b_j^l}\right] \left[\sum_{i=1}^{n} v_{1ij}^u L_i^g \left|\varphi_i^1 - \varphi_i^2\right|_0 + \sum_{i=1}^{m} v_{2ji}^u L_i^k \left|\dot{\psi}_i^1 - \dot{\psi}_i^2\right|_0\right] \\
&\quad \le \left[1 + \frac{b_j^u}{b_j^l}\right] \left[\sum_{i=1}^{n} v_{1ij}^u L_i^g + \sum_{i=1}^{m} v_{2ji}^u L_i^k\right] \left\|\phi^1 - \phi^2\right\|,
\end{aligned} \tag{22}$$

where $i = 1, \ldots, n$ and $j = 1, \ldots, m$.

Hence,

$$\begin{aligned}
&\left\|T(\phi^1) - T(\phi^2)\right\|_0 \\
&\quad = \max_{1\le i\le n, 1\le j\le m} \left\{\left|\Phi_i\left(\phi^1\right) - \Phi_i\left(\phi^2\right)\right|_0, \left|\Psi_j\left(\phi^1\right) - \Psi_j\left(\phi^2\right)\right|_0\right\} \\
&\quad \le \max_{1\le i\le n, 1\le j\le m} \left\{\sum_{j=1}^{m} \left(a_i^l\right)^{-1} w_{1ji}^u L_j^f + \sum_{j=1}^{n} \left(a_i^l\right)^{-1} w_{2ij}^u L_j^h, \sum_{i=1}^{n} \left(b_j^l\right)^{-1} v_{1ij}^u L_i^g + \sum_{i=1}^{m} \left(b_j^l\right)^{-1} v_{2ji}^u L_i^k\right\} \\
&\qquad \times \left\|\phi^1 - \phi^2\right\| \\
&\quad = \theta_1 \left\|\phi^1 - \phi^2\right\|,
\end{aligned}$$

$$
\begin{aligned}
&\left\|\dot{T}(\phi^1)-\dot{T}(\phi^2)\right\|_0\\
&=\max_{1\le i\le n,1\le j\le m}\left\{\left|\dot{\Phi}_i\left(\phi^1\right)-\dot{\Phi}_i\left(\phi^2\right)\right|_0,\left|\dot{\Psi}_j\left(\phi^1\right)-\dot{\Psi}_j\left(\phi^2\right)\right|_0\right\}\\
&\le\max_{1\le i\le n,1\le j\le m}\left\{\left[1+\frac{a_i^u}{a_i^l}\right]\sum_{j=1}^m\left[w_{1ji}^uL_j^f+\sum_{j=1}^n w_{2ij}^uL_j^h\right],\right.\\
&\qquad\left.\left[1+\frac{b_j^u}{b_j^l}\right]\left[\sum_{i=1}^n v_{1ij}^uL_i^g+\sum_{i=1}^m v_{2ji}^uL_i^k\right]\right\}\\
&\quad\times\left\|\phi^1-\phi^2\right\|\\
&=\theta_2\left\|\phi^1-\phi^2\right\|.
\end{aligned}
\tag{23}
$$

Together with the above results, one has

$$
\begin{aligned}
&\left\|T\left(\phi^1\right)-T\left(\phi^2\right)\right\|\\
&\quad=\max\left\{\left\|T\left(\phi^1\right)-T\left(\phi^2\right)\right\|_0,\left\|\dot{T}\left(\phi^1\right)-\dot{T}\left(\phi^2\right)\right\|_0\right\}\\
&\quad\le\max\left\{\theta_1,\theta_2\right\}\left\|\phi^1-\phi^2\right\|\\
&\quad\le\theta\left\|\phi^1-\phi^2\right\|,
\end{aligned}
\tag{24}
$$

where $\theta\in(0,1)$. By Lemma 6, there exists a unique fixed point $\phi_0\in\mathbb{X}$ satisfying $T(\phi_0)=\phi_0$, which implies that system (1) has unique almost periodic solution. This completes the proof. □

Remark 8. Condition (H_3) in Theorem 7 indicates that the neutral terms are harmful for the existence and uniqueness of almost periodic solution of system (1).

4. Global Exponential Stability

Theorem 9. *Assume that (H_1)–(H_3) hold and suppose further the following.*

(H_4) *α_j, β_i, μ_i, and ν_j are differential functions, with $\dot{\alpha}_j^+:=\sup_{s\in\mathbb{R}}\dot{\alpha}_j(s)<1$, $\dot{\beta}_i^+:=\sup_{s\in\mathbb{R}}\dot{\beta}_i(s)<1$, $\dot{\mu}_i^+:=\sup_{s\in\mathbb{R}}\dot{\mu}_i(s)<1$, $\dot{\nu}_j^+:=\sup_{s\in\mathbb{R}}\dot{\nu}_j(s)<1$, where $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$.*

(H_5) *There exists a positive constant $\lambda\le1$ such that*

$$
\begin{aligned}
&-(1-\lambda)a_i^l+(1+\lambda)\sum_{j=1}^m\frac{v_{1ij}^uL_i^g}{1-\dot{\beta}_i^+}<0,\quad i=1,2,\ldots,n,\\
&-(1-\lambda)b_j^l+(1+\lambda)\sum_{i=1}^n\frac{w_{1ji}^uL_j^f}{1-\dot{\alpha}_j^+}<0,\quad j=1,2,\ldots,m,\\
&\sum_{i=1}^n(1+\lambda)w_{2ij}^uL_j^h-\lambda\left(1-\dot{\mu}_j^+\right)<0,\quad j=1,2,\ldots,m,\\
&\sum_{j=1}^m(1+\lambda)v_{2ji}^uL_i^k-\lambda\left(1-\dot{\nu}_i^+\right)<0,\quad i=1,2,\ldots,n.
\end{aligned}
\tag{25}
$$

Then system (1) *has a unique almost periodic solution, which is globally exponentially stable.*

Proof. It follows from Theorem 7 that system (1) has a unique almost periodic solution $\phi=(\varphi_1,\ldots,\varphi_n,\psi_1,\ldots,\psi_m)^T$ with initial value $\phi^*=(\varphi_1^*,\ldots,\varphi_n^*,\psi_1^*,\ldots,\psi_m^*)^T$. We next show that the almost periodic solution ϕ is globally exponentially stable.

Make a transformation for system (1): $x_i=u_i-\varphi_i$, $y_j=v_j-\psi_j$, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$, where $z=(u_1,\ldots,u_n,v_1,\ldots,v_m)^T$ is arbitrary solution of system (1) with initial value $z^*=(u_1^*,\ldots,u_n^*,v_1^*,\ldots,v_m^*)^T$.

By (H_4), there exists a small enough positive constant ω such that

$$
\begin{aligned}
&\omega-(1-\lambda)a_i^l+(1+\lambda)\sum_{j=1}^m\frac{v_{1ij}^uL_i^ge^{\omega\beta_i^u}}{1-\dot{\beta}_i^+}<0,\quad i=1,2,\ldots,n,\\
&\omega-(1-\lambda)b_j^l+(1+\lambda)\sum_{i=1}^n\frac{w_{1ji}^uL_j^fe^{\omega\alpha_j^u}}{1-\dot{\alpha}_j^+}<0,\quad j=1,2,\ldots,m,\\
&\sum_{i=1}^n(1+\lambda)w_{2ij}^uL_j^h-\lambda e^{-\omega\mu_j^u}\left(1-\dot{\mu}_j^+\right)<0,\quad j=1,2,\ldots,m,\\
&\sum_{j=1}^m(1+\lambda)v_{2ji}^uL_i^k-\lambda e^{-\omega\nu_i^u}\left(1-\dot{\nu}_i^+\right)<0,\quad i=1,2,\ldots,n.
\end{aligned}
\tag{26}
$$

Define

$$
V_1(t)=\sum_{i=1}^n e^{\omega t}\left|x_i(t)\right|+\sum_{j=1}^m e^{\omega t}\left|y_j(t)\right|.
\tag{27}
$$

In view of system (1), we have

$$
\begin{aligned}
&D^+V_1(t)\\
&\quad\le\omega\sum_{i=1}^n e^{\omega t}\left|x_i(t)\right|+\omega\sum_{j=1}^m e^{\omega t}\left|y_j(t)\right|\\
&\qquad+\sum_{i=1}^n e^{\omega t}\left[-a_i^l\left|x_i(t)\right|+\sum_{j=1}^m w_{1ji}^uL_j^f\left|y_j\left(t-\alpha_j(t)\right)\right|\right.\\
&\qquad\qquad\left.+\sum_{j=1}^n w_{2ij}^uL_j^h\left|\dot{x}_j\left(t-\mu_j(t)\right)\right|\right]\\
&\qquad+\sum_{j=1}^m e^{\omega t}\left[-b_j^l\left|y_j(t)\right|+\sum_{i=1}^n v_{1ij}^uL_i^g\left|x_i\left(t-\beta_i(t)\right)\right|\right.\\
&\qquad\qquad\left.+\sum_{i=1}^m v_{2ji}^uL_i^k\left|\dot{y}_i\left(t-\nu_i(t)\right)\right|\right].
\end{aligned}
\tag{28}
$$

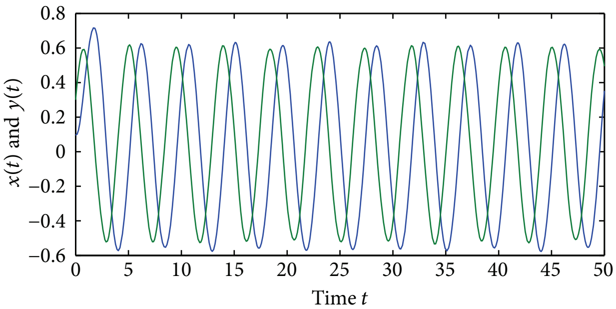

FIGURE 1: Almost periodicity of state variables $x(t)$ and $y(t)$.

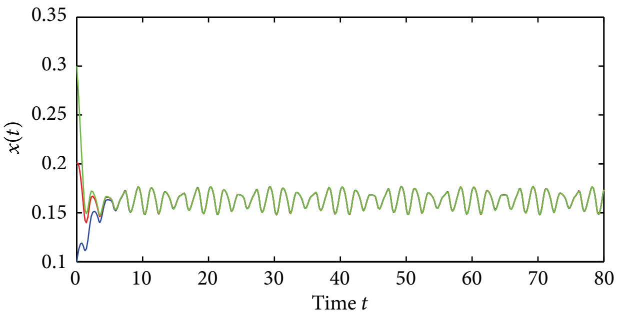

FIGURE 2: Stability of state variables $x(t)$.

Let

$$\begin{aligned} V_2(t) &= \lambda \sum_{i=1}^{n} \int_{t-\mu_i(t)}^{t} e^{\omega s} \left|\dot{x}_i(s)\right| \mathrm{d}s, \\ V_3(t) &= \lambda \sum_{j=1}^{m} \int_{t-\nu_j(t)}^{t} e^{\omega s} \left|\dot{y}_j(s)\right| \mathrm{d}s, \\ V_4(t) &= \sum_{j=1}^{m}\sum_{i=1}^{n} (1+\lambda) \int_{t-\beta_i(t)}^{t} \frac{v_{1ij}^{u} L_i^{g}}{1-\dot{\beta}_i^{+}} e^{\omega(s+\beta_i^u)} \left|x_i(s)\right| \mathrm{d}s, \\ V_5(t) &= \sum_{i=1}^{n}\sum_{j=1}^{m} (1+\lambda) \int_{t-\alpha_j(t)}^{t} \frac{w_{1ji}^{u} L_j^{f}}{1-\dot{\alpha}_j^{+}} e^{\omega(s+\alpha_j^u)} \left|y_j(s)\right| \mathrm{d}s. \end{aligned} \tag{29}$$

So

$$\begin{aligned} D^{+}V_2(t) &\le \lambda \sum_{i=1}^{n} e^{\omega t} \left|\dot{x}_i(t)\right| \\ &\quad - \lambda \sum_{i=1}^{n} e^{\omega(t-\mu_i(t))} \left(1-\dot{\mu}_i(t)\right) \left|\dot{x}_i\left(t-\mu_i(t)\right)\right| \\ &\le \lambda \sum_{i=1}^{n} e^{\omega t} \left[a_i^u \left|x_i(t)\right| + \sum_{j=1}^{m} w_{1ji}^{u} L_j^{f} \left|y_j\left(t-\alpha_j(t)\right)\right| \right] \\ &\quad - \lambda \sum_{j=1}^{n} e^{\omega t} \left[e^{-\omega\mu_j^u}\left(1-\dot{\mu}_j^{+}\right) - \sum_{i=1}^{n} w_{2ij}^{u} L_j^{h} \right] \left|\dot{x}_j\left(t-\mu_j(t)\right)\right|, \end{aligned} \tag{30}$$

$$\begin{aligned} D^{+}V_4(t) &\le \sum_{j=1}^{m}\sum_{i=1}^{n} (1+\lambda) \frac{v_{1ij}^{u} L_i^{g}}{1-\dot{\beta}_i^{+}} e^{\omega(t+\beta_i^u)} \left|x_i(t)\right| \\ &\quad - \sum_{j=1}^{m}\sum_{i=1}^{n} (1+\lambda) \frac{v_{1ij}^{u} L_i^{g}\left(1-\dot{\beta}_i(t)\right)}{1-\dot{\beta}_i^{+}} e^{\omega(t-\beta_i(t)+\beta_i^u)} \\ &\qquad \times \left|x_i\left(t-\beta_i(t)\right)\right| \\ &\le \sum_{j=1}^{m}\sum_{i=1}^{n} (1+\lambda) \frac{v_{1ij}^{u} L_i^{g}}{1-\dot{\beta}_i^{+}} e^{\omega(t+\beta_i^u)} \left|x_i(t)\right| \\ &\quad - \sum_{j=1}^{m}\sum_{i=1}^{n} (1+\lambda) v_{1ij}^{u} L_i^{g} e^{\omega t} \left|x_i\left(t-\beta_i(t)\right)\right|. \end{aligned} \tag{31}$$

Similar to the arguments as that in (30) and (31), we obtain

$$\begin{aligned} D^{+}V_3(t) &\le \lambda \sum_{j=1}^{m} e^{\omega t} \left[b_j^u \left|y_j(t)\right| + \sum_{i=1}^{n} v_{1ij}^{u} L_i^{g} \left|x_i\left(t-\beta_i(t)\right)\right| \right] \\ &\quad - \lambda \sum_{i=1}^{m} e^{\omega t} \left[e^{-\omega\nu_i^u}\left(1-\dot{\nu}_i^{+}\right) - \sum_{j=1}^{m} v_{2ji}^{u} L_i^{k} \right] \left|\dot{y}_i\left(t-\nu_i(t)\right)\right|, \end{aligned} \tag{32}$$

$$\begin{aligned} D^{+}V_5(t) &\le \sum_{i=1}^{n}\sum_{j=1}^{m} (1+\lambda) \frac{w_{1ji}^{u} L_j^{f}}{1-\dot{\alpha}_j^{+}} e^{\omega(t+\alpha_j^u)} \left|y_j(t)\right| \\ &\quad - \sum_{i=1}^{n}\sum_{j=1}^{m} (1+\lambda) w_{1ji}^{u} L_j^{f} e^{\omega t} \left|y_j\left(t-\alpha_j(t)\right)\right|. \end{aligned} \tag{33}$$

Define $V(t) = \sum_{q=1}^{5} V_q$. From (28)–(33), it follows that

$$\begin{aligned} D^{+}V(t) &\le e^{\omega t} \sum_{i=1}^{n} \left[\omega - (1-\lambda) a_i^l + (1+\lambda) \sum_{j=1}^{m} \frac{v_{1ij}^{u} L_i^{g} e^{\omega\beta_i^u}}{1-\beta_i^{\prime +}} \right] \left|x_i(t)\right| \\ &\quad + e^{\omega t} \sum_{j=1}^{m} \left[\omega - (1-\lambda) b_j^l + (1+\lambda) \sum_{i=1}^{n} \frac{w_{1ji}^{u} L_j^{f} e^{\omega\alpha_j^u}}{1-\dot{\alpha}_j^{+}} \right] \end{aligned}$$

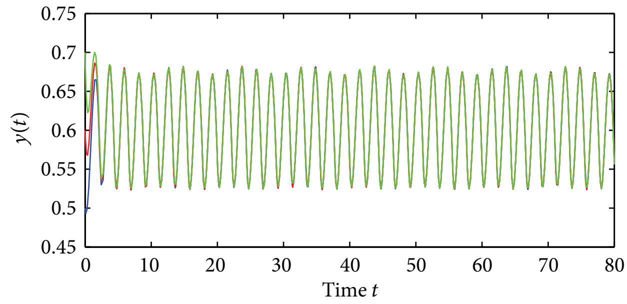

Figure 3: Stability of state variables $y(t)$.

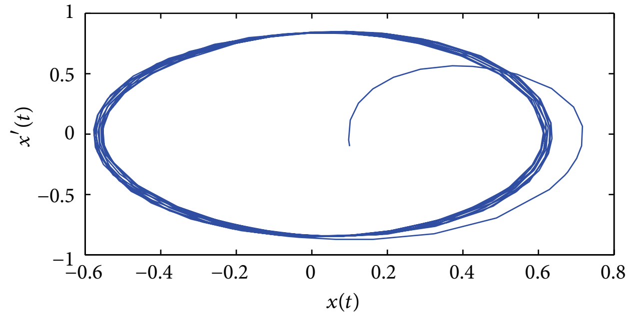

Figure 5: Phase response of state variables $x(t)$ and $x'(t)$.

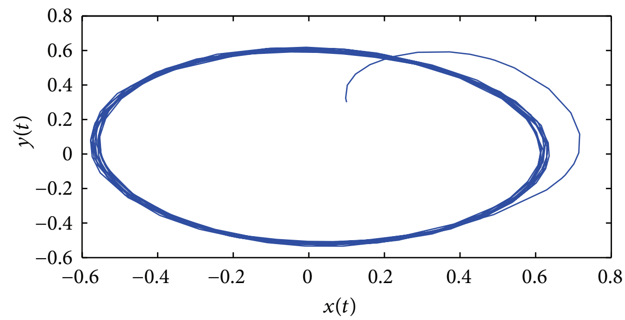

Figure 4: Phase response of state variables $x(t)$ and $y(t)$.

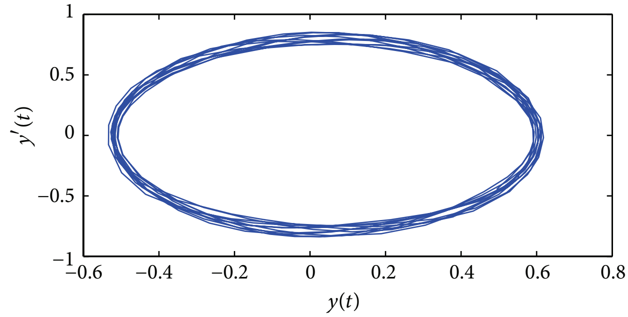

Figure 6: Phase response of state variables $y(t)$ and $y'(t)$.

$$\begin{aligned}
&\times \left|y_j(t)\right| \\
&+ e^{\omega t}\sum_{j=1}^{n}\left[\sum_{i=1}^{n}(1+\lambda)\, w_{2ij}^{u} L_j^{h} - \lambda e^{-\omega\mu_j^{u}}\left(1-\dot{\mu}_j^{+}\right)\right] \\
&\times \left|\dot{x}_j\left(t-\mu_j(t)\right)\right| \\
&+ e^{\omega t}\sum_{i=1}^{m}\left[\sum_{j=1}^{m}(1+\lambda)\, v_{2ji}^{u} L_i^{k} - \lambda e^{-\omega\nu_i^{u}}\left(1-\dot{\nu}_i^{+}\right)\right] \\
&\times \left|\dot{y}_i\left(t-\nu_i(t)\right)\right| \\
\le{}& 0,
\end{aligned} \tag{34}$$

which implies that $V(t) \le V(0)$, for all $t > 0$. Obviously,

$$\sum_{i=1}^{n} e^{\omega t}\left|x_i(t)\right| + \sum_{j=1}^{m} e^{\omega t}\left|y_j(t)\right| \le V(t). \tag{35}$$

On the other hand, we have

$$\begin{aligned}
&V(0) \\
&= \sum_{i=1}^{n}\left|x_i(0)\right| + \sum_{j=1}^{m}\left|y_j(0)\right| + \lambda\sum_{i=1}^{n}\int_{-\mu_i(0)}^{0}\left|\dot{x}_i(s)\right| \mathrm{d}s \\
&\quad + \lambda\sum_{j=1}^{m}\int_{-\nu_j(0)}^{0}\left|\dot{y}_j(s)\right| \mathrm{d}s \\
&\quad + \sum_{j=1}^{m}\sum_{i=1}^{n}(1+\lambda)\int_{-\beta_i(0)}^{0}\frac{v_{1ij}^{u}L_i^{g}}{1-\dot{\beta}_i^{+}} e^{\omega_2(s+\beta_i^{u})}\left|x_i(s)\right| \mathrm{d}s \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m}(1+\lambda)\int_{-\alpha_j(0)}^{0}\frac{w_{1ji}^{u}L_j^{f}}{1-\dot{\alpha}_j^{+}} e^{\omega(s+\alpha_j^{u})}\left|y_j(s)\right| \mathrm{d}s \\
&\le \left\{ m+n+\sum_{i=1}^{n}\mu_i^{u} + \sum_{j=1}^{m}\nu_j^{u} \right. \\
&\quad \left. + \sum_{j=1}^{m}\sum_{i=1}^{n}\left[\frac{2\beta_i^{u}v_{1ij}^{u}L_i^{g}e^{\omega\beta_i^{u}}}{1-\dot{\beta}_i^{+}} + \frac{2\alpha_j^{u}w_{1ji}^{u}L_j^{f}e^{\omega\alpha_j^{u}}}{1-\dot{\alpha}_j^{+}}\right]\right\} \\
&\quad \times \left\|z^{*}-\phi^{*}\right\|,
\end{aligned} \tag{36}$$

which implies from (35) that

$$\sum_{i=1}^{n}\left|x_i(t)\right| + \sum_{j=1}^{m}\left|y_j(t)\right| \le M\left\|z^{*}-\phi^{*}\right\| e^{-\omega t}, \quad \forall t > 0, \tag{37}$$

where

$$\begin{aligned}
M :={}& m+n+\sum_{i=1}^{n}\mu_i^{u} + \sum_{j=1}^{m}\nu_j^{u} \\
&+ \sum_{j=1}^{m}\sum_{i=1}^{n}\left[\frac{2\beta_i^{u}v_{1ij}^{u}L_i^{g}e^{\omega\beta_i^{u}}}{1-\dot{\beta}_i^{+}} + \frac{2\alpha_j^{u}w_{1ji}^{u}L_j^{f}e^{\omega\alpha_j^{u}}}{1-\dot{\alpha}_j^{+}}\right].
\end{aligned} \tag{38}$$

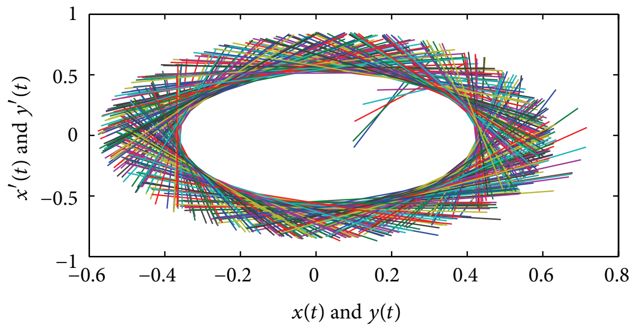

Figure 7: Phase response of state variables $x(t)$, $y(t)$, $x'(t)$, and $y'(t)$.

Thus, the almost periodic solution of system (1) is globally exponentially stable. This completes the proof. □

Remark 10. Condition (H_5) in Theorem 9 indicates that the neutral terms and time delays are harmful for the global exponential stability of almost periodic solution of system (1).

5. An Example

Example 1. Consider the following neutral BAM neural networks with time-varying delays:

$$\begin{aligned}\dot{x}(t) = &-x(t) + \sin\left(\sqrt{2}t\right) f\left(y\left(t-\alpha(t)\right)\right) \\ &+ \cos^2\left(\sqrt{3}t\right) h\left(\dot{x}(t-1)\right) + \sin\left(\sqrt{2}t\right), \\ \dot{y}(t) = &-y(t) + \left|\cos\left(\sqrt{3}t\right)\right| g\left(x\left(t-\beta(t)\right)\right) \\ &+ \sin^2\left(\sqrt{5}t\right) k\left(\dot{y}(t-1)\right) + \cos\left(\sqrt{2}t\right),\end{aligned} \tag{39}$$

where $f(s) = g(s) = 0.1s$,

$$\begin{gathered}\begin{pmatrix} h(s) \\ k(s) \end{pmatrix} = \begin{pmatrix} 0.1\sin(s) \\ 0.1\cos(s) \end{pmatrix}, \\ \begin{pmatrix} \alpha(s) \\ \beta(s) \end{pmatrix} = \begin{pmatrix} 1 + 0.01\sin^2\left(\sqrt{2}s\right) \\ 1 + 0.01\cos^2\left(\sqrt{3}s\right) \end{pmatrix}, \\ \forall s \in \mathbb{R}.\end{gathered} \tag{40}$$

Then system (39) has a unique almost periodic solution, which is globally exponentially stable.

Proof. Corresponding to system (1), $a^l = b^l = 1$, $L^f = L^g = L^h = L^k = 0.1$, $w_1^u = w_2^u = v_1^u = v_2^u = 1$, $\dot{\alpha}^+ \leq 0.02$, $\dot{\beta}^+ \leq 0.02$, and $\dot{\mu}^+ = \dot{\nu}^+ = 0$. Taking $\lambda = 0.5$, it is easy to verify that (H_1)–(H_5) hold and the results follow from Theorems 7–9 (see Figures 1, 2, 3, 4, 5, 6, and 7). This completes the proof. □

For numerical simulation, Figures 1–3 depict the time responses of state variables of $x(t)$ and $y(t)$ with step $h = 0.01$ of system (39), respectively. Figures 4–7 depict the phase responses of state variables $x(t)$, $y(t)$, $x'(t)$, and $y'(t)$, respectively. It confirms that the proposed conditions in our results are effective for system (39).

6. Discussion

In this paper, the neutral BAM neural network is considered. By employing fixed point theory and constructing suitable Lyapunov functional some new sufficient conditions are obtained for the existence and global exponential stability of almost periodic solution of the system. Conditions (H_3) and (H_5) in Theorems 7 and 9 indicate that the neutral terms and time delays are harm for the existence, uniqueness, and global exponential stability of almost periodic solution of the neutral-type system. The method used in this paper provides a possible method to study the existence and global exponential stability of almost periodic solution of other neutral neural networks (with impulses [20–23]).

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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